

Dependence on the Number of Test Fast Ions of Statistical Errors in Neutral Beam Current Drive Calculation

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The integrated plasma transport code requires acceleration. This study evaluates the computation time and calculation error when the number of test fast ions in the neutral beam deposition calculation is reduced. The statistical error increases approximately in proportion to the inverse square root of the number of test fast ions. A more detailed analysis reveals that the statistical error depends on the number of fast ions newly generated per slowing-down time of the fast ions. The results provide useful guidelines for selecting the number of test fast ions in neutral beam current drive calculations according to the required calculation time and accuracy.

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Fast plasma simulators, based on the integrated modeling codes, are required for parameter survey and plasma control in the design and operation of DEMO reactors [1]. Reducing the computation cost in the integrated modeling code TOTAL [2] is planned toward development of a plasma simulator. TOTAL is a simulation code that solves 1D thermal particle transport and 2D magnetic field equilibrium. It is characterized by its low computational cost, as it describes various physical phenomena using simplified models. In TOTAL, NB-CD (Neutral Beam Current Drive) calculations account for approximately 60% of the calculation time, with NB deposition calculations being particularly time-consuming.

NB deposition calculation employs Monte Carlo particle trajectory calculations. The specific calculation procedure involves:

1. Randomly generate a test particle on a disk perpendicular to the beam axis, which is a model of the ion source. The generation point on a disk is randomly sampled from 40,000 points of a grid generated by dividing the area uniformly in both the radial and the azimuthal directions into 200 segments each.
2. Calculating the non-ionization probability of a test particle along the beamline. The point of fast-ion generation is set when the non-ionization probability falls below the ionization threshold determined by a random number in advance.
3. Repeating this process until a prescribed number of fast ions are generated.

The typical prescribed number of fast ions is 10^4 . After completing steps 1 to 3, the velocity pitch angle distributions of fast ions are obtained in each of radial shells. The radial shell is defined by dividing the poloidal flux equally between the

magnetic axis and the plasma surface into 50 zones. From the fast ion distribution function, the function $\langle \mathbf{j}_{\text{NB}} \cdot \mathbf{B} \rangle$ is derived [3]. Here, $\langle \dots \rangle$ means the magnetic surface average. The non-ionization probability must be recalculated for each test particle and at each time step, since the conditions for calculating the non-ionization probability, such as plasma temperature and density, depends on time and location on the ion source cross-section. This is the reason why NB deposition calculations are time-consuming. Reducing the number of fast ions would lower the computation cost but would result in increase in the statistical errors. In this study, the computation time and the statistical errors in the NB-driven current are evaluated with varying the number of fast ions used in the Monte Carlo calculations, and then the trade-off relation between the computation cost and the statistical errors is investigated. The results would be a guideline for the optimum number of fast ions.

The plasma parameters of JA DEMO [4] were used as a reference, and the NB was assumed to be 1 MeV deuterium. The major radius of the plasma is 8.50 m, the minor radius is 2.42 m, and two lines of NB with tangent radii of 8.50 and 8.94 m are employed. The radial profile of density and temperature are shown in Fig. 1(a). In this study, these profiles were kept constant to evaluate the NB-driven current with fixed conditions. The calculated NB-driven current was neither reflected to the total current profile nor to the MHD equilibrium. The time step for NB-CD calculation, including the NB deposition calculation, was fixed at $dt = 0.01$ s. Traditionally, $N = 10^4$ fast ion generation calculations were performed per time step. In this study, the number of calculations was reduced, and the resulting changes in statistical error were investigated. The radial profile of $\langle \mathbf{j}_{\text{NB}} \cdot \mathbf{B} \rangle$ is shown in Fig. 1(b). The mean value $\bar{x}$ and standard deviation σ of

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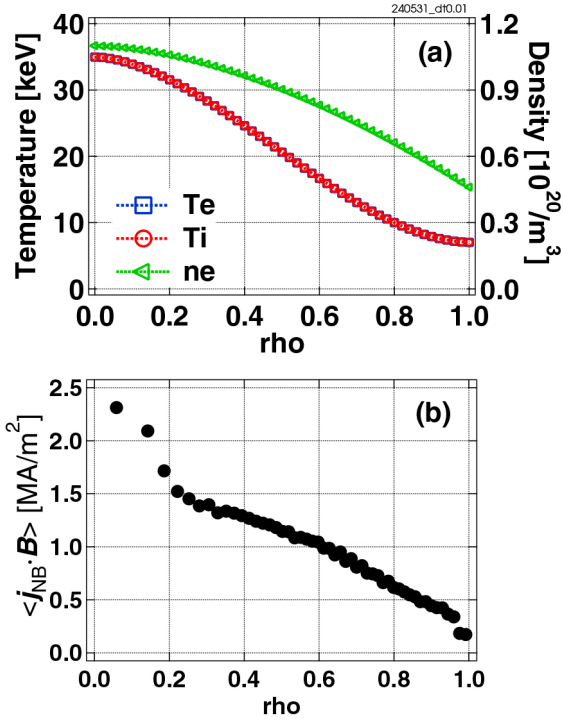


Fig. 1. (a) Calculation conditions. It shows the radial profile of electron and ion temperatures, and the electron density. The ratio of deuterium + tritium density to electron density is constant at $n_{\text{DT}}/n_e = 0.814$ (not shown). (b) Radial profile of $\langle \mathbf{j}_{\text{NB}} \cdot \mathbf{B} \rangle$ at $t = 20$ s with $dt = 0.01$ s and $N = 10^4$. It is calculated for each of the 50 points within normalized minor radius ρ ($0 < \rho < 1$) where the flux function ψ is uniformly divided.

$\langle \mathbf{j}_{\text{NB}} \cdot \mathbf{B} \rangle$ were calculated for each radial position during 20 s (2000 time points), 9.9 s after the start of NB injection, when the NB-driven current reached a steady state. The statistical error was defined and evaluated as $\sigma/\bar{x}$.

Figure 2 shows the changes in statistical error and average deposition calculation time per time step with respect to the number of test fast ions in one time step. When the number of test fast ions was varied, the change in the value of $\bar{x}$ was within the standard deviation σ of $\langle \mathbf{j}_{\text{NB}} \cdot \mathbf{B} \rangle$ ($\bar{x} \approx 1.5 \times 10^6$ [MW/m²] and $\sigma \approx 8.8 \times 10^3$ [MW/m²] at $\rho = 0.222$). Reducing the number of test fast ions resulted in an increase in statistical error approximately proportional to the square root of $1/N$. In Fig. 2(a), only the case of $\rho = 0.222$ is shown, but the same trend is also observed in other radial positions. When N was reduced from 10^4 to $1/10$ (i.e., 10^3), the statistical error increased from approximately 0.5% to about 2%, a fourfold increase. A linear relationship was found between computation time and the number of test fast ions, and reducing N to $0.1N$ similarly reduced the computation time per time step by an average of 55% on a workstation calculation. From Fig. 2(a), we can decide the required number of test fast ions to get a given value of statistical error, but the curve would depend on the plasma parameters. It would be useful if a guideline is available for deciding the required number of test fast ions according to variation of plasma parameters. The test fast ions affect the fast ion distri-

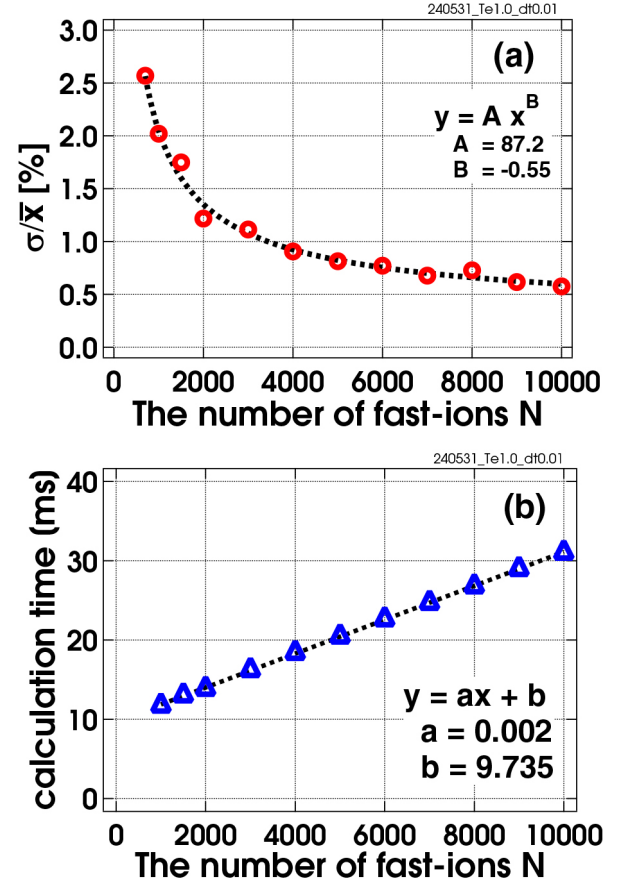


Fig. 2. (a) Relationship between the number of test fast ions in one-step calculation and the statistical error at $\rho = 0.222$. (b) Relationship between the number of test fast ions in one-step calculation and the average deposition calculation time per time step. The dashed line indicate the fitting functions shown in the figures.

bution functions during the period that their velocity slows down to the thermal velocity. Hence it is expected that the statistical errors would be mainly determined by the number of test fast ions generated during the slowing-down time. The slowing-down time τ_s of fast ions is calculated as

$$\tau_s[\text{s}] = \frac{m_b}{m_p} \frac{2(T_e[\text{keV}])^{1.5}}{\ln \Lambda Z_b^2 n_e [10^{19} \text{m}^{-3}]},$$

where m_b is the mass of the beam ion (deuterium), m_p is the proton mass, Z_b is the charge number of the beam ion, and $\ln \Lambda$ is the Coulomb logarithm of electrons. In this study, the fast ion slowing-down time τ_s was varied by changing the electron temperature T_e . When the electron temperature was changed, the value of $\langle \mathbf{j}_{\text{NB}} \cdot \mathbf{B} \rangle$ varied with temperature (the higher the electron temperature, the larger $\langle \mathbf{j}_{\text{NB}} \cdot \mathbf{B} \rangle$). Figure 3 shows the results where the time step dt or electron temperature T_e , which was fixed at the previous study, was varied. The statistical errors increase with increase in the time step and with decrease in the electron temperature (slowing-down time), as expected. In Fig. 4, the statistical errors are plotted as a function of the number of test fast ions per slowing-down time, and the data points are aligned on a single curve. Similar to Fig. 2, the curve is approximately proportional to

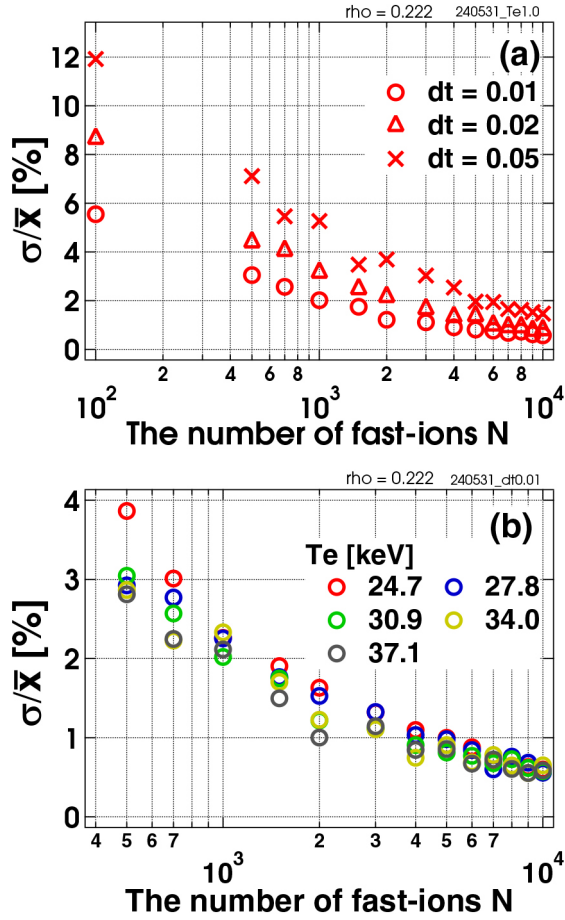


Fig. 3. Statistical error for different (a) time steps and (b) slowing-down time at $\rho = 0.222$. On (b), the values in the legend are the electron temperatures at $\rho = 0.222$.

the inverse square root of $N\tau_s/dt$. These results clearly indicate that the statistical errors are determined by the number of fast ions newly generated per slowing-down time.

In summary, to speed up the calculation of integrated plasma transport code, we investigated the relationship between the number of test fast ions in the NB deposition calculation (Monte Carlo calculations) in TOTAL and the resulting sta-

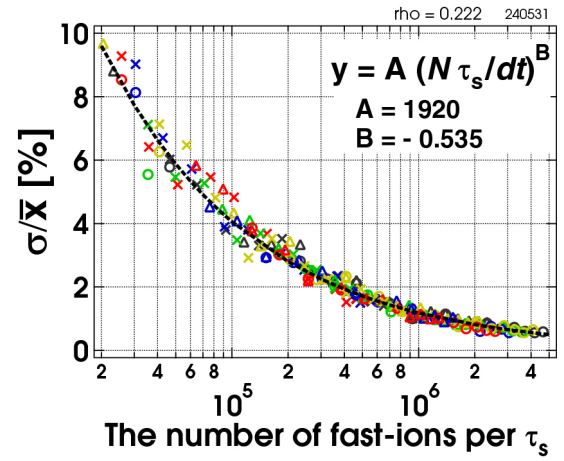


Fig. 4. Dependence of the statistical errors on the number of test fast ions per slowing-down time ($N\tau_s/dt$). The same dataset as in Fig. 3 is used. The dashed line indicate the fitting function shown in the figure.

tistical errors. A linear relationship was found between computation time and the number of test fast ions N per time step, and the statistical errors increased in proportion to the inverse square root of N . By considering the time step dt and the fast ion slowing-down time τ_s , it was revealed that the statistical errors depends on the number of test fast ions per slowing-down time ($N\tau_s/dt$). These results provide useful guidelines for selecting the number of test fast ions in NBCD calculations according to the required calculation time and accuracy.

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