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Discontinuous Transition in the Discharge Process in a Nonlinear Medium
非線形媒質中における放電現象の不連続転移

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We investigate the discharge process by a Monte Carlo simulation on the basis of a coarse-grained model, in which the space is divided into cells mutually connected by resistors and each cell can be ionized or neutral. We show that a nonlinear ionization process induces a discontinuous transition when the potential difference between two electrodes is increased.

1. Introduction

The electric discharge occurs when the potential drop between two electrodes exceeds a critical value and plays fundamental roles in many phenomena which include lightning and ignition of a spark plug. In order to understand mechanism of the discharge, we introduce a coarse-grained model and study effects of nonlinearity on the discharge process.

2. Model

We first divide the space into small cells and assume that a cell can take one of three states; freshly-ionized (F-cell), aged-ionized (A-cell) and neutral (N-cell) states. An F-cell rejuvenates with some probability and when it does not rejuvenate, it stays as an A-cell with older age or it is neutralized by recombination with some probability to become an N-cell. An A-cell can be rejuvenated back to an F-cell or it can be neutralized. It can also stay as an A-cell with older age. An N-cell can be ionized with some probability. Figure 1 shows these processes schematically. We also assume that the adjacent cells are connected by a resistor which is conductive when cells at both ends of the resistor are ionized (F-cell or A-cell) and nonconductive otherwise. This model can be simplified as a resistor network [1] where a lattice point corresponds to a cell and a bond represents the resistor connecting two adjacent cells.

In order to obtain basic properties of this model, we consider a square lattice in the present study. We assume that one set of edges is subjected to a potential difference V and the other set of edges satisfies a periodic boundary condition. Our Monte Carlo simulation proceeds as follows: At the beginning, all cells are set to be neutral and all resistors are assumed to be nonconductive. We solve Kirchhoff's equations to obtain the potential at each cell and the current flowing on each bond. We also calculate the strength of the electric field at each cell from the difference of the potentials. After completing these calculations, we update the state of each cell according to the branching ratios shown in Fig. 1.

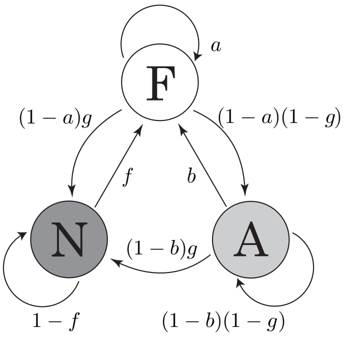

Fig. 1: Transition routes and branching ratios among freshly-ionized, aged-ionized, and neutral state.

We exploit the Townsend form [2] as the ionization probability f,

$$f(E_i) = \exp(-E_0/E_i), \tag{1}$$

where E_i is the electric filed of cell i. The recombination probability g is assumed to

$$g(\alpha_i) = 1 - e^{-\alpha_i t_0/\tau}, \tag{2}$$

where t_0 is the time scale of one Monte Carlo step, and α_i is the age of cell i from the previous ionization or rejuvenation, namely $\alpha_i = 1$ for F-cell and $\alpha_i > 1$ for A-cell. The probabilities, a and b, of rejuvenation of an F-cell and an A-cell are assumed to be the same as f, $f = a = b$.

This completes one Monte Carlo step. We repeated 5.0×10^3 steps of the elementary Monte Carlo process and obtained physical quantities from the time average, taking samples from the last 4.0×10^3 steps, for various potential differences.

3. Result and Discussion

Figures 2 and 3 show the V dependence of the fraction of the ionized cells and the current flowing from the electrode for $\tau/t_0 = 0.5$ in a 20×20 square lattice. There is a discontinuous transition from nonconductive to conductive state when the potential difference is increased.

We noticed coexistence of a conductive state and a poorly conductive state for a given value of V and thus we averaged physical quantities among the same states which are shown in Figs. 4 and 5. When the rejuvenation probabilities are set to zero ($a = b = 0$), we found coexistence of the two states even if potential difference is increased. Therefore the discontinuous transition is induced by the rejuvenation process.

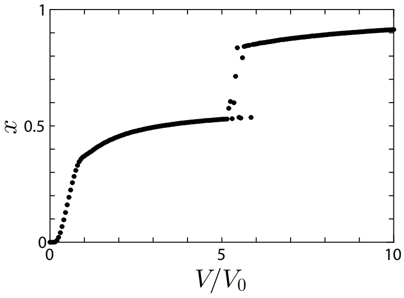

Fig. 2: The fraction of ionized cells as a function of potential difference for $\tau/t_0 = 0.5$.

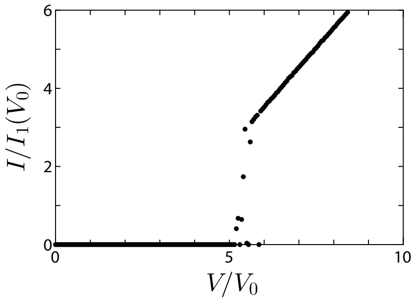

Fig. 3: Current as a function of potential difference for $\tau/t_0 = 0.5$.

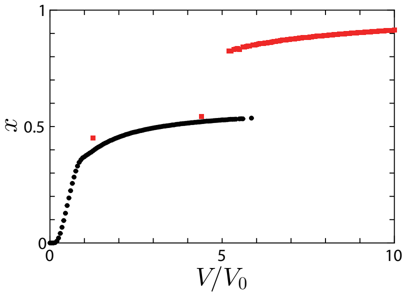

Fig. 4: The fraction of ionized cells as a function of potential difference, distinguishing poorly conductive state from conductive state, for $\tau/t_0 = 0.5$. ○ (black) : nonconductive state and □ (red) : conductive state.

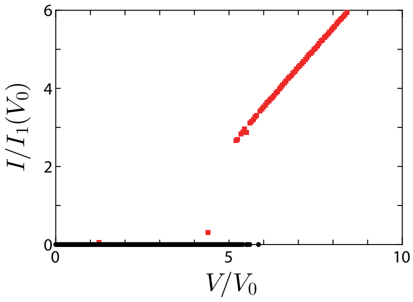

Fig. 5: Current as a function of potential difference, distinguishing poorly conductive state from conductive state, for $\tau/t_0 = 0.5$. ○ (black) : nonconductive state and □ (red) : conductive state.

4. Conclusion

We presented a Monte Carlo simulation of the discharge in a coarse-grained model and showed that there is a discontinuous transition from nonconductive to conductive states, and the transition is induced by the rejuvenation of ionized cells.

References

[1] A. Sasaki: Phys. Rev. Lett. **105** (2010) 075004.

[2] Yuri P. Raizer: *Gas Discharge Physics* (Springer, Berlin Heidelberg, 1991), Chap. 4, p.56.