

## Modeling of Impurity Neoclassical Transport in Tokamak

トカマク型核融合炉における不純物の新古典輸送のモデリング

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Understanding impurity transport in tokamaks is one of most important issues in fusion reactor development. In this study, we have extended our numerical model of impurity transport, which is based on Binary Collision Monte-Carlo Model (BCM), for the application to more wide range of collisionality, i.e., Pfirsch-Schluter (PS), Plateau and Banana regimes. Its validity has been checked by some test calculations for a simple tokamak geometry.

### 1. Introduction

Understanding impurity transport in tokamaks is one of the important issues to reduce the impurity in fusion plasmas. Using numerical simulation model is very effective. Recently, a new numerical scheme of impurity classical/neoclassical transport<sup>[1,2,3]</sup> have been developed. The numerical scheme make it possible to take into not only collisional self-diffusion (SD), but also the inward pinch (IWP) and temprature screening effect (TSE) for impurity ions.

However, impurity neoclassical transport has been modeled in the Pfirsch-Schluter (PS) regime.

The final goal is to extend our previous model above so as to be applied to more wide range of neoclassical collisionality regimes, i.e., not only PS regimes, but also Plateau and Banana regimes. In this paper, we focus on the SD and IWP. Validity of our numerical scheme has been checked by comparing the numerical results with neoclassical theory. A wide range of parameter dependence of diffusion coefficient and IWP velocity has been studied.

### 2. Classical/neoclassical Transport<sup>[4]</sup>

Now consider two types of ion denoted by subscript 1 and 2. Subscript 1 corresponds to the impurity ions, while subscript 2 corresponds to the background fuel ions. The collisional flux  $\Gamma$  of impurity ions across the magnetic field is given by,

$$\Gamma_1 = -D\nabla N_1 + N_1 V_1, \quad (1)$$

$$V_1 = D \frac{Z_1}{Z_2} \frac{\nabla N_2}{N_2}, \quad (2)$$

where,  $D$ ,  $N$ ,  $V$ ,  $Z$  are the diffusion coefficient, density, velocity, and charge state, respectively. The first term in eq. (1) denotes SD effect relaxing the density gradient of impurity ions.

The second term in eq. (1) denotes the IWP effect, which is in the direction parallel to the density gradient of the background fuel ions. In neoclassical transport theory, diffusion coefficient  $D$  depends on the collisionality parameter, and is given as follows;

$$\text{Banana regime } \nu^* < 1 \quad D \approx \varepsilon^{-3/2} q^2 r_L^2 \nu$$

$$\text{Plateau regime } 1 < \nu^* < \varepsilon^{-3/2} \quad D \approx q^2 r_L^2 \nu_t$$

$$\text{PS regime } \varepsilon^{-3/2} < \nu^* \quad D \approx q^2 r_L^2 \nu$$

Here the collisionality parameter  $\nu^*$  and the transit frequency  $\nu_t$  are defined by,

$$\nu^* = \nu v_{th} / \varepsilon^{3/2} q R, \quad (3)$$

$$\nu_t = \nu_{th} / q R, \quad (4)$$

where,  $\nu$ ,  $\nu_{th}$ ,  $r_L$ ,  $q$ ,  $\varepsilon$ ,  $R$  are Coulomb collision frequency, thermal velocity, Lamor radius, safety factor, inverse aspect ratio, and major radius, respectively.

### 3. Numerical Scheme

In this study, kinetic model<sup>[1,2,3]</sup> with the Binary Collision Monte-Carlo Model (BCM)<sup>[5]</sup> has been adopted. The trajectory of each test impurity ion in the magnetic field  $\mathbf{B}$  is followed by Boris-Buneman algorithm<sup>[6]</sup>. Furthermore, to calculate the effects of Coulomb collisions, the BCM has been used. In the BCM, the scattering angle  $\phi = 2\pi U$  and  $\theta = 2\arctan \delta$  are given by using random numbers.  $U$  is a uniform random number.  $\delta$  is sampled from the normal distribution with the mean value  $\langle \delta \rangle$  and the variance  $\langle \delta^2 \rangle$ ,

$$\langle \delta \rangle = 0, \quad \langle \delta^2 \rangle = \frac{N_2 (q_1 q_2)^2 \ln \Lambda}{8\pi \varepsilon_0^2 M^2 u^3} \Delta t, \quad (5)$$

where  $\ln \Lambda$ ,  $M$ ,  $\varepsilon_0$ ,  $u$ , and  $\Delta t$  are Coulomb logarithm, the relative mass of the species 1 and 2, permittivity of vacuum, the relative velocity between the species 1 and 2, the simulation time step respectively.

In addition to these, for modeling classical transport, background ions involved in the binary collision is sampled from a Maxwell velocity distribution not at the position of test impurity ion, but at the background ion's guiding center.

Moreover, taking the PS current into this model enables to represent neoclassical transport IWP in PS regime, because the Boris-Buneman algorithm includes the effect of drift naturally.

#### 4. Results and Discussion

Firstly, in order to check the effects of SD of neoclassical transport, calculations with a simple torus magnetic configuration have been done. Test impurity ions are tungsten whose charge state  $Z = 3$ . Background ions are deuterium. The temperatures of tungsten and deuterium are the same,  $T_1 = T_2 = 100$  eV. The initial position of impurity ions is  $r_{t=0} = 0.8$  m and uniform in poloidal angle.

$r$  is minor radius. Figure 1 shows the dependence of neoclassical diffusion coefficient on collisionality. In order to change value of collisionality parameter, density of background ions is varied. The calculated simulation values of diffusion coefficient is given by,

$$D = \langle (\Delta r)^2 \rangle / 2\Delta t. \quad (6)$$

Theoretical values of  $D$  in PS regime and Banana regime are showed by straight lines in Fig. 1. Then calculated values roughly agree with  $D_{\text{banana}} = \varepsilon^{-\frac{3}{2}} q^2 r_L^2 \nu$  when collisionality parameter is low, and agree with  $D_{\text{PS}} = q^2 r_L^2 \nu$  when collisionality is high.

Secondly, calculations of the effect of IWP in PS regime have been done. Figure 2 shows dependence of IWP velocity towards core plasmas on safety factor. The remaining conditions except for the safety factor are the same as above. The density of background ions is  $n_d = 4 \times 10^{19} \text{ m}^{-3}$  at the initial position, and its gradient is  $|\nabla n_d| = 1.6 \times 10^{20} \text{ m}^{-4}$  in the negative  $r$  direction. In order to change the value of safety factor, plasma current is varied. As seen from Fig. 2, the calculated values of the IWP velocity agree well with theoretical ones.

#### 5. Conclusion

As for SD, our scheme reproduces the

dependence on collisionality parameter in full range of collisionality. Our numerical scheme of IWP correctly reproduces the parameter dependences (safety factor, characteristic length of density, temperature, and charge state). In the near future, we will extend this model so as to be applied to more wide range of neoclassical collisionality regimes, i.e., Plateau and Banana regimes.

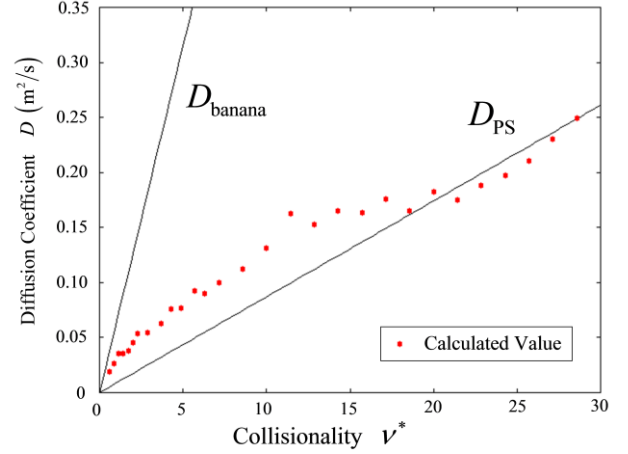


Fig.1 Dependence of neoclassical diffusion coefficient on collisionality.

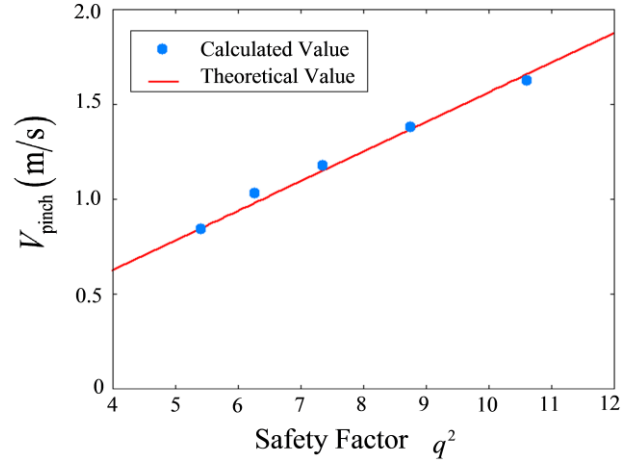


Fig.2 Dependence of neoclassical inward pinch on safety factor.

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