18PB-115

Development of linearized collision operator for multiple ion species in gyrokinetic simulation

ジャイロ運動論的シミュレーションにおける多イオン種衝突オペレータの開発

Masanori Nunami[1)], Motoki Nakata[2)], Tomo-Hiko Watanabe[3)], and Hideo Sugama[1)]
沼波政倫[1)], 仲田資季[2)], 渡邉智彦[3)], 洲鎌英雄[1)]

[1)]*National Institute for Fusion Science, 322-6 Oroshi-cho, Toki 509-5292, Japan,*
[2)]*JAEA, 2-166 Omotedate, Obuchi, Rokkasho 039-3212, Japan,*
[3)]*Nagoya Univ., Furo-cho, Chikusa, Nagoya 464-8602, Japan*
[1)]核融合研 〒509-5292 土岐市下石町322-6,
[2)]原子力機構 〒039-3212上北郡六ヶ所村尾駮字表舘2-166,
[3)]名古屋大 〒464-8602名古屋市千種区不老町

Linearized model collision operators for multiple ion species are implemented to the gyrokinetic local flux-tube code, which satisfy the conservation laws of particles, momentum, and energy, and also satisfy adjointness relations even for collisions between different particle species under unequal temperatures. It is confirmed that the developed operator works well in the simulations within certain resolution of the velocity space.

1. Introduction

Nowadays, the magnetically confined plasma experiments should contain different ion species including deuterium, carbon, and so on. The collisions between each ion species play a significant role in the plasma transport with kinetic approaches. In the gyrokinetic simulation, especially, the collision operator is needed to realize the steady turbulent state. Therefore, it is desirable to employ a good collision operator in the kinetic simulations, which is easy to treat numerically satisfying physical properties such as several conservation laws. In this work, we implement the linearized model collision operator for multiple ion species plasmas [1] into the gyrokinetic Vlasov local flux-tube code, GKV/GKV-X [2,3], and it is confirmed that the operator works well with several conservation laws and physical constraints within certain allowable numerical errors. Using the developed collision operator, it is confirmed that the correct collisional processes appears in the simulations.

2. The linearized collision operator

The turbulent transport driven by micro instabilities needs collisional processes for the achieving the steady state. Therefore, the collision model that satisfies correct properties such as physical conservation laws should be included in the kinetic simulations.

2.1 *Properties*

In our work [1], the linearized model collision operator for multiple ion species plasmas were constructed, satisfying conservation laws for particle, momentum, and energy, and satisfies also adjointness relations and Boltzmann's H-theorem including unequal temperature plasmas with different particle species. Several properties satisfied by the linear collision operator for collisions between species a and b are discussed in explicit way. The conservation of particle should be satisfied for the test-particle part C_{ab}^T and the field-particle part C_{ab}^F, where the collision operator is given by $C_{ab} = C_{ab}^T + C_{ab}^F$. The conservations of particle are represented by $\int d^3v\, C_{ab}^T(\delta f_a) = \int d^3v\, C_{ab}^F(\delta f_a) = 0$, while momentum conservation, $\int d^3v\, m_a \boldsymbol{v} C_{ab}^T(\delta f_a) + \int d^3v\, m_b \boldsymbol{v} C_{ba}^F(\delta f_a) = 0$, and the energy conservation, $\int d^3v\, \frac{1}{2} m_a v^2 C_{ab}^T(\delta f_a) + \int d^3v\, \frac{1}{2} m_b v^2 C_{ba}^F(\delta f_a) = 0$ should be satisfied. For the adjointness relations for the test-particle part, $\int d^3v \left(\frac{\delta f_a}{F_{aM}}\right) C_{ab}^T(\delta g_a) = \int d^3v \left(\frac{\delta g_a}{F_{aM}}\right) C_{ab}^T(\delta f_a)$, and for the field-particle part, $T_a \int d^3v \left(\frac{\delta f_a}{F_{aM}}\right) C_{ab}^T(\delta f_a) = T_b \int d^3v \left(\frac{\delta f_b}{F_{bM}}\right) C_{ba}^F(\delta f_a)$. The H-theorem can be written by $T_a \int d^3v \left(\frac{\delta f_a}{F_{aM}}\right) [C_{ab}^T(\delta f_a) + C_{ab}^F(\delta f_b)] + T_b \int d^3v \left(\frac{\delta f_b}{F_{bM}}\right) [C_{ba}^T(\delta f_b) + C_{ba}^F(\delta f_a)] \le 0$.

2.2 *Implementation*

The GKV/GKV-X, which were used in ITG turbulence simulations [4] in the Large Helical Device [5] plasma, is extended recently to multi-species plasma simulations. In the implement of the collision operator in the code, we should have the velocity moments for different species ions to calculate the field-particle operator. If the parallelization in species direction is employed, the communications for the moments may be time-consuming. Therefore, in the implementations, we optimize the code for the moment calculations and the communications.

3. Numerical test

In order to validate the developed operator, we check the thermal equilibration by the operator and the numerical errors for the conservation laws and the adjointness relations. From here, we consider the drift kinetic limit of the model operator except because the discussions under the limit are equivalent to that in the gyrokinetic form.

3.1 *Thermal equilibration*

We conduct a test of thermal equilibration processes with the collision operator in the three-ion species plasmas with deuterium (D), helium (He) and carbon (C). If each species ion has distributions with different temperature of the perturbed Maxwellian, $\delta f_a = F_{aM}\left(\left(\frac{\delta n_a}{n_a}\right) + \left(\frac{m_a}{T_a}\right)\boldsymbol{u}_a \cdot \boldsymbol{v} + \left(\frac{\delta T_a}{T_a}\right)\left(\frac{v^2}{v_{ta}^2} - \frac{3}{2}\right)\right)$, the thermal equilibrations between each species ions will be proceeded due to $\frac{\partial \delta f_a}{\partial t} = \sum_b \left(C^T(\delta f_a) + C^F(\delta f_b)\right)$ and the temperatures will be relaxed to the equal value. Figure 1(a) shows the time evolutions of each temperature fluctuation with the initial distributions with $\boldsymbol{u}_a = 0$ in the equilibration process, where the initial temperatures are given by different values. It is found that the differences between each temperature vanish as a result of the thermal relaxation brought by the collision operator. Due to the equilibration process, the distribution functions should go to same function for sufficiently long time. In Fig.1(b), the distribution functions at initial state t=0, and at t/(R_0/v_{tD})=1000. Indeed, it can be seen that each distribution function goes to same function.

3.2 *Error estimates*

Based on the conservation laws, we estimate the errors of the collision operator for the conservations in the three-ion species case. In the evaluations, we estimates following errors,

$$\Delta_{ab}^{(0)T,F} \equiv \tau_{ab} \int d^3 v \; C_{ab}^{T,F}(h_a) / \int d^3 v \; h_a \; ,$$

$$\Delta_{ab}^{(1)} \equiv \tau_{ab}\left(\int d^3 v \; m_a v_\parallel \; C_{ab}^T(h_a) + \int d^3 v \; m_b v_\parallel \; C_{ba}^F(h_a)\right) / \int d^3 v \; m_a v_\parallel h_a \; ,$$

$$\Delta_{ab}^{(2)} \equiv \tau_{ab}\left(\int d^3 v \; m_a v^2 \; C_{ab}^T(h_a) + \int d^3 v \; m_b v^2 \; C_{ba}^F(h_a)\right) / \int d^3 v \; m_a v^2 h_a \; .$$

Calculation duration t_{sim}~10^2(R_0/v_{tD}) is necessary in the turbulence simulations. Therefore, we set an acceptable error less than 1% for the duration, i.e., the upper limit $|\Delta_{lim}|$ should be less than 10^{-4}. The results for each error show that $|\Delta^{(0)}| < 6.5 \times 10^{-5}$, $|\Delta^{(1)}| < 1.7 \times 10^{-5}$, and $|\Delta^{(2)}| < 2.9 \times 10^{-5}$. For the adjointness relations, we also confirmed that the error is in the acceptable level. Therefore, it is confirmed that all errors of the operator are in the acceptable level.

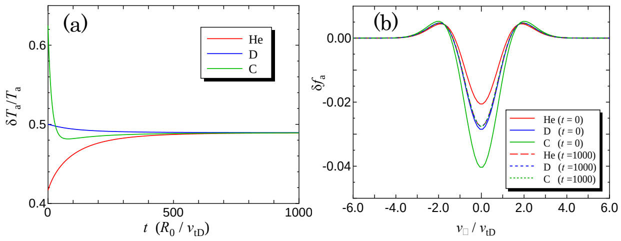

Fig.1. (a) Time evolutions of the temperature fluctuations for three-species plasma. (b) The distribution functions at t=0 (solid curves) and t/(R_0/v_{tD})=1000 (dashed curves).

4. Summary

In this work, we developed a numerical collision operator for multiple ion species are implemented to the gyrokinetic local flux-tube code satisfying the conservation laws of particles, momentum, and energy, and also adjointness relations even for collisions between different particle species under unequal temperatures. We confirmed that the operator works well with the conservation laws and physical constraints within certain allowable numerical errors.

Acknowledgments

This work is supported by JSPS KAKENHI Grant Nos. 26820398 and 26820401, in part by the NIFS collaborative Research Programs (13KNS057, 14KNT026, and 14KNST065), and by use of Helios system at IFERC-CSC (Project code: VLDGK_ST and GTNAXIS). Part of the results is obtained by using the K computer at the RIKEN AICS (Proposal number hp140053).

References

[1] H. Sugama, T.-H. Watanabe, and M. Nunami: Phys. Plasmas **16** (2009) 112503.

[2] T.-H. Watanabe, and H. Sugama: Nucl. Fusion **46** (2006) 24.

[3] M. Nunami, T.-H. Watanabe, and H. Sugama: Plasma Fusion Res. **5** (2010) 016.

[4] M. Nunami, T.-H. Watanabe, and H. Sugama: Phys. Plasmas **19** (2012) 042504.

[5] A. Komori, et al., Fusion Sci. Technol. **58** (2010) 1.