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Gyrokinetic simulations in helical plasmas with the hybrid method of semi-Lagrangian and semi-implicit Runge-Kutta schemes
セミラグランジアン法と半陰的ルンゲクッタ法の複合解法を用いたヘリカルプラズマのジャイロ運動論的シミュレーション

Shinya Maeyama[1], Akihiro Ishizawa[2], Tomohiko Watanabe[2], Noriyoshi Nakajima[2], Shunji Tsuji-Iio[1], Hiroaki Tsutsui[1]
前山伸也[1], 石澤明宏[2], 渡邉智彦[2], 中島徳嘉[2], 飯尾俊二[1], 筒井広明[1]

[1]*Tokyo Institute of Technology*
2-12-1, Ookayama, Meguro-ku, Tokyo 152-8550, Japan
東京工業大学 〒152-8550 東京都目黒区大岡山2-12-1
[2]*National Institute for Fusion Science*
322-6, Oroshi-cho, Toki, Gifu 509-5292, Japan
核融合科学研究所 〒509-5292 岐阜県土岐市下石町322-6

A hybrid method of semi-Lagrangian and additive semi-implicit Runge-Kutta schemes for gyrokinetic δf Vlasov simulations is presented. This method is free from the Courant-Friedrichs-Lewy condition for the linear terms in the gyrokinetic equation. Simulations of parallel dynamics and of the ion-temperature-gradient instability in fusion plasmas confined by helical magnetic fields are carried out by means of this method. It is demonstrated that their results show good agreements with those obtained by using the explicit Runge-Kutta-Gill scheme, while the new numerical method has no time-step restrictions for the linear terms and substantially reduces the computational cost.

1. Introduction

Three-dimensional configurations of helical plasmas bring numerical difficulties to gyro-kinetic simulations of micro-instabilities. To investigate the ion-temperature-gradient (ITG)-driven turbulence in helical plasmas, Watanabe *et al.* employ a large number of grid points along a magnetic field line in the gyrokinetic Vlasov simulation code GKV [1]. As a result, the Courant-Friedrichs-Lewy (CFL) condition on the parallel advection severely restricts time steps. To address this problem, we propose an efficient numerical method for linear analysis of micro-instabilities in helical plasmas by means of gyro-kinetic Vlasov simulations, employing semi-Lagrangian and additive semi-implicit Runge-Kutta schemes. The new scheme is free from the CFL restrictions for the linear terms.

2. Equations and schemes

Employing the flux tube model, the linearized gyrokinetic Vlasov equation for the perturbed ion gyrocenter distribution function $f_{\mathbf{k}}(z,v_\parallel,\mu)$ in the electrostatic limit is given by

$$\left[\frac{\partial}{\partial t}+v_\parallel\nabla_\parallel+i\omega_{\mathrm{d}}-\frac{\mu\nabla_\parallel B}{m_{\mathrm{i}}}\frac{\partial}{\partial v_\parallel}\right]f_{\mathbf{k}} = -\frac{eF_{\mathrm{M}}}{T_{\mathrm{i}}}\left[v_\parallel\nabla_\parallel+i\omega_{\mathrm{d}}+i\omega_*\right]J_0(k_\perp\rho_{\mathrm{ti}})\phi_{\mathbf{k}}, \qquad (1)$$

where B, m_i, e, F_M, T_i, ω_d, ω_* and ρ_{ti} are the magnetic field strength, ion mass, elementary charge, Maxwell distribution function, ion temperature, magnetic and diamagnetic drift frequencies and ion Larmor radius, respectively. The radial and poloidal wavenumber k_x and k_y, the field-aligned coordinate z, the parallel velocity $v_\parallel$ and the magnetic moment μ are employed as the phase space coordinates. The electrostatic potential $\phi_{\mathbf{k}}$ is given by the quasi-neutrality condition with an adiabatic electron response.

By means of the operator splitting method [2], Eq. (1) can be split into the parallel motions and the others. The former consists of two linear advection equations in z and $v_\parallel$. They are easily computed by a semi-Lagrangian scheme [3]. Practically, one has to evaluate the value of $f_{\mathbf{k}}$ by using one-dimensional interpolations. For more details, see Ref. [4]. The latter is regarded as an additive operator of the perpendicular drift, source and collision terms. To solve this problem, we employ additive semi-implicit Runge-Kutta schemes (ASIRK) [5] and treat the magnetic drift term implicitly. Since the coefficient matrix of the magnetic drift term is diagonal, one can easily compute its semi-implicit time integration without using matrix solvers.

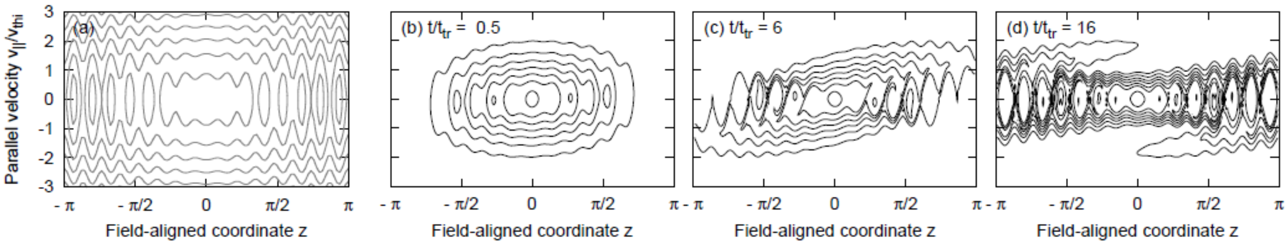

Fig 1. (a) Contour lines of the particle kinetic energy $mv_{||}^2/2+\mu B$ with the helical field, and (b)-(d) Snapshots of equi-contours of the distribution function $f(z,v_{||})$ in parallel phase space (where $\mu B_0/T_i = 4.0$). Horizontal and vertical axes are defined by z and $v_{||}$, respectively.

3. Parallel dynamics in helical plasmas

Here we compute only the parallel dynamics by a semi-Lagrangian scheme with a second-order operator splitting method. We employ 192×256 grid points and a time step size $\Delta t/t_{tr} = 0.1$, where $t_{tr} = L_n/v_{ti}$ with the density scale length L_n and the ion thermal velocity v_{ti}.The initial profile is given by $f(z,v_{||};t=0) = F_M(mv_{||}^2/2+\mu B)(1+\cos z)$, and the periodic boundary condition is employed in z. Characteristic curves of the dynamics are given as contour lines of the particle kinetic energy, as shown in Fig. 1 (a). There are trajectories of helical-ripple-trapped particles as well as those of passing particles. Snapshots of the contour lines of the distribution functions are shown in Fig. 1 (b)-(d). The distribution function is advected along the contour lines of the particle kinetic energy. While passing particles elongate the profile, trapped particles stay in the trapped regions. Thus, fine-scale structures appear at the trapped-passing boundary.

4. Linear ion-temperature-gradient instabilities in helical plasmas

Employing the hybrid method of semi-Lagrangian and additive semi-implicit Runge-Kutta schemes (SLASIRK), we have carried out linear ITG simulations of a helical plasma. Physical and numerical settings are the same as those for the inward-shifted LHD case shown in Ref. [1].

The linear growth rates and the real frequencies are plotted as a function of the poloidal wave number k_y in Fig. 2. The results agree well with the results obtained by using the fourth-order Runge-Kutta-Gill scheme (RKG), while taking the time step size larger than that of RKG. The presented numerical method gives sufficiently accurate results for $\Delta t/t_{tr}<0.4$, which is comparable to the transit time of passing particles through one helical ripple.

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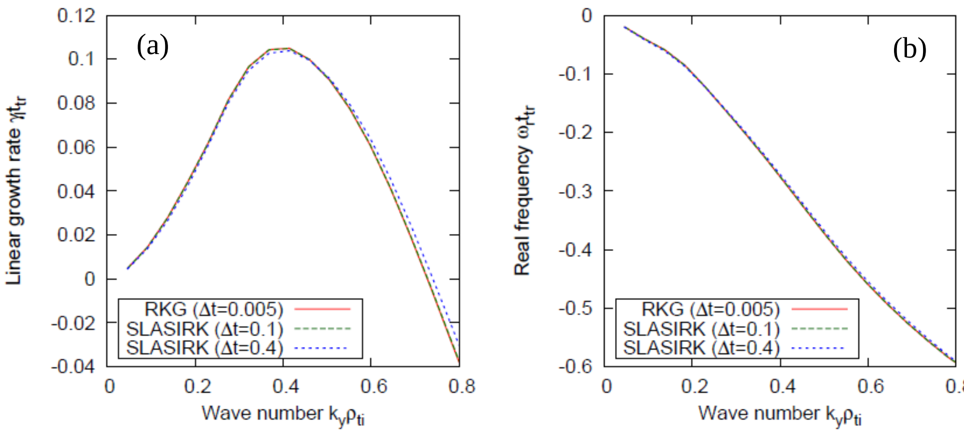

Fig. 2. (a) Linear growth rates γ_l and (b) real frequencies ω_r as a function of the poloidal wave number $k_y\rho_{ti}$. The solid, dashed and dotted lines represent the results obtained by RKG with $\Delta t/t_{tr} = 0.005$ and SLASIRK with $\Delta t/t_{tr} = 0.1, 0.4$, respectively.