Revisit to Cormack Inversion Using Function Modification Technique for Plasma Tomography

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The Cormack inversion is a well-known procedure for tomography. However, the Fourier-Bessel function expansion could be preferably used for plasma tomography, since plasma images obtained with the Cormack inversion tend to produce a false plasma image in the boundary without regards to its advantage: the direct structural analysis of plasma is possible using a set of line-integrated data. Here we propose an application of function modification technique to revive the advantage of the Cormack inversion and present successful experimental result in a tomography system of a cylindrical plasma.

Keywords: plasma diagnostics, tomography, Cormack inversion, image reconstruction, Fourier-Zernike function

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Tomography has been used in the field of plasma research to investigate its structure and dynamics. The Fourier-Bessel function (FBF) expansion is a widely used technique [1] to reconstruct and analyze its images, other than Fourier-Zernike function (FZF) expansion that Cormack proposed in its initial stage of its invention [2]. This is because the Cormack inversion using a finite number of the bases shows ghost values around the plasma edge to give a wrong and deceptive impression [1]. This paper proposes a concise method to revive the advantages of the Cormack inversion, by solving the ghost value problem with function modification technique [3].

Figure 1 shows a concept of tomography reconstruction: in the tomography the emissivity $f(r,\theta)$ is related to line-integrated emission, $g(p,\varphi)=\int_{L(p,\varphi)} f(r,\theta)\,dl$. These functions are expanded as,

$$f(r,\theta) = \sum_{m,n}\{\alpha_{m,n}\cos(m\theta)+\beta_{m,n}\sin(m\theta)\}\phi_{m,n}(r), \tag{1}$$

$$g(p,\varphi) = \sum_{m,n}\{\alpha_{m,n}\cos(m\varphi)+\beta_{m,n}\sin(m\varphi)\}\psi_{m,n}(p), \tag{2}$$

where m and n indicate azimuthal and radial mode number, respectively. The functions, $\phi_{m,n}(r)$ and $\psi_{m,n}(p)$ are expressed as,

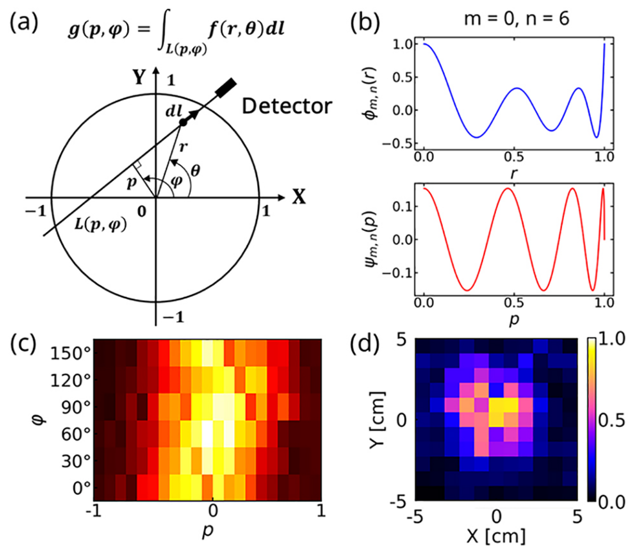

Fig. 1. (a) Concept of tomography reconstruction. (b) $\phi_{m,n}(r)$ and $\psi_{m,n}(p)$ for $m=0$ and $n=6$. (c) The sinogram obtained from tomography and (d) the local emission using MLEM method.

$$\phi_{m,n}(r) = \frac{m+2n+1}{2}R^n_m(r), \tag{3}$$

$$\psi_{m,n}(p) = \sqrt{1-p^2}\,U_{m+2n}(p), \tag{4}$$

where $R^n_m(r)$ and $U_{m+2n}(p)$ are the Zernike polynomials and the Chebyshev polynomials of the second kind, respectively; the definitions of $R^n_m(r)$ and $U_{m+2n}(p)$ are,

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$$R_m^n(r) = \sum_{k=0}^{n} \frac{(-1)^k (m+2n-k)!\, r^{m+2n-2k}}{k!\,(m+n-k)!\,(n-k)!}, \tag{5}$$

$$U_{m+2n}(p) = \frac{\sin\left[(m+2n+1)\cos^{-1}(p)\right]}{\sqrt{1-p^2}}. \tag{6}$$

As for the orthogonal bases, $\Phi^{C,S}_{m,n}(r,\theta) = (\cos m\theta, \sin m\theta)\,\phi_{m,n}(r)$, $\Psi^{C,S}_{m,n}(p,\varphi) = (\cos m\varphi, \sin m\varphi)\,\psi_{m,n}(p)$, these functions satisfy the following relation,

$$\Psi^{C,S}_{m,n}(p,\varphi) = \int_{L(p,\varphi)} \Phi^{C,S}_{m,n}(r,\theta)\, dl. \tag{7}$$

The advantage of the Cormack inversion is that both basis functions in real and sinogram space satisfy orthonormality and they are mutually related with each other in the Radon inversion. Thus, the expansion coefficients (here termed Cormack coefficients) are calculated directly from the set of line-integrated data $g_{obs}(p,\varphi)$ using the following inner product,

$$\alpha_{m,n},\ \beta_{m,n} = \int_0^{2\pi}\int_{-1}^{1} g_{obs}(p,\varphi)\,\Psi^{C,S}_{m,n}(p,\varphi)\,\omega(p)\,dp\,d\varphi, \tag{8}$$

where $\omega(p) = 2(1+\delta_{m0})^{-1}\pi^{-2}(1-p^2)^{-1/2}$, with δ_{m0} being the Kronecker delta. This formula suggests that the Cormack coefficients can be calculated in linear algebra as the inner product of vectors if the bases and images are discretized (or vectorized) on grids of (p_i, φ_i) or the sinogram space. The Cormack inversion with modified Zernike function (MZF) [3], termed here MZ-Cormack inversion, is proposed to eliminate the ghost image problem and is expressed as,

$$\phi^{MZF}_{m,n}(r) = M_{m,n}(r)\,\phi^{ZF}_{m,n}(r), \tag{9}$$

where

$$M_{m,n}(r) = \frac{1}{2}\left\{1 - \tanh\left(\frac{r - r_{0,(m,n)}}{\Delta r_{(m,n)}}\right)\right\}, \tag{10}$$

with $r_{0,(m,n)}$ and $\Delta r_{(m,n)}$ being the modification parameters (position and width), respectively.

Figure 2 shows an example of a determination process of modification parameters with a modified Zernike function; the parameters can be optimized to provide a minimum residual when they are fitted to a Bessel function.

The MZ-Cormack inversion is applied to 126 channels of line-integrated emission in a tomography system on a cylindrical plasma produced in PANTA. The system covers a square region of 10 × 10 cm², including the entire plasma cross-section whose radius is 5 cm. As the standard process of reconstruction, the sinogram data is inverted to local emission with Maximum Likelihood Expectation Maximization (MLEM) [4] (see Figs. 1(c) and (d)). Then, structural analysis of plasma emission is performed by applying the FBF expansion to MLEM images. In contrast, the Cormack inversion has the advantage that the structural analysis is possible directly on the sinogram (or raw data) space.

Figure 3 gives an instance of the Cormack inversion applied to the experimental sinogram shown in Fig. 1(c). Here

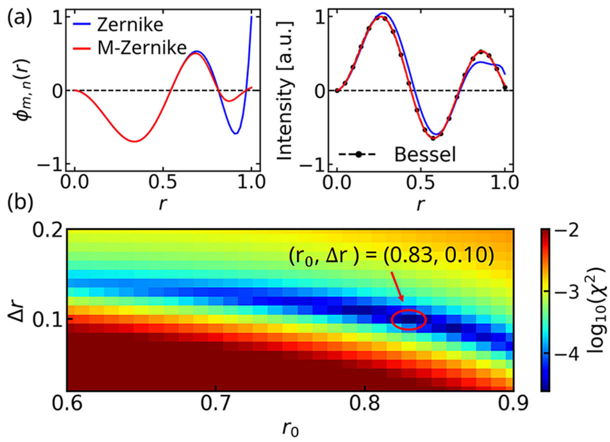

Fig. 2. An example of determination process of modification parameters. (a) The Zernike and M-Zernike function with m = 2 and n = 3. The Bessel function of m = 2 and n = 2 expanded with five M-Zernike and Zernike functions. (b) The χ^2-residuals as a function of position and width in the fitting.

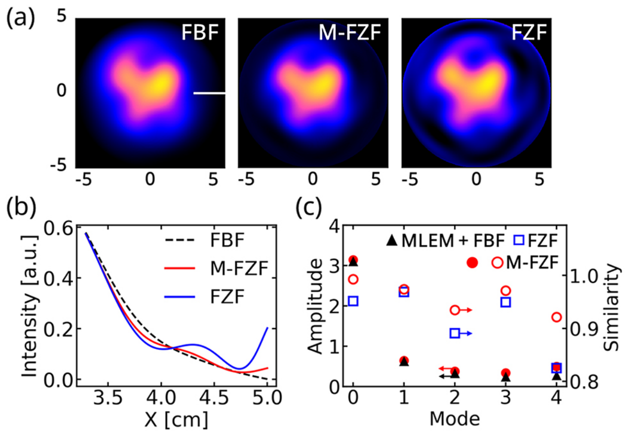

Fig. 3. (a) Comparison between the reconstructed images. An FBF image reconstructed from the MLEM one in Fig. 1(c), and those of M-FZF and FZF. Here 27 and 45 bases are used for the expansion of FBF and both Cormack inversion, respectively. (b) The profiles along the white line in (a). (c) Comparison between the images of FBF and M-FZF in similarity and amplitude for each azimuthal mode.

optimized modification parameters are set in advance individually for FZFs: an averaged function over radial number, n, or $\langle M_m(r)\rangle = \sum_{n=1}^{N} M_{m,n}(r)/N$, is used for the same azimuthal number, m. It is obvious that the Cormack inversion with MZF provides better image than the original one; particularly in the boundary region of Fig. 3(b). To quantify the difference, the degree of similarity between the images is evaluated with normalized inner product as,

$$S_m(r_s, r_e) = \frac{\int_0^{2\pi}\int_{r_s}^{r_e} f_x(r,\theta)\, f_y(r,\theta)\, r\,dr\,d\theta}{\sqrt{\int_0^{2\pi}\int_{r_s}^{r_e} f_x(r,\theta)^2\, r\,dr\,d\theta}\,\sqrt{\int_0^{2\pi}\int_{r_s}^{r_e} f_y(r,\theta)^2\, r\,dr\,d\theta}}. \tag{11}$$

Then similarity is evaluated as $S_m(3.5\ \mathrm{cm},\ 5.0\ \mathrm{cm}) = 0.78$ and 0.94 with the entire region similarity, $S_m(0.0\mathrm{cm}, 5.0\mathrm{cm})$ = 0.99 and 0.98 for the original and MZ-Cormack inversion, respectively. Thus, the result shows that the MZ-Cormack inversion provide an improved image in the edge. Moreover, detailed comparison is made for each azimuthal mode in similarity and amplitude defined as, $A_m = \sqrt{\sum_n(\alpha_{m,n}^2 + \beta_{m,n}^2)}$. The result shows that the modal similarity is reasonably high (larger than 0.9) and that the modal amplitudes are well agreed with those of the standard process using FBFs.

Therefore, the structural analysis of the MZ-Cormack inversion produces sufficiently identical results to those of our standard analysis. Moreover, the calculation times for an image are 0.0013 and 0.93 ms for the Cormack inversion with vectorization technique and MLEM, respectively; the Cormack inversion is 715 times faster than our standard procedure if appropriate modification parameters are set in advance. In the Cormack inversion using vectorized bases, the optimized set of bases can be easily determined from the absolute values of the inner product between the sinogram and the bases. In conclusion, the introduction of boundary modification technique to the Cormack inversion can realize fast and direct structural analysis with reproducing an excellent image without any ghost values in the boundary.

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