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Collisional Kinetic-Fluid Closure Model for Zonal Flows
ゾーナルフローに対する衝突性運動論的流体クロージャーモデル

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A novel kinetic-fluid closure model is presented, which describes collisional damping of zonal flows in the ion temperature gradient turbulence in tokamaks. The closure relations representing the parallel heat fluxes are derived from the Laplace transform of the solution to the bounce-averaged collisional ion gyrokinetic equation for the radial wave number vector corresponding to the zonal-flow component. Approximate expressions for the closure relations are also obtained, which are suitable for numerical simulation.

1. Introduction

Zonal flows are intensively investigated in the fusion research as an attractive mechanism for realizing a good plasma confinement [1]. An accurate theoretical description of zonal-flow evolution is a key issue for correctly predicting the turbulent transport of fusion plasmas. In fact, unless the residual zonal flow [2] is properly treated in a gyrofluid model, the gyrofluid simulation cannot reproduce the same turbulent transport as given by the gyrokinetic simulation. In order for a set of gyrofluid fluid equations to describe the residual zonal-flow level given by Rosenbluth and Hinton [1], a novel collisionless kinetic-fluid closure model of zonal flows in tokamaks was presented by Sugama *et al*.[3], which differs from the zonal-flow closure model by Beer and Hammett [4]. It is confirmed from the fluid simulations of zonal-flow damping [5] that the collisionless kinetic-fluid closure model of zonal flows [3] correctly reproduces the residual zonal flow level predicted by the kinetic theory, in which effects of the noncircular tokamak cross section can be included. Behaviors of zonal flows are also influenced by collisions [6]. In fact, the gyrokinetic simulation of the ion temperature gradient (ITG) turbulence shows that the turbulent transport level significantly depends on the collision frequency through the collisional damping of zonal flows [7]. In the present work, we extend the closure model to take account of collisional effects on zonal flows.

2. Kinetic-Fluid Equations

In the same way as in [3], we take the velocity moments of the electrostatic gyrokinetic equation for the perturbed gyrocenter distribution function $\delta f^{(g)}_{\mathbf{k}_\perp}$ in order to obtain the equations which govern time evolution of the fluid variables defined by

$$
\begin{aligned}
&[\delta n^{(g)}_{\mathbf{k}_\perp}, n_0 u_{\parallel\mathbf{k}_\perp}, \delta p_{\parallel\mathbf{k}_\perp}, \delta p_{\perp\mathbf{k}_\perp}] \\
&= \int d^3v\, \delta f^{(g)}_{\mathbf{k}_\perp}[1, v_\parallel, m v_\parallel^2, m v_\perp^2/2]\,.
\end{aligned} \tag{1}
$$

Consequently, we obtain the perturbed gyrocenter density equation,

$$
\begin{aligned}
&\frac{\partial \delta n^{(g)}_{\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla\left(\frac{n_0 u_{\parallel\mathbf{k}_\perp}}{B}\right) \\
&= -i\frac{c}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)[\delta p_{\parallel\mathbf{k}_\perp} + \delta p_{\perp\mathbf{k}_\perp} \\
&\quad + n_0 e\phi_{\mathbf{k}_\perp} e^{-b/2}(2-b/2)] + N_{0\mathbf{k}_\perp}\,,
\end{aligned} \tag{2}
$$

the parallel momentum balance equation,

$$
\begin{aligned}
&m n_0 \frac{\partial u_{\parallel\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla\left(\frac{\delta p_{\parallel\mathbf{k}_\perp}}{B}\right) + \frac{\delta p_{\perp\mathbf{k}_\perp}}{B}\mathbf{b}\cdot\nabla B \\
&= -i\frac{mc}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)(q_{\parallel\mathbf{k}_\perp} + q_{\perp\mathbf{k}_\perp} + 4p_0 u_{\parallel\mathbf{k}_\perp}) \\
&\quad - n_0 e\mathbf{b}\cdot\nabla(\phi_{\mathbf{k}_\perp}e^{-b/2}) + \frac{n_0 e}{2B}\phi_{\mathbf{k}_\perp}e^{-b/2}\mathbf{b}\cdot\nabla B + N_{1\mathbf{k}_\perp}\,,
\end{aligned} \tag{3}
$$

the perturbed parallel pressure equation,

$$
\begin{aligned}
&\frac{\partial \delta p_{\parallel\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla[(q_{\parallel\mathbf{k}_\perp} + 3p_0 u_{\parallel\mathbf{k}_\perp})/B] \\
&\quad + \frac{2}{B}(q_{\perp\mathbf{k}_\perp} + p_0 u_{\parallel\mathbf{k}_\perp})\mathbf{b}\cdot\nabla B \\
&= -i\frac{c}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)[m(\delta r_{\parallel,\parallel\mathbf{k}_\perp} + \delta r_{\parallel,\perp\mathbf{k}_\perp}) \\
&\quad + p_0 e\phi_{\mathbf{k}_\perp}e^{-b/2}(4-b/2)] + N_{2\parallel\mathbf{k}_\perp}\,,
\end{aligned} \tag{4}
$$

and the perturbed perpendicular pressure equation,

$$
\begin{aligned}
&\frac{\partial \delta p_{\perp\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla[(q_{\perp\mathbf{k}_\perp} + p_0 u_{\parallel\mathbf{k}_\perp})/B] \\
&\quad - \frac{1}{B}(q_{\perp\mathbf{k}_\perp} + p_0 u_{\parallel\mathbf{k}_\perp})\mathbf{b}\cdot\nabla B \\
&= -i\frac{c}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)[m(\delta r_{\parallel,\perp\mathbf{k}_\perp} + \delta r_{\perp,\perp\mathbf{k}_\perp}) \\
&\quad + p_0 e\phi_{\mathbf{k}_\perp}e^{-b/2}(3-3b/2+b^2/8)] + N_{2\perp\mathbf{k}_\perp}\,,
\end{aligned} \tag{5}
$$

where $N_{0\mathbf{k}_\perp}$, $N_{1\mathbf{k}_\perp}$, $N_{2\parallel\mathbf{k}_\perp}$, and $N_{2\perp\mathbf{k}_\perp}$ are the nonlinear terms defined in [3]. The right-hand sides of (2)-(4) contain the third-order fluid variables (or parallel heat fluxes),

$$[q_{\|\mathbf{k}_\perp}, q_{\perp\mathbf{k}_\perp}] = \int d^3v\ \delta f^{(g)}_{\mathbf{k}_\perp} v_\| [(mv_\|^2 - 3T), (mv_\perp^2/2 - T)], \tag{6}$$

and the fourth-order fluid variables

$$[\delta r_{\|,\|\mathbf{k}_\perp}, \delta r_{\|,\perp\mathbf{k}_\perp}, \delta r_{\perp\perp\mathbf{k}_\perp}] = \int d^3v\ \delta f^{(g)}_{\mathbf{k}_\perp} m[v_\|^4, v_\|^2 v_\perp^2/2, v_\perp^4/4]. \tag{7}$$

In order to construct a closed system of kinetic-fluid equations, we need closure relations, which express the higher-order fluid variables in (6)-(7) in terms of the lower-order variables in (1).

3. Closure Model

Here, we consider the ion temperature gradient (ITG) turbulence for typical perpendicular wave numbers of fluctuations are given by $k_\perp \rho_{ti} \leq 1$, where ρ_{ti} denotes the ion thermal gyroradius. The ITG turbulence are governed by the ion gyrokinetic equation for $\delta f^{(g)}_{i\mathbf{k}_\perp}$, the relation of the perturbed electron density to the electrostatic potential $\delta n_{e\mathbf{k}_\perp} = n_0 e\left(\phi_{\mathbf{k}_\perp} - \langle \phi_{\mathbf{k}_\perp}\rangle\right)/T_e$ and the quasineutrality condition $\delta n_{i\mathbf{k}_\perp} = \delta n_{e\mathbf{k}_\perp}$. We now derive the closure relations for the high-order fluid variables in the fluid equations (2)-(5) for ions instead of the ion gyrokinetic equation to accurately describe the zonal-flow behaviors in the ITG turbulence. The collionless case was already treated in [3]. We assume the weakly collisional case, in which the collisional effects on the short-time behaviors of the fluid variables are neglected. Then, the same closure relations as (61)-(62) in [3] can be used for the shot-time evolution parts of the parallel heat fluxes $(q^{(s)}_{\|\mathbf{k}_\perp}, q^{(s)}_{\perp\mathbf{k}_\perp})$ and the fourth-order variables $(\delta r_{\|,\|\mathbf{k}_\perp}, \delta r_{\|,\perp\mathbf{k}_\perp}, \delta r_{\perp,\perp\mathbf{k}_\perp})$. The long-time evolution parts of the parallel heat fluxes $(q^{(l)}_{\|\mathbf{k}_\perp}, q^{(l)}_{\perp\mathbf{k}_\perp})$ are derived from the solution of the bounce-averaged collisional ion gyrokinetic equation for the radial wave number vector $\mathbf{k}_\perp = k_r \nabla r$ corresponding to the zonal-flow component. The resultant closure relations are given in terms of the Laplace transform $[q^{(l)}_{\|\mathbf{k}_\perp}(p), q^{(l)}_{\perp\mathbf{k}_\perp}(p)] = \int_0^\infty [q^{(l)}_{\|\mathbf{k}_\perp}(t), q^{(l)}_{\perp\mathbf{k}_\perp}(t)] \times \exp(-pt)dt$ as

$$q^{(l)}_{\|\mathbf{k}_\perp}(p) = -2q^{(l)}_{\perp\mathbf{k}_\perp}(p) = 2n_0 T_i [U_{1\mathbf{k}_\perp}(p)B - U_{2\mathbf{k}_\perp}(p)B^2], \tag{8}$$

where $U_{j\mathbf{k}_\perp}(p)(j=1,2)$ are given by

$$U_{j\mathbf{k}_\perp}(p) = \left\{\beta_1(p) - \langle B^{-2}\rangle\right\}^{-1} \beta_j(p) \times \left[\left\langle \frac{u_{\|\mathbf{k}_\perp}(p)}{B}\right\rangle - \frac{\langle B^{-2}\rangle}{p}\left\langle \left\{u_{\|\mathbf{k}_\perp}(t=0) + u^{(s)}_{\|\mathbf{k}_\perp}(p)\right\} B\right\rangle\right]. \tag{9}$$

Here, $\beta_j(p)(j=1,2)$ are defined by

$$[\beta_1(p), \beta_2(p)] = (m_i/n_0 T_i)\int d^3v\ G(p)(v_\|/B)[1, \mu] \tag{10}$$

with $G(p)$ being the solution of

$$G(p) - p^{-1} C_{ii}(G(p)) = \overline{(v_\|/B)} f_{iM}. \tag{11}$$

For installing the closure relations into numerical simulation codes, it is useful to express $\beta_j(p)$ $(j=1,2)$ approximately as

$$\beta_j(p) = \beta_j\ p\tau/(1+p\tau), \tag{12}$$

where $\beta_j = \lim_{p\to\infty} \beta_j(p)\,(j=1,2)$ are defined in [3] and τ represents the typical time scale of the collisional damping of the zonal-flow potential. The approximate expressions in (12) are shown to enable the closure relations to be written in the form of the first-order differential equations in time.

4. Summary

In this paper, the new kinetic-fluid model is derived, which describes behaviors of zonal flows in the ITG turbulence for the weakly collisional case. The collisional effects are included in the closure relations for the long-time evolution part of the parallel heat fluxes. The approximate expressions for the closure model are also shown and numerical simulation using them remains as a future task to confirm their validity.

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