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On Stability Property of the Numerical Cherenkov Radiation and Its Application to Relativistic Shock Simulations

相対論的プラズマシミュレーションにおける数値チェレンコフ不安定の特性とその抑制

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We studied stability property of the numerical Cherenkov radiation in relativistic plasma flows by means of particle-in-cell simulations. With the implicit FDTD method for the Maxwell equations, we found that the instability was greatly inhibited with the CFL number of 1.0. The present result contrasts with the recently reported results [1,2,3] in which the magical CFL numbers were 0.5 and 0.7, respectively for their different explicit field solvers. In addition, we found higher order shape functions and a specific implicitness factor further suppressed the non-resonant type of the numerical instability. Applications of the obtained results enabled to examine a long-time evolution of a relativistic collisionless shock without significant numerical noise in the upstream. This property will allow us to investigate particle accelerations in relativistic shocks associated with, for example, gamma-ray bursts.

1. Introduction

Particle-in-cell (PIC) simulations have been used to study plasma dynamics in laboratory, space, and astrophysical phenomena. In particular, PIC simulations have been powerful tools to investigate particle accelerations associated with explosive phenomena in astrophysical objects, such as supernova remnant shocks and gamma-ray bursts.

In PIC simulations, the finite-difference time-domain (FDTD) method has been employed as a standard numerical solver for the electromagnetic fields. The FDTD method is very simple and flexible, however it has been known that the phase speed of the electromagnetic wave is reduced numerically less than the light speed in large wavenumber regions. In relativistic plasma flows, the numerical dispersion induces a non-physical numerical instability which is now known as the numerical Cherenkov radiation [4]. This is one of the critical issues in examining relativistic collisionless shocks by multidimensional PIC simulations with the FDTD method.

To suppress the numerical instability, several methods have been developed. In particular, it has been recently reported that careful choices of the CFL number greatly inhibited growth of the numerical Cherenkov instability [1,2,3]. For the FDTD method, this magical CFL number was 0.5. It has been shown, however, that this stability property depends on algorithms of solving the Maxwell equations, and interpolations for the electromagnetic fields and the current deposit.

We examined the stability property of the numerical Cherenkov instability by means of two-dimensional PIC simulations. We used a PIC simulation code package, pCANS, employing the momentum conserving field interpolation, the density decomposition method [5], and the implicit FDTD method [6] for the Maxwell equations.

2. Numerical Dispersion Relation of Implicit FDTD scheme

A numerical dispersion relation of the implicit FDTD method in two dimensions (x-y) is obtained as

$$\left[\frac{e^{i\omega\Delta t}-1}{\theta(e^{i\omega\Delta t}-1)+1}\right]^2+i\frac{\omega_{pe}^2\Delta t}{\omega}\left[\frac{e^{i\omega\Delta t}-1}{\theta(e^{i\omega\Delta t}-1)+1}\right]+4\left(\frac{c\Delta t}{\Delta x}\sin(\frac{k_x\Delta x}{2})\right)^2+4\left(\frac{c\Delta t}{\Delta y}\sin(\frac{k_y\Delta y}{2})\right)^2=0.$$

For the light wave with $\omega_{pe}/\omega \ll 1$ and $\omega_{pe}\Delta t \ll 1$ the dispersion relation can be simplified to

$$\Re[\omega\Delta t]=2\arctan\left[\frac{1}{4}\frac{\sqrt{1/(S_x^2+S_y^2)}}{(\theta-1/2)^2+\frac{1}{4}\sqrt{1/(S_x^2+S_y^2)}}\right],$$

$$\Im[\omega\Delta t]=2\arctan\left[\frac{1}{2}\frac{(\theta-1/2)}{(\theta-1/2)^2+\frac{1}{4}\sqrt{1/(S_x^2+S_y^2)}}\right],$$

where $S_x=c\frac{\Delta t}{\Delta x}\sin(\frac{k_x\Delta x}{2})$,

$S_y=c\frac{\Delta t}{\Delta y}\sin(\frac{k_y\Delta y}{2})$.

Since the imaginary part is always positive, the implicit FDTD method damps high-frequency waves except for the case with θ = 0.5.

3. Results

We examined two-dimensional (x-y) PIC simulations with periodic boundary conditions with 128×128 cells. We initially set a relativistic plasma flow in the x direction with a Lorentz factor of Γ = 100 with 100 particles per cell for each species (electron and positron). The thermal velocity in the flow frame is 10% of the light speed.

First, we examined with various CFL numbers ranging from 0.4 to 1.0. Fig.1 shows that the numerical Cherenkov instability is remarkably inhibited with the CFL number of 1.0. Difference in the field solver (implicit vs. explicit) resulted in the different magical CFL numbers (1.0 and 0.5-0.7).

We also found in Fig.2 that with CFL = 1.0, a non-resonant mode [7] still grows with a small growth rate. This mode can be suppressed by adopting higher order shape functions (Fig.2(a)), and an optimal implicitness factor of θ = 0.501 (Fig.2(b)).

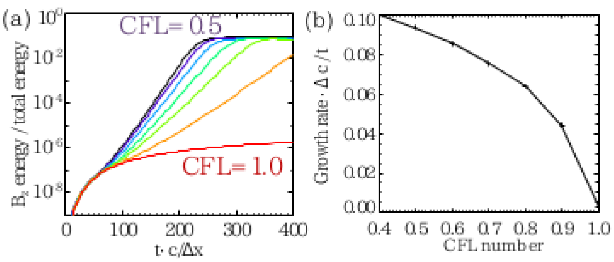

Fig.1. (a) Time evolutions of the magnetic energy and (b) growth rate of the numerical instability with various CFL number.

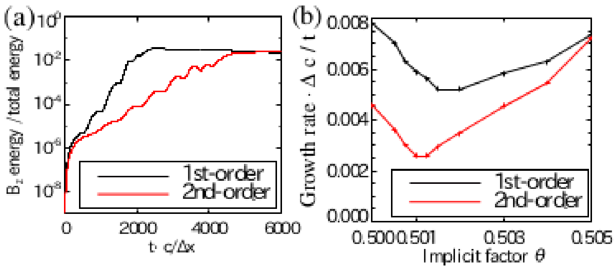

Fig.2. (a) Time evolutions of the magnetic energy for the first (black) and second (red) order shape functions. (b) Growth rate for different implicitness factors.

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We found that the magical CFL number for the pCANS code with the implicit field solver is 1.0, and the adoption of the higher order shape function and the implicitness factor of θ = 0.501 further suppressed the slowly-growing mode of the numerical Cherenkov instability. This stability property was successfully applied to multidimensional PIC simulations of relativistic collisionless shocks.

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References

[1] J.-L. Vay, C.G.R. geddes, E. Cormier-Michel, D.P. Grote: J. Comp. Phys, 230 (2011) 5908-5929

[2] B. Godfrey, J.-L. Vay: J. Comp. Phys, 248 (2013) 33-46

[3] X. Xu, P. Yu, S.F. Martins, F.S. Tsung, V.K. Decyk, J. Vieira, R.A. Fonseca, W. Lu, L.O. Silva, W.B. Mori: Comput. Phys. Comm, 184 (2013) 2503-2514

[4] B. Godfrey: J. Comp. Phys. 15 (1974) 504-52

[5] T. Zh. Esirkepov: Comput. Phys. Comm, 135 (2001) 144-153

[6] M. Hoshino: ApJ, 672 (2008) 940-956

[7] B. Godfrey, J.-L. Vay: J. Comp. Phys, 267 (2014)1-6