Spectroscopy of Hydrogenic Spectral Lines in Plasmas: Review of the Latest Analytical Progresses*)

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Broadening of hydrogenic spectral lines is an important tool in spectroscopic diagnostics of various laboratory and astrophysical plasmas. We review recent analytical advances in five areas. First, we review a new method for spectroscopic diagnostics of tokamak edge plasmas based on a peculiar Stark broadening of hydrogen or deuterium spectral lines emitted by the injected neutral beam. Second, we review the analytical solution for the magnetic-field-caused narrowing of hydrogenic spectral lines under a circularly polarized electromagnetic wave. Third, we review analytical results concerning the Stark-Zeeman broadening of the Lyman-alpha line in plasmas. Forth, we review the effect of helical trajectories of electrons in strongly magnetized plasmas on the width of hydrogen/deuterium spectral lines. Fifth, we review recent analytical advances in the area of the intra-Stark spectroscopy: three different new methods, based on the emergent phenomenon of the Langmuir-wave-caused structures ("L-dips") in the line profiles, for measuring super-strong magnetic fields of the GigaGauss range developing during relativistic laser-plasma interactions. We also review the rich physics behind the L-dips phenomenon – because there was a confusion in the literature in this regard.

Keywords: broadening of hydrogenic spectral lines in plasmas, strong magnetic field, intra-Stark spectroscopy, Langmuir-wave-caused structures, relativistic laser-plasma interactions, electron cyclotron waves

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1. Introduction

Lineshape-based diagnostics of laboratory and astrophysical plasmas use the spectral line broadening as an important instrument – see, e.g., books [1–7] (in the reverse chronological order) published in the last three decades and references in these books. Hydrogenic spectral lines are particularly perceptive to various fields inside plasmas (the fields developing inside plasmas or caused by external sources). These lines yield a valuable diagnostic information.

We concentrate on the analytical progresses in this research area for the following reasons: 1) analytical studies offer the physical insight in the phenomena – in distinction to simulations; 2) analytical studies can capture emergent phenomena, while simulations usually cannot; 3) codes can fail due to the inadequate verification and validation – see paper [8] for examples.

2. New Method for Spectroscopic Diagnostics of Tokamak Edge Plasmas Based on a Peculiar Stark Broadening of Hydrogen or Deuterium Spectral Lines Emitted by the Injected Neutral Beam

In paper [9] one of us analyzed the situation (vital for the magnetic fusion devices, most importantly for ITER), where a highly energetic beam of atoms of hydrogen or deuterium is injected into a strongly-magnetized plasma. In paper [9] there was employed an advanced formalism for obtaining analytical results for the anisotropic Stark broadening of the corresponding spectral lines emitted by the atoms of the beam. As a result, there were revealed *three fundamental new features*, as follows.

First, it was demonstrated that in the case of ITER, both the electrons and ions of the plasma *contribute equally* to the Stark broadening, *as if the electron density were doubled*. This *counterintuitive result* was never previously obtained in the literature.

Second, it was revealed that in the case of ITER, there is *no dependence* of this anisotropic Stark broadening *on the reduced mass* of the "radiator-perturber" pair, while the isotropic Stark broadening (to which the overwhelm-

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ing majority of the literature was devoted) depended on the reduced mass of the pair.

Third, it was demonstrated that in the case of ITER, there is a *strictly linear* dependence of the anisotropic Stark broadening on the electron density N_e, while the isotropic Stark broadening (previously investigated in the variety of the literature) was not strictly-linear with respect to N_e. This is *yet another counterintuitive result*. Below are just few details.

The design of ITER contains a deuterium beam, having the energy E = 1 MeV, and a hydrogen beam, having the energy E = 0.87 MeV. If the velocity of the beam atoms is **v**, then from the viewpoint of the beam atoms, the velocity of the plasma electrons and the velocity of the plasma ions has the directional component −**v**: the spectral lines of the beam atoms are subjected to the anisotropic Stark broadening (first brought up by Seidel [10], who used the formalism of conventional theory of the Stark broadening to calculate the broadening of the spectral lines of the beam atoms by plasma ions). Later in paper [11] the authors utilized an advanced analytical formalism.

In paper [9] of year 2024, there was analyzed the case relevant to ITER. In the reference frame moving with the beam atoms, the anisotropic Stark broadening of their spectral lines takes place at the presence of the magnetic field **B** and of the Lorentz field $\mathbf{F} = \mathbf{v}\mathrm{x}\mathbf{B}/c$. Compared to the setup in paper [11], there were two important distinctions in the setup of paper [9]: 1) the presence of the two fields, **F** and **B**; 2) the analysis included the case where the beam atoms are much lighter than the plasma ions.

From the practical viewpoint, in paper [9] there was put forward a new spectroscopic method for deducing the electron density N_e from the experimental width of the Paschen-alpha line radiated by the atoms of the injected beam. For the conditions of the ITER egde plasmas and for the planned beams, the Stark broadening of the Paschen-alpha line, radiated by the beam atoms, occurs in the regime of the isolated components. So, for deducing N_e it should be sufficient to determine experimentally just the Stark width $\Delta\lambda_S$ of the most intense component of the Paschen-alpha line, which is its central Stark component. Then the experimental electron density N_e can be deduced from the following simple formulas derived in paper [9]: $\Delta\lambda_S(\mathrm{nm}) = 0.0124\,[N_e(\mathrm{cm}^{-3})/10^{14}]$ for the deuterium beam of the energy 1 MeV or $\Delta\lambda_S(\mathrm{nm}) = 0.0129\,[N_e(\mathrm{cm}^{-3})/10^{14}]$ for the hydrogen beam of the energy 0.87 MeV.

This *new method has an advanta*ge over the traditional method based on measuring the isotropic Stark width $\Delta\lambda_{s,\mathrm{isotropic}}$ of the spectral lines radiated by the deuterium or hydrogen atoms not linked to the beam. Namely, for the conditions at the edge plasmas of ITER, $\Delta\lambda_{s,\mathrm{isotropic}}$ generally depends on three parameters (N_e, T_e, and T_i, the latter two quantitites being the electron and ion temperature, respectively), so that for deducing N_e it would be necessary to measure the width of *three* spectral lines. In distinction, in the new method, it should be sufficient to determine experimentally the width of just one spectral line (the Pashen-alpha line). This is because in the case of ITER, the width $\Delta\lambda_S$ depends only on N_e.

3. Magnetic-Field-Caused Narrowing of Hydrogenic Spectral Lines under a Circularly Polarized Electromagnetic Wave: the Analytical Solution

In paper [12] there was considered a hydrogen atom subjected to the electric field $\mathbf{F}_0$ rotating with the constant by magnitude angular velocity $\mathbf{\Omega}$. The analytical solution for the splitting of hydrogen lines was obtained in paper [12] by analyzing the situation in the reference frame rotating with the angular velocity $\mathbf{\Omega}$. In this reference frame, the problem got reduced to a hydrogen atom in the static electric field orthogonal to a fictitious static magnetic field $\mathbf{B}_{\mathrm{fict}} = -2\,m_e c\mathbf{\Omega}/e$. The analytical solution for the latter problem was known from paper [13].

In paper [14] the authors introduced in the above physical system a *true magnetic field* **B** parallel or antiparallel to the angular velocity vector $\mathbf{\Omega}$. Then the interaction term took the form $V = e\mathbf{r}\mathbf{F}_0 + \mu_B\mathbf{L}\mathbf{B}_{\mathrm{eff}},$ where the effective magnetic field consisted of the following two terms: $\mathbf{B}_{\mathrm{eff}} = \mathbf{B}_{\mathrm{fict}} + \mathbf{B}$. So, the problem came to be mathematically equivalent to the hydrogenic atom or ion subjected to a circularly polarized wave of the following effective angular velocity: $\mathbf{\Omega}_{\mathrm{eff}} = -(\mu_B/\hbar)(\mathbf{B}_{\mathrm{fict}} + \mathbf{B}) = \mathbf{\Omega} - \mu_B\mathbf{B}/\hbar$.

In paper [14] there was found the analytical solution, according to which the separation between the split components of the line in the frequency scale is equal either to Ω or to the following quantity $\omega = \{[\Omega - (\mu_B B/\hbar)\mathrm{sign}(\mathbf{B\Omega})]^2 + 9n^2\hbar^2F_0^2/(4Z^2m_e^2e^2)\}^{1/2}$. The primary separation is by this quantity ω.

It is easy to see that the separation ω *decreases* if the true magnetic field **B** is parallel to $\mathbf{\Omega}$ (so that it is antiparallel to $\mathbf{B}_{\mathrm{fict}} = -\hbar\mathbf{\Omega}/\mu_B$). This is a *counterintuitive* result.

When the true magnetic field **B** is parallel to the angular velocity vector $\mathbf{\Omega}$ and has the absolute value $B = 2\,m_e c\Omega/e$, the primary separation between the components of the spectral line diminishes to $3n\hbar F_0/(2Zm_e e)$. In other words, in this case the primary separation decreases to its minimum equal just to the Stark splitting – because the true magnetic field cancels out the fictitious magnetic field. Interestingly enough, in this case the total intensity of the hydrogen line Lyman-alpha diminishes by 40%, what is another *counterintuitive* result.

Potential practical applications can be relevant to the spectroscopic diagnostic of the electron cyclotron waves in plasmas of the magnetic fusion machines and to the relativistic laser plasma interactions.

4. Analytical Results for the Stark-Zeeman Broadening of the Lyman-Alpha Line in Plasmas

The focal point of this section is the π-component of the hydrogenic Ly-alpha line – because the properties of this component are unique. In paper [15] it was shown that for tokamaks edge plasmas, the π-component of the Ly-alpha line allows to determine experimentally the effective charge of ions Z_{eff} – in distinction to the σ-component of this line or any component of any other hydrogenic spectral line.

In [16] it was demonstrated that the π-component of the Ly-alpha line, radiated by the atoms of the injected neutral beam of deuterium or hydrogen, allows the experimental determination of both Z_{eff} and the pitch angle (i.e., the angle between the magnetic field and the velocity of the beam.

In paper [17] of 2023, there was revealed another important property of the π-component of the Ly-alpha line. In the situation where the Zeeman effect predominates over the Stark effect, the broadening of this line component came out to be controlled by the Stark effect (specifically, by the linear Stark effect) - almost without the dependence on the field **B** (in distinction to the σ-component of this line or any component of any other hydrogenic spectral line).

Thus, the experimental measurements of the Stark width of the π-component of the Lyman-alpha line can be employed in this situation for deducing the ion/electron density or the root-mean-square field of a Low-frequency Electrostatic Plasma Turbulence, LEPT, (examples of the latter being, e.g., Bernstein modes, or ion-acoustic waves, or lower-hybrid waves) – despite the Zeeman effect predominates over the Stark effect. This is a *counterintuitive* result.

We note in passing that after paper [17] was published in January 2023, paper [18] was published by another author in September 2023, where there was an unintentional overlap with some equations from paper [17].

5. Effect of Helical Trajectories of Electrons in Strongly Magnetized Plasmas on the Width of Hydrogen/Deuterium Spectral Lines: Beyond the Perturbation Theory

This effect was first investigated analytically in papers [19, 20] for plasmas where the magnetic field **B** is strong enough for the dynamical Stark width of these spectral lines to be controlled by the so-called adiabatic Stark width, while the so-called nonadiabatic Stark width is completely suppressed by the field **B**. This is the case, e.g., for DA and DBA white dwarfs.

In papers [19,20], the analytical results were produced under the conventional theory of the impact Stark broadening: by conducting calculations in the 2nd order of the Dyson *perturbation* expansion. The main result: the dynamical Stark broadening had no dependence on the B (for appropriately strong B).

Then in paper [21] of 2024, there were conducted the corresponding *non-perturbative* analytical calculations – the calculations equivalent to taking into account all orders of the Dyson perturbation expansion. The obtained results turned out to be different from those from papers [19, 20]: the dynamical Stark broadening was found to depend on B even for the strong B.

These new analytical results were produced for strong magnetic fields $B > B_{cr} = 2T_e/(3|X_{\alpha\beta}|e\lambda_c) = 9.2x\ 10^2 T_e(eV)/|X_{\alpha\beta}|$ Tesla, where $X_{\alpha\beta} = n_a q_\alpha - n_b q_\beta$ is expressed via the parabolic quantum numbers of the upper ($n_a q_\alpha$) and lower ($n_b q_\beta$) Stark sublevels. According to the astrophysical observations, in the white dwarfs plasmas, B is in the range from 10^3 Tesla to 10^5 Tesla, thus clearly surpassing the above critical value. Indeed, since in the white dwarfs plasmas radiating hydrogen lines, the electron temperature is typically $T_e \sim 1$ eV, then $B_{cr} \sim 10^3/|X_{\alpha\beta}|$ Tesla $< 10^3$ Tesla.

Therefore, the above new analytical results call for the revised interpretation of the hydrogen lines, from the DA and DBA white dwarfs. Below we also point out some false statements in the literature on the role of helical trajectories of the perturbing electrons.

In paper [22] its author performed some very limited simulations of the effect of Helical Trajectories of the Perturbing Electrons (HTPE) on some hydrogen lines and made several false statements about the analytical results of our earlier paper [19] (not concerning our latest paper [21]) – as listed below. First, the analytical results of the effect of HTPE on the Stark width of hydrogen spectral lines, published in our paper [19] and then represented in our review article [20], were produced under the *conventional* theory of the impact Stark broadening. In spite of this obvious fact, the author of paper [22] falsely claimed that the results in paper [19] were produced under the so-called *generalized* theory of Stark broadening.

Second, but most importantly, he falsely claimed in paper [22] that presumably our paper [19] predicted the HTPE-caused increase of the Stark width of the lines Balmer-beta, Balmer-delta, and Balmer-epsilon at high densities, while his simulations yielded a decrease of those stark widths. To back up his false claim, the author of paper [22] used specific examples of the above three Balmer lines at the electron density $N_e = 2x10^{17}$ cm^{-3} and the electron temperature $T_e = 1$ eV.

However, in fact, in paper [19] there was clearly stated that whether taking into account the HTPE will increase or decrease the width of the Stark components of any hydrogen line, depends on the value of the following dimensionless controlling parameter $D = 5.57x10^{-11}|X_{\alpha\beta}|\ [N_e(cm^{-3})]^{1/2}/T_e(eV)$. Namely, in paper [19] there was clearly stated that the allowance for the HTPE will increase the width of a Stark component if $D > 0.44$, but would decreases its width if D < 0.44. For the plasma parameters chosen by the author of paper [22], the above formula reduces to $D = 0.025\ |X_{\alpha\beta}| < 0.44$. For the overwhelming majority of the Stark components of the above three Balmer lines, it is easy to find that D < 0.44. Thus, the actual prediction from paper [19] for these lines at the plasma parameters chosen in paper [22] will be the *decrease* of the Stark width, rather than the increase of the Stark width falsely stated in paper [22] about the prediction from paper [19].

6. Recent Analytical Advances in the Area of the Intra-Stark Spectroscopy: Three Different New Diagnostic Methods, Based on the Emergent Phenomenon of the Langmuir-Wave-Caused Structures ("L-dips") in the Line Profiles, and Review of the Rich Physics Behind the L-Dips Phenomenon

The research area of the *Intra-Stark Spectroscopy* (ISS) has in its foundation the Langmuir-wave-caused "dips" in spectral line profiles. The word "dip", used for briefness, relates to a highly-localized *structure* in the line profile – namely, to the structure having a local minimum of the intensity surrounded by two local peaks ("bumps").

This is an *emergent phenomenon* stemming from *Multifrequency Multiquantum Nonlinear Dynamic Resonances* (MMNDR) – as explained in papers [23, 24] and books [2, 7]. The analytical expectations of this emergent phenomenon were proven to occur in copious experiments performed by variety of experimental groups working at diverse types of plasma machines, as well as in astrophysical observations. These experiments and observations covered the span of the electron densities of approximately 10 orders of magnitude: from $10^{13}\ \mathrm{cm}^{-3}$ to $3\mathrm{x}10^{22}\ \mathrm{cm}^{-3}$. In all of them (including, in particular, the high-precision experiments at the gas-liner pinch [25, 26], where plasma parameters have been determined by the coherent Thomson scattering independently of the experimental spectral line profiles), the Langmuir-wave-caused highly-localized structures were consistently detected, identified, and utilized for plasma diagnostics.

Despite this, a recent paper [27] showed that there is still the confusion on the ISS. For this reason, in our review paper [28] we clarified that confusion for the benefit of the entire research community and brought up to its attention new analytical results on the ISS. Here are some details.

For the Langmuir-wave-caused structures in spectral line profiles, the underlying physics is the following. The starting point is the electric field $\mathbf{E}(t) = \mathbf{F} + \mathbf{E}_0 \cos(\omega t)$, where $\mathbf{F}$ is the quasistatic part of the plasma electric field. The contributors to the field $\mathbf{F}$ are the LEPT and/or the quasistatic part of the ion microfield.

For the overwhelming majority of the angles between vectors $\mathbf{F}$ and $\mathbf{E}_0$, the total field $\mathbf{E}(t)$ is *librating*. The frequency spectrum of the field $\mathbf{E}(t)$ is $u\omega$; $u = 1, 2, 3, \ldots$.

Let F_{eff} be the absolute value of $\mathbf{E}(t)$ averaged over the period of the libration: $F_{\mathrm{eff}} = \langle|\mathbf{E}(t)|\rangle$. If the libration of $\mathbf{E}(t)$ would be first neglected, the energy levels of a radiating hydrogenic atom or ion (the radiator) would undergo the splitting in 2n – 1 Stark sublevels, the separation between the sublevels being (in the atomic units) $\Omega = 3nF_{\mathrm{eff}}/(2Z_r)$, where Z_r is the nuclear charge of the radiator and n is the principal quantum number. The Stark sublevels are labeled by the electric quantum number $q = n_1 - n_2$, n_1 and n_2 being the parabolic quantum numbers. The composite system "radiator + field" can be most conveniently analyzed in terms of *quasienergy states* (introduced by Zeldovich [29] and Ritus [30]) of the whose quasienergies $Q = \Omega + v\omega$, $v = 0, \pm1, \pm2, \pm3, \ldots$.

Now let us allow for the time-dependent component of the librating field $\mathbf{E}(t)$. In the situation, where $\Omega = u\omega$, $u = 1, 2, \ldots$, there happen numerous resonances between the harmonics of the librating field and all quasienergy states of the quasienergies $Q = \Omega + v\omega$. Thus, the resonances are *multifrequency* and *multiquantum* (in terms of the the Langmuir field quanta). These resonances lead to the *degeneracy of all quasienergy states*. Namely, the quasienergy harmonics, originating from 2n – 1 Stark substates of the radiator, get superimposed.

In review [28] we wrote:

"In this multiquantum multifrequency resonance, each degenerate quasienergy state is a superposition of several quasienergy harmonics originating from Stark sublevels of different values of the electric quantum number q.

The Stark sublevel of some value of q is coupled by the dipole matrix element with the sublevels of q + 1 and q – 1.

As a result of this coupling, there occurs an additional splitting of *all* quasienergy harmonics. This splitting has an analogy with the Rabi splitting, but it is its generalization for the case of the multiquantum multifrequency resonances. The additional splitting of all quasienergy harmonics is generally a *nonlinear* function of the Langmuir field amplitude E_0."

The resonance values of F_{eff}, determined by the resonance condition $3nF_{\mathrm{eff}}/(2Z_r) = u\omega$, $(u = 1, 2, 3, \ldots)$, translate into an array of specific locations $\Delta\lambda_{\mathrm{dip}}$ in the hydrogenic spectral line profile. The positions $\Delta\lambda_{\mathrm{dip}}$ are well-defined functions of the Langmuir wave frequency $\omega = (4\pi e^2 N_e/m_e)^{1/2}$ and thus, of the electron density: $\Delta\lambda_{\mathrm{dip}} = aN_e^{1/2} + bN_e^{3/4}$, the coefficients a and b being controlled by the charges of the radiating and perturbing ions and by the quantum numbers. From the experimental locations of these structures, one can deduce a very accurate value of N_e.

We point out that in the high-precision experiments by the Kunze group at the gas-liner pinch [25, 26], where plasma parameters were determined by the coherent

Thomson scattering unrelated to the experimental line profiles, first, it was reliably confirmed the existence of the Langmuir-wave-caused structures, the evolution of their positions, as the electron density varied, being consistent with the theory. Second, the detailed bump-dip-bump structure was confirmed experimentally for the first time. Third, it was also shown that the experimental positions $\Delta\lambda_{dip}$ yielded just as accurate values of N_e, as measured by the coherent Thomson scattering. All theoretical expectations concerning the MMNDR and the ensuing bump-dip-bump structures in profiles of hydrogenic lines, have been confirmed and utilized for plasma diagnostics at copious experiments around the world by various experimental groups.

In recent papers [31–33] there were produced new analytical results on the ISS: namely three new, L-dip-based spectroscopic methods for the experimental determination of the *GigaGauss magnetic fields* developing at the process of the relativistic laser-plasma interactions. The idea of the 1st new method was described in paper [34], as follows.

"It is possible to select a pair of the L-dips, such that the location of one of the two L-dips is unaffected by the magnetic field while the location of the other of the two L-dips is shifted by the magnetic field. Then from the *relative separation* of the two L-dips it is possible to determine the magnetic field."

The next paper [35] concentrated on the effect of the GigaGauss magnetic fields on the *halfwidth* of the L-dips. On strongly magnetized plasmas, the L-dip halfwidth, which at B = 0 was controlled only by the Langmuir wave amplitude E_0, becomes controlled by B, E_0, and F.

The 3rd new method [36] was based on the B-field-caused shift of the *mid-point* between the pair (or pairs) of the experimental L-dips in the line profile. This shift grows with the increasing N_e, reaching several tens of mÅ for sub-solid-state densities. This happens at the laser intensities greater than $2\text{x}10^{20}$ W/cm^2.

Thus, the research area of the ISS is expanding. The implementation of the above three new methods is planned for the near future.

It is important to note that in paper [33] it was emphatically stressed that in many cases, modulations in some experimental profile of a hydrogenic line have nothing to do with the Langmuir-wave-caused structures. For establishing whether the modulations in some experimental hydrogenic spectral line profile are relevant to the Langmuir-wave-caused structures, *six tests of self-consistency* should be passed (for details we refer to paper [33] and review [28]).

Finally, we clarify the confusion by the author of paper [27] on the physics of the ISS. In paper [27] there were published simulations ignoring the actual physics of the Langmuir-wave-caused structures. This led to the inability of the author of paper [27] to reproduce them for the experimental spectral line profiles published in paper [34] communicating the project on spectroscopic diagnostics of the relativistic laser-plasma interactions – the project stemming from the collaboration of 21 experimentalists and theorists from seven countries (Japan, the UK, France, Germany, Hungary, the USA, and Russia). Since there was the confusion on this matter in paper [27], then for the benefit of all others working in the area of plasma spectroscopy we detail below the various aspects of the confusion from paper [27].

The primary deficiency of the simulations from paper [27]: no understanding of the physics of the *emergent phenomenon* of Langmuir-wave-caused structures as originating from the MMNDR. The MMNDR was not included in the code used in paper [27]. From the start, that code was limited to just Blochinzew satellites [35]. What was called "dips" in paper [27] were are in fact just *random troughs between the peaks*, where the peaks were (shifted) Blochinzew satellites.

The definition of the "dips" in paper [27] had no relation to the highly-localized structures in spectral line profiles caused by the MMNDR. For a specific hydrogenic spectral line, a specific charge of the perturbing ions, and a specific nuclear charge of the radiator, the locations of the Langmuir wave-caused structures are controlled by the electron density N_e, this being the reason why from their experimental locations, the electron density N_e was deduced with same accuracy as from the Thomson scattering.

In contrast, the positions of the *random* troughs between the Blochinzew peaks (the troughs confused in paper [27] with the above structures caused by the MMNDR) arise from the interplay of many plasma parameters, which is why it is practically impossible to use them for deducing N_e. Just this primary fallacy from paper [27] would be the sufficient for the failed attempts by the author of paper [27] to fit the experimental data from paper [34]. When the settings of any simulation do not contain the capability to model the specific physical phenomenon, such as the MMNDR, the results of the simulation are not adequate to the corresponding physical reality. For this reason, the code from paper [27] flunked to capture the emergent phenomenon of the MMNDR-caused structures in spectral line profiles.

Second, in paper [27], the experimental spectra presented in Fig. 3 from paper [34] (reproduced below) were called "noisy". This indicates once again that the author of paper [30] did not understand that the Langmuir-wave-caused "dip" is the *structure* involving the local minimum of the intensity (at the location controlled by N_e) encircled by two local maxima plus the secondary minimum. While understanding this structure of Langmuir "dips", the authors of paper [34] showed that *all* the experimental local minima and maxima of the intensity in Fig. 3 from [34], without any exception, were accounted for, i.e., identified and clearly indicated in Si XIV Ly-beta.

We note that the bump-to-dip ratio of the intensities was reaching up to 45%, surpassing the level of the noise by at least one order of magnitude. Not surprisingly, the

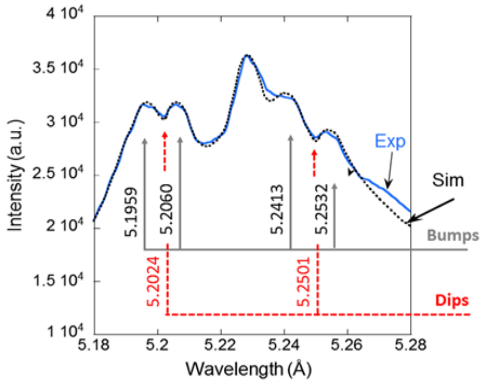

Fig. 1 Profile of Si XIV Ly$_\beta$ line from Fig. 3 of [34].

experimental spectra, recorder with a high spectral resolution ($\lambda/\delta\lambda \sim 3000$), clearly allowed the unambiguous, reliable identification of the Langmuir-wave-caused structures. We also note that for shot D the laser intensity was intentionally much lower than for shots A, B, and C – to demonstrate the disappearance of the L-dip structures.

Third, the author of paper [27] did not understand that irrespective of the particular distribution of magnitude and the direction of the quasistatic field **F**, there is always present a small group of radiating atoms or ions in the ensemble, for which the Stark splitting by the field **F** is in the resonance (namely, in the MMNDR) with the frequency of the Langmuir field $\mathbf{E}_0 \cos(\omega t)$ and its harmonics. Therefore, the positions of the ensuing structures in the spectral line profiles are independent of the specific distribution of the quasistatic field F – in contrast to the *random* locations of the troughs between (shifted) Blochinzew satellites to which paper [27] was actually limited.

Fourth, in paper [27] it was declared that "distribution functions of the turbulent fields are not known". However, in fact the distribution functions of the quasistatic turbulent fields were derived analytically in paper [36] as early as in 1976.

Fifth, the author of paper [27] declared that he was the first to discover (in paper [27]) that there are directional/polarization effects in spectral line profiles emitted from plasmas containing Langmuir waves. However, in fact, as early as in 1977 in paper [37] it was demonstrated analytically that the Langmuir-wave caused structures in spectral line profiles, caused by the MMNDR, show the directional/polarization effects. Moreover, later on in 1977, the corresponding polarization effects were exhibited experimentally and employed for plasma diagnostics in paper [38].

7. Conclusions

1. There is a new advanced method for the spectroscopic diagnostic of the edge plasmas of ITER.

2. There is a new method relevant to the spectroscopic diagnostic of the electron cyclotron waves in plasmas of the magnetic fusion machines and to the relativistic laser plasma interactions.

3. Even when the Zeeman effect predominates over the Stark effect, the experimental measurements of the Stark width of the π-component of the Lyman-alpha line can be employed in for deducing N_e or the rms field of a low-frequency electrostatic plasma turbulence.

4. The non-perturbative analytical study of the spectroscopic effect of helical trajectories of the perturbing electrons calls for revised interpretation of the hydrogen lines from the DA and DBA white dwarfs.

5. There are three new L-dip-based spectroscopic methods for measuring GigaGauss magnetic fields developing during relativistic laser-plasma interactions.

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