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Fully-implicit MHD Simulation Using Nonconforming Vector Finite Elements
非適合型ベクトル有限要素法による陰的 MHD シミュレーション

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A novel vector finite element method is proposed to ensure that vector variables in the MHD equations satisfy the divergence-free and the curl-free constraints exactly in a general coordinate system. The method called 'Nonconforming Vector Finite Element Method' were implemented in a single-fluid resistive MHD code and validations were performed.

1 Introduction

For long time simulations of MHD instabilities, it is essential to use a scheme satisfying the divergence-free constraint on the magnetic field ($\nabla \cdot \boldsymbol{b} = 0$). Another constraint is also important for the analysis of MHD modes close to the marginal stability. Fluid is nearly incompressible and necessary to satisfy the condition $\nabla \cdot \boldsymbol{v} \sim 0$ below the marginal stability. These constraints are quite important because unphysical spurious modes emerge and interact with physical modes as violation of such a constraint. In this paper, we propose a novel finite element method to ensure that vector variables in the MHD equations satisfy the divergence-free and the curl-free constraints exactly in a general coordinate system. The method, named 'Nonconforming Vector Finite Element Method', consists of two ideas. The first idea is that different types of formulations (basis functions) are used to describe covariant and contravariant components of a vector. A basis function of a covariant vector is determined in accordance with the discrete 'curl' operator, and that of a contravariant vector is in accordance with the discrete 'divergence' operator to ensure both div-curl and curl-grad operators are identically zero. Since the basis function is different among covariant and contravariant components, transform between them is not defined by the local metric tensor. Therefore, we introduce the second idea that the equation of the covariant-contravariant metric transformation is substituted into the weak form in which norm conserving condition is imposed. We call this method 'nonconforming' in the same manner as in FEM theory. The proposed formulation were implemented in a single-fluid resistive MHD code and validations were performed.

2 Simulation Model

2.1 Nonconforming Vector Finite Element Formulation

We focus attention in this study on the cylindrical tokamak. Vector variables in the MHD equations are represented as a combination of radial basis functions. We adopt different basis functions for each component of a covariant vector (a_μ for $\mu = s, \theta, \phi$) and a contravariant vector (A^μ),

$$a_s = \sum_j a_{s,j-1/2} c_{j-1/2}(s), \tag{1}$$

$$\begin{pmatrix} a_\theta \\ a_\phi \end{pmatrix} = \sum_j \begin{pmatrix} a_{\theta,j} \\ a_{\phi,j} \end{pmatrix} e_j(s), \tag{2}$$

$$A^s = \sum_j A^s_j e_j(s), \tag{3}$$

$$\begin{pmatrix} A^\theta \\ A^\phi \end{pmatrix} = \sum_j \begin{pmatrix} A^\theta_{j-1/2} \\ A^\phi_{j-1/2} \end{pmatrix} c_{j-1/2}(s), \tag{4}$$

where $e_j(s)$ is the linear finite element and $c_{j-1/2}(s)$ is the piecewise constant element. Such a formulation makes the divergence-free and the curl-free constraints satisfiable everywhere.

2.2 Fully Implicit Method

Explicit time stepping schemes are computationally expensive for the free boundary simulation in which functions have a fairly robust profile through the plasma surface. Fully implicit approach is more appropriate, and the backward differentiation (BDF) algorithm is implemented in this work for the linear MHD stability analysis.

3 Simulation Results

Numerical simulations of MHD instabilities are carried out to check the validity of our code. The mode structure and growth rate are confirmed to be consistent with that obtained from the linear MHD eigenvalue solver using the conventional FEM formulation in Ref. [1] as shown in Fig. 1. The divergence constraint of the magnetic field, $\nabla\cdot\boldsymbol{b}=0$, is examined during the mode growth. The measurement in Fig. 2 indicates that it is satisfied with an error of only $\sim 10^{-12}$ due to the discretization of the spatial derivatives. Another divergence constraint is examined for the velocity field in Fig. 3. The incompressibility condition, $\nabla\cdot\boldsymbol{v}=0$, is violated around the resonant surface position, but its amplitude tends towards zero as Suydam index parameter is varied towards the marginal stability limit.

In the free boundary simulation by using the pseudo-vacuum model [2], the vacuum is modeled as a highly resistive, low density plasma, and the density and resistivity profiles have a steep gradient at the plasma surface. Fully-implicit method allows the use of as large time steps as Alfven time scale given by the core plasma parameters even when the simulation is run with the pseudo-vacuum model. It is verified that the simulation with the density ratio of 1/100 and the resistivity ratio of 10^6 does not fail.

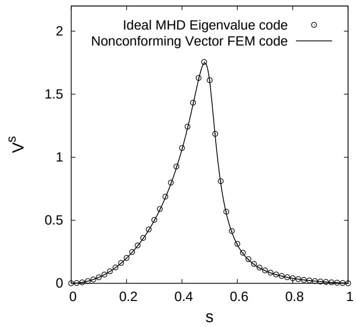

Fig. 1: The radial profile of the m/n=2/1 Suydam mode. The resonant surface, q=2, is located at s=0.5, where the Suydam index D=0.588. 500 radial grid points are used and the amplitude is normalized by the norm $\langle\boldsymbol{v},\boldsymbol{v}\rangle^{1/2}$. The growth rate -5.632×10^{-3} for the ideal MHD parameter case is shown to be consistent with -5.858×10^{-3} obtained from the ideal MHD eigenvalue code.

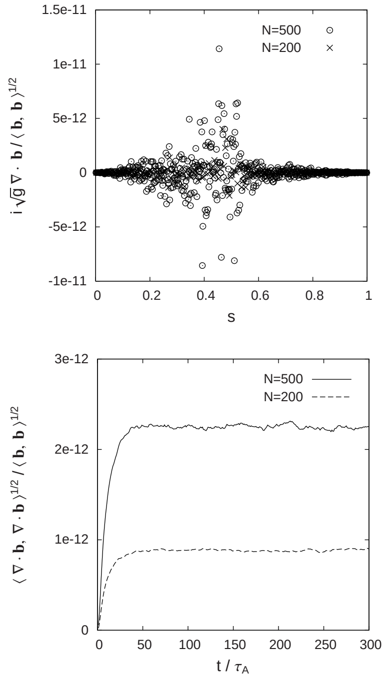

Fig. 2: The radial profile of the divergence of the magnetic field (upper fig.) and the temporal variation of their norm(lower fig.) for two different cases of grid points(N).

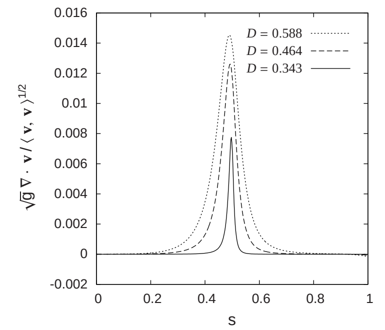

Fig. 3: The radial profile of the divergence of the flow velocity. The incompressibility condition is violated around the resonant surface position, $s=0.5$, but the amplitude tends towards zero as Suydam index parameter(D) is varied towards 1/4, i.e., Suydam criterion.

References

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[2] G.Kurita, M.Azumi, et al.: Nucl. Fusion **26** (1986) 449.