23P133-B

Numerical analysis of axisymmetric toroidal equilibria with flow
流れをもつ軸対称トロイダル平衡の数値解析

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Numerical analysis of axisymmetric toroidal equilibria with flow is performed based on single-fluid and two-fluid magnetohydrodynamic (MHD) models. Effects of toroidal and poloidal flow comparable to the poloidal sound velocity, two-fluid, ion finite Larmor radius (FLR), pressure anisotropy and parallel heat fluxes on high-beta toroidal equilibrium is studied by solving reduced MHD equilibrium equations. Higher order terms of quantities like the pressures and the stream functions show the shift of their isosurfaces from the magnetic surfaces due to effects of flow, two-fluid and pressure anisotropy.

1. Introduction

We investigate small scale effects on equilibrium with flow based on reduced magnetohydrodynamic (MHD) models. At the sharp boundary of a well-confined region in magnetically confined plasmas where high-beta is achieved by shear-flow suppression of instability and turbulent transport, small-scale effects cannot be neglected. A reduced set of Grad-Shafranov (GS) type equilibrium equations for high-beta tokamaks with flow comparable to the poloidal sound velocity, ion finite Larmor radius (FLR), pressure anisotropy and parallel heat fluxes on high-beta tokamaks equilibrium has been derived from the fluid moment equations for collisionless, magnetized plasmas [1]. We show the results of numerical solutions of the reduced GS equations by means of the finite element method. The two-fluid effects induce the diamagnetic flows, which result in asymmetry of the equilibria with respect to the sign of the $E\times B$ flow. Higher order terms of quantities like the pressures and the stream functions show the shift of their isosurfaces from the magnetic surfaces due to effects of flow, two-fluid and pressure anisotropy.

2. Equilibrium Equations

The reduced set of equilibrium equations are derived from the fluid moment equations for collisionless, magnetized plasmas with the slow-dynamics (drift) ordering [2] by the asymptotic expansions for large aspect ratio, high-beta tokamaks with poloidal sonic flow. The FLR effects appear as the gyroviscosity and the perpendicular (diamagnetic) heat fluxes in the fluid moment equations. The equilibrium equations consist of the first two orders of the Grad-Shafranov (GS) equation of which the first order is same as that for static equilibria [1],

$$\left(\frac{\partial^2}{\partial R^2}+\frac{\partial^2}{\partial Z^2}\right)\psi_1 = -\mu_0 R_0^2\left[\left(\frac{x}{R_0}\right)\sum_{s=i,e}(p'_{s\parallel 1}+p'_{s\perp 1})+g'_*\right]-\left(\frac{I_1^2}{2}\right)', \tag{1}$$

and the second order is given by

$$\begin{aligned}&\left(\frac{\partial^2}{\partial R^2}+\frac{\partial^2}{\partial Z^2}\right)\psi_2 + \left[\mu_0 R_0^2\left(\frac{x}{R_0}\right)\sum_{s=i,e}\left(p''_{s\perp 1}+p''_{s\parallel 1}\right)+\mu_0 R_0^2 g''_* + \left(\frac{I_1^2}{2}\right)''\right]\psi_2\\ &=\frac{1}{R}\frac{\partial\psi_1}{\partial R}+F\left(\frac{\partial^2}{\partial R^2}+\frac{\partial^2}{\partial Z^2}\right)\psi_1+F'\frac{\left|\nabla\psi_1\right|^2}{2}\\ &\quad-\mu_0 R_0^2\left[E'_*+\left(\frac{x}{R_0}\right)\sum_{s=i,e}\left(P'_{s\perp 2*}+P'_{s\parallel 2*}\right)\right.\\ &\quad\left.+\frac{1}{2}\left(\frac{x}{R_0}\right)^2\sum_{s=i,e}\left(p'_{s\perp 1}+p'_{s\parallel 1}+C'_{s\perp 1}+C'_{s\parallel 1}\right)\right],\end{aligned} \tag{2}$$

where

$$F(\psi_1)=\left(\frac{B_0^2}{\mu_0}\right)^{-1}\left[m_i n_0 R_0^2\left(\Phi'_1+\frac{\lambda_H-\lambda_i}{en_0}p'_{i\perp 1}\right)\times\left(\Phi'_1+\frac{\lambda_H}{en_0}p'_{i\perp 1}\right)+\sum_{s=i,e}(p_{s\parallel 1}-p_{s\perp 1})\right] \tag{3}$$

includes contributions from the $E\times B$ and the ion diamagnetic poloidal flows with the gyroviscous cancellation and the pressure anisotropy. The coefficients $C_{\ldots}(\psi_1)$ are obtained by solving the equations for the higher-order quantities as functionals of the lowest order quantities while those in the second terms, denoted by '*', are arbitrary functions of ψ_1. The pressure and other quantities are determined once Eqs. (1) and (2) are solved. This set of equilibrium equations is an extension of previous models [3,4] to include pressure anisotropy with parallel heat flux. The GS equation for ψ_1, (1), is same as for the single-fluid, static case**.** In spite of its complexity, the GS equation for ψ_2, (2), is a linear, elliptic partial differential equation once the solution for ψ_1 of (1) is substituted and, thus, is easy to solve. However, singularity in the GS equation for ψ_2 occurs when the poloidal flow velocity equals the poloidal sound velocities. It arises because higher order terms not negligible in its vicinity are ordered out in the asymptotic expansions. The present model is to study the extension of regular, elliptic solution for single-fluid MHD equilibria with flow. One has to choose the profiles of free functions that do not include the vicinity of singularity to get regular solutions.

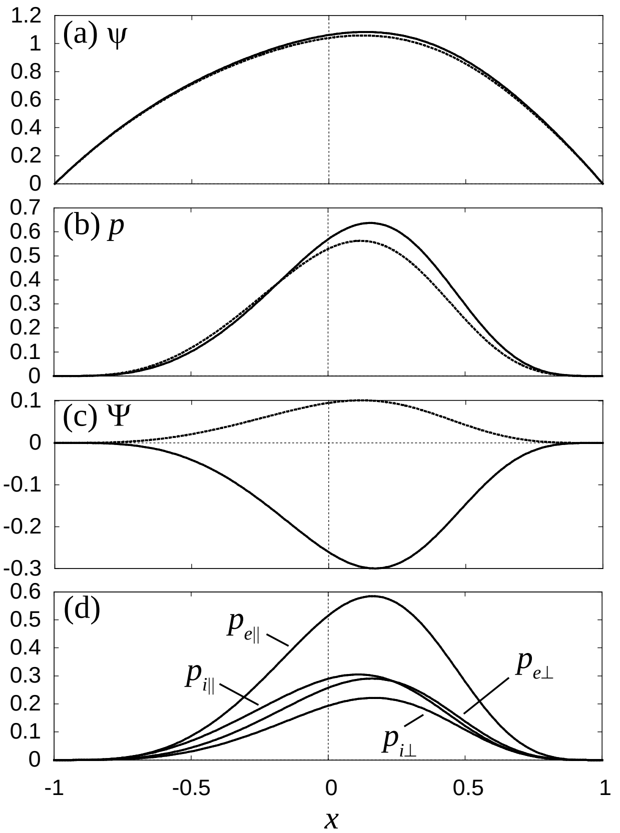

Fig. 1. Radial profiles in the midplane obtained from numerical solutions for the FLR two-fluid model. The solid (dashed) lines show the case where the poloidal directions of the $E\times B$ and the ion diamagnetic flows are the same (opposite).

3. Numerical Solutions

The reduced GS equations are solved numerically by means of the finite element method. The circular cross-section, the fixed boundary condition at the normalized minor radius r=1 and the up-down symmetry are assumed. Figure 1 shows the radial profiles at the midplane of (a) the magnetic flux ψ, (b) the total pressure p, (c) the ion stream function Ψ and (d) the parallel and perpendicular pressures for ions and electrons. The two-fluid effects induce the diamagnetic flows, which result in asymmetry of the equilibria with respect to the sign of the $E\times B$ flow. Higher order terms of quantities like the pressures and the stream functions show the shift of their isosurfaces from the magnetic surfaces due to effects of flow, two-fluid and pressure anisotropy. The parallel and perpendicular components of the pressures for ions and electrons [Fig. 1 (d)] are self-consistently determined and show their peaks in different positions from those of the magnetic flux and each other. The shift of the isosurfaces of the ion stream function from the magnetic flux is caused by the breaking of the frozen-in condition due to the two-fluid effect. However, we have found that it also depends on the FLR effect. Figure 2 shows that these shifts in the FLR two-fluid and the Hall MHD models are in opposite radial directions.

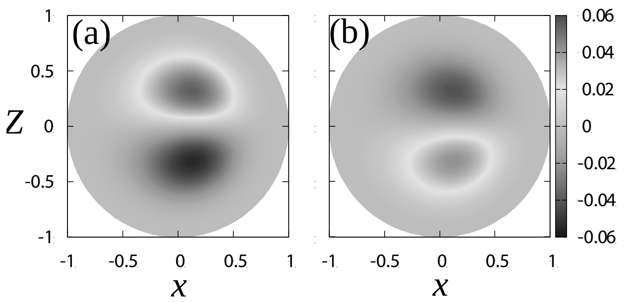

Fig. 2. Profiles of $(\nabla\psi\times\nabla\Psi)\cdot(R\nabla\varphi)$ in the poloidal cross-section for (a) the FLR two-fluid and (b) the Hall MHD models. This quantity is always zero in the single-fluid MHD model since $\Psi=\Psi(\psi)$.

References

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