23P053-P

Simulation of two-dimensional transport in tokamak plasmas for integrated analysis of core and peripheral plasmas
トカマクプラズマの統合解析に向けた二次元輸送シミュレーション

Haruki Seto, Atsushi Fukuyama
瀬戸春樹, 福山淳

Department of Nuclear Engineering, Kyoto University, Sakyo-ku, Kyoto 606-8501, JAPAN
京都大学工学研究科原子核工学専攻 〒606-8501　京都市左京区吉田本町

In order to describe the behavior of tokamak plasmas in both core and peripheral regions self-consistently, two-dimensional transport modeling is desirable and becoming feasible. In the present study, we have formulated more rigorous transport equations with poloidal-angle dependence from Braginskii's equations for two-dimensional transport analysis. The set of equations is composed of continuity equation, equation for velocity including the neoclassical viscosity, and equation for energy transport for each species. Preliminary numerical results of two-dimensional transport analysis will be presented.

1 Introduction

In most of conventional core transport simulations in tokamaks, the particle density and the temperature are assumed to be almost constant on a magnetic surface, and flux-averaging method is employed to describe transport phenomena as a one-dimensional problem, since the transport along the field line is very fast. On the other hand, transport in a peripheral SOL-divertor plasma is usually described as a two-dimensional problem with simplified transport models and plasma flow, for example SOLDOR [1] and B2.5 [2], since variation of quantities along the field line is large and important to understand transport process in peripheral region. Recent remarkable progress in computational resources, however, has made more rigorous two-dimensional simulation of tokamak plasmas feasible. In the present study, we formulate a set of two-dimensional transport equations with the neoclassical viscosity [3] in magnetic flux coordinate system (MFCS) $(\xi_1, \xi_2, \xi_3) = (\rho, \chi, \zeta)$ from Braginskii's equations [4] to develop an integrated two-dimensional transport simulation code for the entire tokamak plasmas including both core and peripheral plasmas.

2 Assumptions

In the present study, the following four assumptions have been made to derive the transport equations. The first is that the plasma has toroidal axisymmetry, which means that all physical quantities are independent of the toroidal angle variable. The second is that the quantities related to MHD equilibrium depend only on the flux label ρ; the poloidal flux function ψ, the toroidal flux function ψ_T, the toroidal current function I, the electrostatic potential ϕ, the total plasma pressure p, and the rotational transform ι. This assumption makes it possible to reduce the magnetic equilibrium problem to two-dimensional. The third is that phenomena with the Alfvén time scale are much faster than the relaxation process such as diffusion of magnetic field and transport phenomena, which implies that the MHD equilibrium is attained much faster than the relaxation processes. Finally, the fourth is that the time derivatives of basis vectors are small enough to be ignored for simplicity when we take the time derivative of vector quantities.

3 Modeling of transport equations

The transport equations are derived from Braginskii's equations [4] and consist of the equation for particle density, momentum, and energy transport for both electron and ion in MFCS. The equation for particle density is the equation of continuity of Braginskii's equations.

The equation for momentum is derived from equation of motion of Braginskii's equation. For the compatibility with the neoclassical transport theory [3], we take three components of vector quantities $(\xi_1^{\mathrm{N}}, \xi_2^{\mathrm{N}}, \xi_3^{\mathrm{N}}) = (\rho, \|, \zeta)$, which indicates the radial direction, the field line direction and the toroidal direction respectively, $(\boldsymbol{e}^{\xi_1^{\mathrm{N}}}, \boldsymbol{e}^{\xi_2^{\mathrm{N}}}, \boldsymbol{e}^{\xi_3^{\mathrm{N}}}) = (\nabla\rho, \boldsymbol{B}/B, \nabla\zeta)$. And we assume that the force balance is attained in the radial and toroidal direction in the transport time scale. Therefore, taking the scalar product of the equation of motion and $\boldsymbol{e}^{\xi_i^{\mathrm{N}}}$, we obtain equations for momentum in each direction as follows.

1. radial direction

$$\begin{aligned} 0 = &-F_a^{\mathrm{kin},1} - \sum_{i=1}^{3} g^{1i}\frac{\partial p_a}{\partial \xi_i} - \sum_{i=1}^{3}\frac{1}{3}g^{1i}\frac{\partial N_a^{\mathrm{neo}}}{\partial \xi_i} \\ &+ N_a^{\mathrm{neo}}\kappa^{\rho} - g^{11}e_a n_a\frac{\partial\phi}{\partial\rho} + \sum_{i=1}^{3} C_a^{\mathrm{Lor},i} n_a u_a^{\xi_i^{\mathrm{N}}} \\ &\mp \frac{3}{2\Omega_{\mathrm{e}}\tau_{\mathrm{e}}}\frac{I}{\sqrt{g}B} n_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial\chi} + m_a S_a u_a^{\rho} \end{aligned} \tag{1}$$

2. parallel direction

$$\begin{aligned} \frac{\partial}{\partial t}\left(m_a n_a u_{a\|}\right) = &-\sum_{i=1}^{3} C_a^{\mathrm{kin},i} F_a^{\mathrm{kin},i} - \frac{\psi'}{\sqrt{g}B}\frac{\partial p_a}{\partial\chi} \\ &- \frac{\psi'}{\sqrt{g}B} N_a^{\mathrm{neo}}\frac{\partial \ln B}{\partial\chi} + \frac{2}{3}\frac{\psi'}{\sqrt{g}B}\frac{\partial N_a^{\mathrm{neo}}}{\partial\chi} \\ &\mp \frac{m_{\mathrm{e}} n_{\mathrm{e}}}{\tau_{\mathrm{e}}}\left(u_{\mathrm{e}\|} - u_{\mathrm{i}\|}\right) \mp 0.71\frac{\psi'}{\sqrt{g}B} n_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial\chi} \\ &+ m_a S_a u_{a\|} \end{aligned} \tag{2}$$

3. toroidal direction

$$\begin{aligned} 0 = &-F_a^{\mathrm{kin},3} - \frac{I\psi'}{\sqrt{g}B^2R^2} N_a^{\mathrm{neo}}\frac{\partial \ln B}{\partial\chi} + \frac{I\psi'}{\sqrt{g}B^2R^2}\frac{\partial N_a^{\mathrm{neo}}}{\partial\chi} \\ &+ N_a^{\mathrm{neo}}\kappa^{\zeta} + \frac{e_a\psi'}{R^2} n_a u_a^{\rho} \\ &\mp \frac{m_{\mathrm{e}} n_{\mathrm{e}}}{\tau_{\mathrm{e}}}\left\{\left(u_{\mathrm{e}}^{\zeta} - u_{\mathrm{i}}^{\zeta}\right) - 0.49\frac{I}{BR^2}\left(u_{\mathrm{e}\|} - u_{\mathrm{i}\|}\right)\right\} \\ &\mp \left[0.71\frac{I\psi'}{\sqrt{g}B^2R^2} n_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial\chi} + \sum_{i=1}^{3}\frac{3}{2\Omega_{\mathrm{e}}\tau_{\mathrm{e}}}\frac{\psi' g^{1i}}{BR^2} n_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial\xi_i}\right] \\ &+ m_a S_a u_a^{\zeta} \end{aligned} \tag{3}$$

where $F_a^{\mathrm{kin},i}$, g^{ij}, $\sqrt{g}$, N_a^{neo}, $C_a^{\mathrm{Lor},i}$ and $C_a^{\mathrm{kin},i}$ are the contravariant component of kinetic stress force, the contravariant metric tensor, Jacobian, quantity from the neoclassical viscosity force, the coefficient of Lorentz force term and the coefficient of kinetic stress force in MFCS respectively.

The equation for energy transport is obtained by transforming Braginskii's equation for energy transport into the advection-diffusion form

$$\frac{3}{2}\frac{\partial p_a}{\partial t} = -\nabla\cdot\left(p_a\boldsymbol{u}_{p_a} - n_a\overleftrightarrow{\chi}_a\cdot\nabla T_a\right) + Q_{p_a} \tag{4}$$

where $\boldsymbol{u}_{p_a} \equiv \frac{5}{2}\boldsymbol{u}_a + p_a^{-1}\boldsymbol{q}_{u_a}$ is the energy flow velocity, $Q_{p_a} \equiv \frac{3}{2}T_a S_a + \boldsymbol{u}_a\cdot\vec{\nabla}p_a - \overleftrightarrow{\pi}_a\cdot\nabla\cdot\boldsymbol{u}_a + Q_a$ the energy source, and $\overleftrightarrow{\chi}_a$ the diffusion tensor.

The set of transport equations is coupled with a set of equations for electromagnetic field in order to develop an integrated two-dimensional transport simulation code. The set of electromagnetic equations consists of Grad-Shaftranov equation, magnetic diffusion equation [5], and Poisson equation for static electric field. The two sets of equations are reduced to two-dimensional with axisymmetry and the finite element method is used to discretize the differential equations.

4 Preliminary simulation results

We will present preliminary numerical results of two-dimensional transport analysis, which we employ toroidal coordinate system $(\varrho, \theta, \varphi)$ instead of magnetic flux coordinate system.

Acknowledgment

This work is supported by Grant-in-Aid for Science Research (S) (202260171) from JSPS and G-COE program (J-051) from MEXT

Reference

[1] K. Shimizu *et. al*, J. of Nucl. Mater. 313-316(2003) 1277-1281

[2] V. A. Rozhansky *et. al*, Nucl. Fusion Vol. 41 387-401 (2001)

[3] S.P. Hirshman and D.J. Sigmar, Nucl. Fusion Vol.21 No.9 (1981) 1079

[4] S. I. Braginskii, in Reviews of Plasma Physics, Vol. 1 (Leontovich, M. A., Ed.), Consultants Bureau, New York (1965) 205.

[5] F. L. Hinton and R. D. Hazeltine, Rev. of Mod. Phys. Vol. 48 (1976) 239