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Analytical Formua of offset toroidal rotation in NTV
新古典トロイダル粘性によるオフセットトロイダル回転の解析式

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An analytic formula of offset toroidal rotation due to NTV is derived for e-ion-impurity plasma assuming negligible impurity viscous forces in parallel and toroidal directions. The formula is expressed in the radial coordinate $\rho=(\phi/\phi_a)^{1/2}a$ to see parametric dependence.

1. Introduction

Tokamak has toroidal symmetry and symmetry breaking via application of non-axisymmetric field produces dissipation of toroidal momentum by NTV [1] and produces offset toroidal rotation [2]. For toroidal rotation measurement, impurity (typically carbon) charge exchange recombination spectroscopy (CXRS) is used. In this case, we expect some difference of toroidal rotation between ion and impurity [3].

An analytic formula of offset toroidal rotation is derived assuming negligible impurity viscous forces in parallel and toroidal directions [4], [5]. In this talk, we show these expressions in the radial coordinate $\rho=(\phi/\phi_a)^{1/2}a$, where ϕ and a are toroidal flux and minor radius, respectively, to show its importance in high β_p regime, which is relevant for steady state operation.

2. Analytical form of offset ion toroidal rotation

The 0-th order ion force balance is give by $0 = eZ_i n_i(\boldsymbol{E} + \mathbf{u}_i \times \boldsymbol{B}) - \nabla P_i$. The radial component of above equation can be obtained by taking inner product with tangent vector $\partial\boldsymbol{x}/\partial\psi$ and using the identity $\partial\boldsymbol{x}/\partial\psi\cdot\nabla\psi=1$ in the flux coordinates (ψ, θ, ζ) [6], where $\boldsymbol{x}$ is position vector as follows,

$$\boldsymbol{u}_i \cdot \nabla\zeta = -\left[\frac{d\Phi}{d\psi} + \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi}\right] + q\boldsymbol{u}_i \cdot \nabla\theta \quad (1)$$

Here, $q=d\Phi/d\psi$ is safety factor. This equation is expressed by using $B_\theta=R^{-1}d\psi/d\rho$, $u_{i\zeta}=R\boldsymbol{u}_i\cdot\nabla\zeta$, $u_{i\theta}=R\boldsymbol{u}_i\cdot\nabla\theta$ as follows,

$$u_{i\zeta} = -\frac{1}{B_\theta}\left(\Phi' + \frac{P_i'}{eZ_i n_i}\right) + qu_{i\theta} \quad (2)$$

Here, ' denotes derivative with respect to ρ. This equation implies that role of pressure gradient and radial electric field in the flow balance becomes more important in high β_p regime, where B_θ is relatively small.

Tokamak is an axisymmetric system and the particle diffusion is ambipolar and its radial electric field $-d\Phi/d\rho$ is free, so that it needs to be determined from the toroidal momentum diffusion equation. When symmetric breaking occurs by the application of non-axisymmetric magnetic field, particle diffusion is no longer ambipolar. And the radial electric field is determined so that non-ambipolar flux becomes zero. Shaing [7] derived non-ambipolar flux=0 condition as follows,

$$\Phi' + \frac{1}{eZ_i}\frac{P_i'}{n_i} = -\frac{\lambda_2}{eZ_i\lambda_1}T_i' \quad (3)$$

This leads to following formula for offset toroidal rotation $u_{i\zeta 0}$,

$$u_{i\zeta 0} = \frac{\lambda_2}{eZ_i\lambda_1}\frac{T_i'}{B_\theta} + qu_{i\theta 0} \quad (4)$$

Here $u_{i\theta 0}$ is a residual poloidal rotation mainly driven by thermal force dT_i/dr and is given by Kim [3] as follows,

$$u_{i\theta 0} = -\frac{K_1 F(\boldsymbol{B}\cdot\nabla\theta)}{eZ_i\langle B^2\rangle B_\theta}T_i' \quad (5)$$

Substitution of (5) to (4) gives following formula of offset toroidal rotation of bulk ion,

$$u_{i\zeta 0}=\left[\frac{\lambda_2}{eZ_i\lambda_1}-\frac{qK_1F(\boldsymbol{B}\cdot\nabla\theta)}{eZ_i\langle B^2\rangle}\right]\frac{T_i'}{B_\theta} \qquad (5)$$

In the cylindrical approximation, $q=rB_t/RB_\theta$, $F=RB_t$, $<B^2>=B_t^2$. So, following approximate analytical formula is obtained considering $\lambda_2/\lambda_1=3.54$ as given in [8].

$$u_{i\zeta 0}=\frac{3.54-K_1}{eZ_iB_\theta}\frac{dT_i}{dr} \qquad (6)$$

The approximate cylindrical expression in [5] is not correct. Since dT_i/dr is negative, offset toroidal rotation in NTV is counter to the plasma current. The value of K_1 in collisionless plasma is ~1.1 near center and 0.5 at r/R~0.3 as seen in Fig.2 of [3]. So, this changes theoretical coefficient in 1/ν regime $k_c=u_{i\zeta 0}/[(dT_i/dr)/eZ_iB_\theta]$ becomes closer to experimental observation including this residual poloidal flow correction.

Since the measurement of roroidal rotation corresponds to impurity toroidal rotation, there is finite difference between ion and impurity parallel flow as follows,

$$\langle Bu_{//I}\rangle=\langle Bu_{//i}\rangle-\frac{1.5K_2B_\zeta}{eZ_iB_\theta}T_i' \qquad (7)$$

The local toroidal flow $u_{a\zeta}=R\boldsymbol{u}_a\cdot\nabla\zeta$ of ion ($a=i$) and impurity ($a=I$) is given by,

$$u_{a\zeta}=\frac{B_\zeta\langle Bu_{//a}\rangle}{\langle B^2\rangle}-\left[1-\frac{B_\zeta^2}{\langle B^2\rangle}\right]\frac{1}{B_\theta}\left(\Phi'+\frac{P_a'}{eZ_an_a}\right) \qquad (8)$$

We obtain following form of offset troidal rotation for impurity species.

$$u_{I\zeta 0}=\left[\frac{\lambda_2}{eZ_i\lambda_1}-\frac{qK_1F(\boldsymbol{B}\cdot\nabla\theta)}{eZ_i\langle B^2\rangle}-\frac{1.5K_2B_\zeta^2}{eZ_i\langle B^2\rangle}\right]\frac{T_i'}{B_\theta}$$
$$-\left[1-\frac{B_\zeta^2}{\langle B^2\rangle}\right]\frac{1}{B_\theta}\left(\frac{P_I'}{eZ_In_I}-\frac{P_i'}{eZ_in_i}\right) \qquad (9)$$

In the large aspect ratio cylindrical plasma, offset toroidal rotation measurement corresponds to following impurity offset toroidal rotation.

$$u_{I\zeta 0}=\frac{3.54-K_1-1.5K_2}{eZ_iB_\theta}\frac{dT_i}{dr} \qquad (10)$$

If we plot these value in Garofalo's paper [2], measurement becomes closer as shown below.

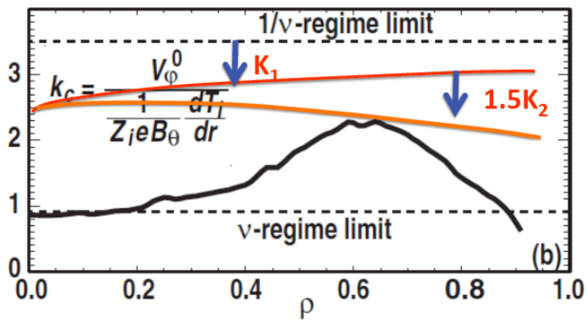

Fig. 1 Offset toroidal rotation in DIII-D [2] and correction of theoretical value in this talk.

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