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Effect of Electron Degeneracy on Ion Distribution Function in Ultra-High Density Plasmas

超高密度プラズマ中のイオン分布関数に対する電子縮退の影響

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In ultra-high density plasmas as realized in inertial confinement fusion, the electron degeneracy can arise and it is thought to affect the energy distribution of coexisting ions through Coulombic ion-electron interaction. We evaluate these effects by developing and solving the model equation for the distribution function of ions coexisting with degenerate electrons. It is shown that the ion distribution maintains a Maxwellian form at a temperature equal to that of degenerate electrons.

1. Introduction

In inertial confinement fusion, the fuel is compressed to ultra-high density as 1000 times the solid density and the wave nature of electron becomes conspicuous in such plasmas. Since electron is fermion, the energy transition is restricted by Pauli's exclusion principle and such situation is called "electron degeneracy". The degree of degeneracy can be evaluated by the degeneracy parameter $\Theta \equiv kT_e / E_F$ (E_F is Fermi energy); the smaller it is, the stronger the degeneracy becomes. The consequences of electron degeneracy are as follows:

(a) The electron distribution function becomes to follow the Fermi-Dirac statistics.
(b) Scattering between an electron and other particle is restricted (Pauli blocking).

In addition, these events can also affect the energy distribution of coexisting ions through Coulombic ion-electron interaction. These effects can be evaluated by calculating the distribution function of ions, but there has been no proper equation in which are perfectly incorporated the above two effects due to electron degeneracy. Thus we develop the model equation to describe the distribution function of ions coexisting with degenerate electrons and estimate the magnitude of these effects by solving it.

2. Derivation of the equation followed by the ion distribution function

We start from the balance equation consisting of the scattering collision term (in- and out-scattering rates), the loss rate due to nuclear reactions and the independent source. Usually, small-angle Coulomb scattering term is written in the Fokker-Planck (FP) form, but the FP term is not suitable for describing individual scattering; it is difficult to incorporate Pauli blocking, which is effect on the individual scattering, into the final form of the FP term. Therefore we get back to the Boltzmann integral [1] and incorporate Pauli blocking into it. After that, we recover the FP-like form for small-angle Coulomb scattering. Instead of verbosity $\boldsymbol{v}$ we use energy E as an independent variable and adopt the flux $\Psi(E)$ defined by $\Psi(E) = v f(E)$, where $f(E)$ is the ion energy distribution function. The final form of the equation in steady-state is

$$\begin{aligned}\sum_{j\neq e} n_j\left[\sigma_j^{NI}(E)+\sigma_j^{abs}(E)\right]\Psi(E) &= \sum_j \frac{\partial}{\partial E}\left[S_j(E)\Psi(E)\right]+\sum_j \frac{\partial^2}{\partial E^2}\left[D_j(E)\Psi(E)\right] \\ &\quad +\sum_{j\neq e}\int n_j\sigma_j^{NI}(E'\to E)\Psi(E')dE' + Q(E),\end{aligned} \tag{1}$$

where n_j is the number density of "background" species j. On the left-hand side, the first and second terms are respectively the removal rate due to large-angle scattering and the loss rate due to absorption. The first and second terms on the right-hand side are the expansions of the small-angle scattering term. The third term represents the large-angle in-scattering rate. The last term is the independent source. The (differential) cross sections in Eq. (1) are the averages over the thermal motion of target j [2] and the symbol "*NI*" represents "elastic nuclear plus interference scattering". Additionally S_j and D_j are quantities defined during processing the small-angle Coulomb scattering term; the former is the Coulombic stopping power while the latter is the energy dispersion coefficient. The effects of electron degeneracy are incorporated in these coefficients.

3. Results and discussion

Supposing steady-state DT plasmas at various electron temperatures T_e and degeneracy parameters Θ, and fixing the distribution function of electrons, we solved Eq. (1). Figure 1 presents, as an example, the deuteron distribution function when T_e = 0.5keV, Θ = 0.1. The solid curve shows the distribution function calculated by fully considering the effect of electron degeneracy, while the dashed curve is obtained by partially considering the degeneracy effect, that is the electron distribution function based on the Fermi-Dirac statistics was used but Pauli blocking was ignored. The dotted curve (almost falling on the solid one) is the result in the case ignoring the electron degeneracy. It can be seen that each ion distribution function forms the Maxwellian distribution. However, the ion temperature gets higher when the degeneracy effect is patially considered.

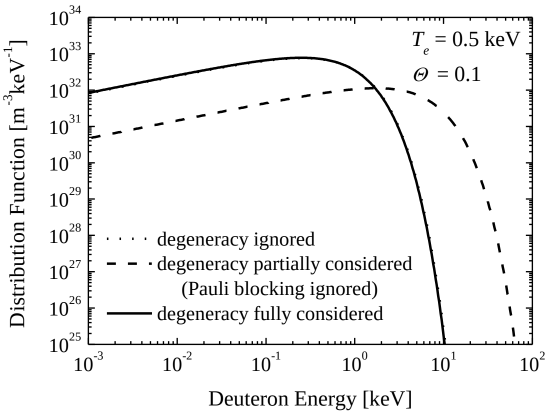

Fig.1. Deuteron distribution function

From the calculated distribution function we evaluated the ion temperature:

$$T = \frac{2}{3} < E > = \frac{2}{3} \frac{\int_0^\infty E\, f(E) dE}{\int_0^\infty f(E) dE}. \quad (2)$$

Figure 2 presents Θ– dependency of the ion temperature when T_e = 0.5keV. In the case where the electron degeneracy is ignored, the ion temperature is equal to the electron temperature.

As for how to incorporate the electron degeneracy, we obtain the following results:

(a) The ion temperature gets higher when the degeneracy effect is partially included (ignoring Pauli blocking), and this is significant at low Θ region.

(b) However, the result of (a) disappears when we consider Pauli blocking.

The event like (a) can be explained as follows. When the electron degeneracy arises, the number of electrons in lower energy region, which are susceptible to Coulomb interaction, gets smaller and Coulomb interaction they undergo gets weakened. Therefore the energy being transferred to ions from the electrons gets smaller than that in the case of non-degenerate plasmas. As a result, the ion distribution function spreads toward high-energy side in order to weaken Coulomb interaction that the ions undergo, and the ion temperature gets higher, because the distribution of ions is determined so that the energy flows between the ions and the electrons are canceled each other.

Meanwhile the event like (b) happens for the following reasons. Pauli blocking mainly restricts energy gaining of the electrons because of higher probability of electron occupation in the lower energy region, and the energy moving to electrons from the ions lessens. In other words, the energy transferred to ions from electrons relatively becomes larger. This effect counteracts the result of (a), so the ion temperature does not change from the result without considering the electron degeneracy.

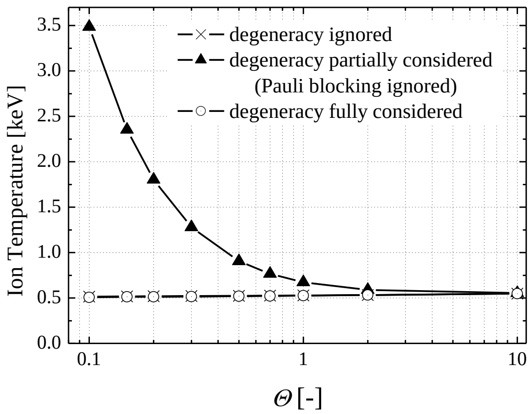

Fig.2. Ion temperature

4. Conclusion

By properly incorporating the effect of electron degeneracy, we have found the following results; the ion distribution function maintains Maxwellian form and the ion temperature becomes equal to the electron temperature even if electrons are in degenerate state. This is because two effects of electron degeneracy counteract each other.

References

[1] For example, F. Reif: *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill, New York, 1965).

[2] Y. Nakao, K. Tsukida and V. T. Voronchev: *Phys. Rev. D.* **84**, (2011) 063016.