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Magnetized electron plasma turbulence is investigated by using gyrokinetic simulations in a single helicity shearless slab configuration. A quasi two dimensional (2-D) feature of magnetized plasma leads to self-organization of zonal flows under normal enstrophy cascade and inverse energy cascade. Series of decaying turbulence simulations with different density gradients show that scale lengths of zonal flows follow the Rhines scale, which is determined by the competition between inverse energy cascade at high wavenumber and linear drift wave dispersion at low wavenumber. Anisotropic spectral condensation in 2-D turbulent energy spectra is first identified by gyrokinetic simulations.

## 1. Introduction

Turbulence in magnetically confined plasma can be regarded as a quasi two dimensional (2-D) problem due to anisotropy imposed by strong ambient magnetic fields. Therefore, magnetized plasma turbulence has common properties as 2-D neutral fluid turbulence. In particular, the similarity of drift wave turbulence and Rossby wave turbulence is well known, and their turbulent spectra and self-organization processes have been studied based on the Hasegawa-Mima (H-M) equation [1]. On the other hand, with increasing computational resources, gyrokinetic simulations have been established as standard tools for analyzing magnetized plasma turbulence. While gyrokinetic simulations disclosed various structure formations such as streamers and zonal flows, basic properties of the H-M equation have not been examined in gyrokinetic simulations. In this study, we revisit this classical issue, and study self-organization processes of zonal flows in gyrokinetic simulations of decaying electron turbulence.

## 2. Spectral structure of quasi-2D turbulence

We consider electrostatic plasma turbulence described by a gyrokinetic Vlasov-Poisson system,

$$\frac{\partial F}{\partial t} + \left( v_{\parallel} \mathbf{b} + \frac{c}{B_0} \mathbf{b} \times \nabla \langle \phi \rangle \right) \cdot \nabla F - \frac{e}{m_e} \mathbf{b} \cdot \nabla \langle \phi \rangle \frac{\partial F}{\partial v_{\parallel}} = C(F) \quad (1)$$

$$-\left( \nabla^2 + \frac{\rho_{te}^2}{\lambda_{De}^2} \nabla_{\perp}^2 \right) \phi + \frac{1}{\lambda_{Di}^2} \phi = \int F \delta(\mathbf{R} + \boldsymbol{\rho} - \mathbf{x}) d^6 \mathbf{Z} - n_0 \quad (2)$$

where  $F$  is the electron distribution and  $C$  is a collision operator. Here, adiabatic ions and the long wavelength approximation  $k_{\perp} \rho_{te} \ll 1$  are assumed for simplicity. While an adiabatic electron response, which is often assumed in ion turbulence simulation, vanishes for ion zonal flows, adiabatic ions respond to electron zonal flows in Eq.(2). Because of this feature, the electron gyrokinetic system has similar features as the H-M equation, while the ion gyrokinetic system has stronger zonal flow response than the H-M equation. By neglecting parallel dynamics (2-D limit) and finite Larmor radius (FLR) effects, Eqs. (1) and (2) yield the H-M equation [2],

$$\frac{\partial}{\partial t} (\phi - \nabla^2 \phi) - (\mathbf{b} \times \nabla \phi) \cdot \nabla (\nabla^2 \phi - \ln n_0) = 0, \quad (3)$$

which describes a rotating 2-D fluid turbulence. This kind of 2-D fluid conserves two quantities, the energy,  $E = \int [(\nabla \phi)^2 + \phi^2] dV/2$ , and the enstrophy,  $W = \int [(\nabla^2 \phi)^2 + (\nabla \phi)^2] dV/2$ , which lead to different power laws for turbulent energy spectral cascade. Kraichnann [3] argued that if there is selective dissipation of the enstrophy, normal cascade of the enstrophy occurs, while the energy inversely cascades toward longer wavelength. However, in rotating fluids such as drift wave turbulence, the structure formation due to inverse energy cascade may be avoided by linear drift wave dispersion. Equating linear drift wave frequency and nonlinear turbulent transfer rate in Eq. (3) yields a critical wavenumber characterized by the so-called Rhines scale, where energy spectra condensate and self-organization occurs [4],

$$k^c = \left(\frac{\beta}{U}\right)^{\frac{1}{2}}. \quad (4)$$

Here,  $\beta=|\nabla \ln n_0|=1/L_n$ ,  $U=\varepsilon^{1/2}$  is the average turbulence velocity, and  $\varepsilon=[(\nabla \phi)^2 dV/V]$ . If one takes account of the anisotropy in the linear wave term, an anisotropic expression of  $k^c$  is given as [5]

$$\begin{aligned} k_x^c &= \left(\frac{\beta}{U}\right)^{\frac{1}{2}} \cos \theta \sin^{\frac{1}{2}} \theta, \\ k_y^c &= \left(\frac{\beta}{U}\right)^{\frac{1}{2}} \sin^{\frac{3}{2}} \theta, \end{aligned} \quad (5)$$

where  $\theta=\tan^{-1}(k_y/k_x)$ .

### 3. Calculation model.

The full-f gyrokinetic Vlasov code G5D [6] is used to solve Eqs. (1) and (2) in a single helicity shearless slab configuration (see Fig.1). Box size and grid size for  $(x, y)$  are respectively chosen as  $(600\rho_{te}, 300\rho_{te})$  and  $(256, 128)$ , so that the minimum wavenumber is much smaller than a typical wavenumber of zonal flows  $k_{ZF}$ , while the maximum wavenumber is much larger than the wave number of initial perturbation. The initial density perturbation is set such that random fluctuation is uniform in configuration space and its isotropic wavenumber spectrum peaks at  $k_{\perp}\rho_{te}\sim 0.3$ . The correspondence between  $k_{ZF}$  and  $k_c\sim\varepsilon^{-1/4}L_n^{-1/2}$  is investigated by varying the density gradient  $L_n$ , which is expected to affect zonal flow structures.

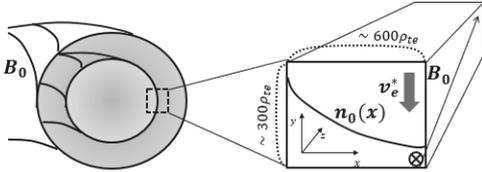


Fig.1 Shearless slab configuration.

### 4. Results

Figure 2 (a) plots time evolution of relative change of the energy and the enstrophy at  $L_n/\rho_{te}=3\times 10^3$ . In this simulation, the enstrophy decays faster than the energy, which infers selective dissipation of the enstrophy through normal enstrophy cascade and thus, inverse energy cascade.

Figure 2 (b) shows the scaling of  $k_{ZF}\varepsilon^{1/4}$  against  $L_n^{-1/2}$  for  $L_n/\rho_{te}=3\sim 12\times 10^3$ , where  $k_{ZF}$  is defined as

$$k_{ZF} = \int k_x E_k(k_x, k_y = 0) dk_x / \int E_k(k_x, k_y = 0) dk_x. \quad (7)$$

Numerical results show good agreements with Eq.(4), and zonal flow structures are well characterized by the Rhines scale.

Figure 3 plots normalized electrostatic potential  $e\phi/T_e$  on the configuration space at  $t v_{ti}/L_n=0, 20$ .

Initial perturbations evolve to longer wavelength structures, which are slightly elongated along  $y$  direction. In Fig.4, the corresponding 2-D energy spectrum clearly shows a dumb-bell shaped anisotropic structure around  $k_{\perp}\rho_{te}\sim 0.1$  as given in Eq.(5).

### 5. Conclusion

Decaying electron turbulence is investigated by using gyrokinetic simulations, and the self-organization process of zonal flows is confirmed in several aspects: selective dissipation of the enstrophy, the Rhines scale, and the anisotropic 2-D spectral structure around  $k_c$ . In the future work, detailed analyses on power laws of turbulent spectra will be discussed.

### References

- [1] A. Hasegawa: Adv. Physics. **96** (1985) 1.
- [2] Y. Idomura: Phys. Plasmas. **13**(2006) 080701.
- [3] R. H. Kraichnann,: Phys. Fluids. **10** (1967) 1417.
- [4] P. B. Rhines: J. Fluid Mech. **69**(1975) 417.
- [5] G. K. Vallis, and M. E. Maltrud: J. Phys. Oceanogr **23**(1992) 1346.
- [6] Y. Idomura, et al.:J. Compt. Phys. **226**(2007) 244.

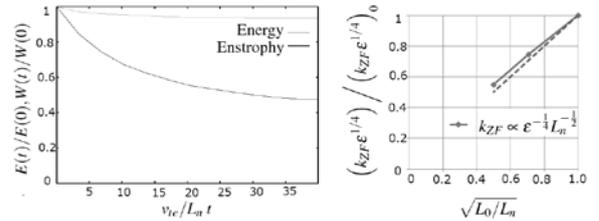


Fig.2 (a) Time evolution of the energy and the enstrophy. (b) Comparisons of  $k_{ZF}$  and the Rhines scale  $k_c$ .

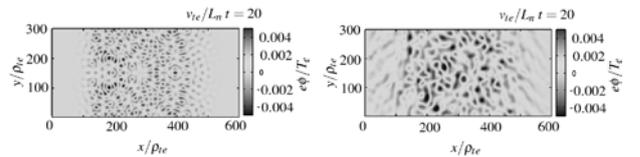


Fig.3 The electrostatic potential  $e\phi/T_e$  at  $t v_{ti}/L_n=0$  (left) and  $t v_{ti}/L_n=20$  (right)

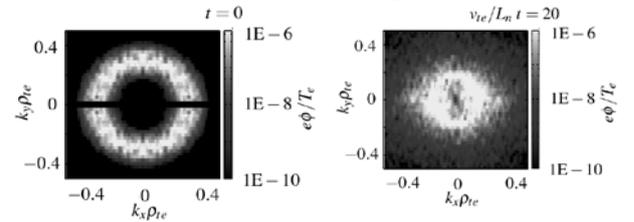


Fig.4 The 2-D energy spectrum  $E_k(k_x, k_y)$  at  $t v_{ti}/L_n=0$  (left) and  $t v_{ti}/L_n=20$  (right)