A Method to Analyze Plasma Images Using Modified Fourier-Bessel Functions

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A new method is proposed to analyze plasma image of tomography using modified Fourier-Bessel Functions (FBF) instead of the original FBF analysis. The application of the method to an assumed plasma image shows that the difference between the original and the fitting image is improved considerably to that of FBF, owing to giving a better fitting inside the plasma and eliminating the ghost values outside the plasma, without any disadvantages in analysis of plasma structures and patterns.

Keywords: plasma, turbulence, tomography, Fourier-Bessel function, Modified Fourier-Bessel Function

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The entire cross-section image of local plasma emission is successfully reconstructed for turbulence studies in PANTA (Plasma Assembly for Nonlinear Turbulence Analysis), using tomography with an algorithm, Maximum Likelihood Expectation Maximization [1]. The Fourier-Bessel functions (FBF) expansions, amongst various methods [2], are used to characterize global structure of steady state and coherent fluctuation patterns [3, 4]. However, the FBF images often show ghost values outside of the plasma, giving a wrong and deceptive impression where no emission is expected. Moreover, the internal values of plasma, which are practically accurate, may be affected in the fitting procedure to try to eliminate the external value with a finite number of the bases. The paper proposes a simple method using modified FBFs to give a better fitting to solve the problem.

In the analysis of structure and pattern, the tomography image of plasma is expanded or fitted with FBFs as

$$\epsilon_{FBF}(r,\theta) = \sum_n \alpha_{0,n}\phi_{0,n}(r,\theta) + \sum_m \sum_n \left\{\alpha_{m,n}\phi^c_{m,n}(r,\theta) + \beta_{m,n}\phi^s_{m,n}(r,\theta)\right\}, \quad (1)$$

where $\phi^c_{m,n}(r,\theta)$ and $\phi^s_{m,n}(r,\theta)$ are basis functions and $\phi^c_{m,n}(r,\theta) = J_m(k_{mn}r)\cos(m\theta)$, $\phi^s_{m,n}(r,\theta) = J_m(k_{mn}r)\sin(m\theta)$, with $J_m(r)$, k_{mn}, and m being the m-th order Bessel function, n-th value to satisfy $J_m(k_{mn}L) = 0$ and azimuthal mode number, respectively. The ghost values emerge outside the plasma in the fitting of a finite number of FBF bases.

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The modified bases are defined as

$$\psi^{M-FBF}_{m,n}(r,\theta) = M(r)\phi^{FBF}_{m,n}(r,\theta), \quad (2)$$

where the modification function $M(r)$ is written as

$$M(r) = \frac{1}{2}\left\{1 - \tanh\left(\frac{r - r_0}{\Delta r}\right)\right\}, \quad (3)$$

where r_0 and Δr represent the modification parameters (position and width), respectively. The fitting using the modified functions is expected to keep the inside value almost the same, however, eliminates the outside values of the plasma or remove the ghost values. In addition, the modification should provide the same advantageous if the original function is Zernike polynomials used in the Cormack inversion [2], although the modified functions lose the orthogonal and complete properties that the original functions possess. Both expansion coefficients of FBF and M-FBF expansion are determined by minimizing the following residual using the least square fitting,

$$\chi^2 = \sum_j \left\{\epsilon_{obs}(r_j,\theta_j) - \epsilon_{FIT}(r_j,\theta_j)\right\}^2, \quad (4)$$

where $\epsilon_{obs}(r_j,\theta_j)$ is the tomography image of plasma emission, with $\epsilon_{FIT}(r_j,\theta_j)$ being the one expanded with FBF or M-FBF. In the case of M-FBF expansion, an extra process is necessary to determine the position r_0 and width Δr.

A comparison between the fittings of FBF and M-FBF is made for an assumed plasma image of tomography (or original image) shown in Fig. 1 (a). The radius of the plasma image and the square region that the tomography covers are assumed to be 5 cm, similarly in PANTA, and

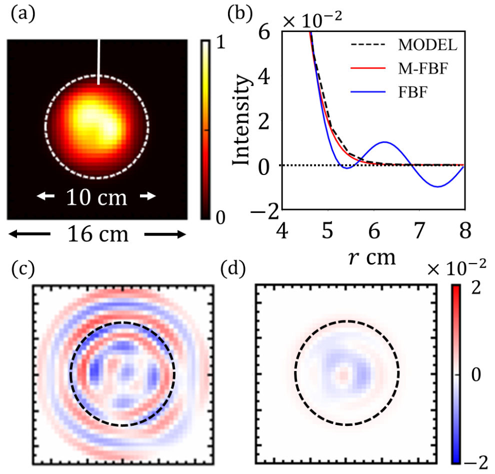

Fig. 1 (a) An assumed plasma image of tomography, (b) the differences along the white line in (a), (c) difference between the original and FBF images, and (d) difference between the original and M-FBF. The circles in figures indicate the plasma region.

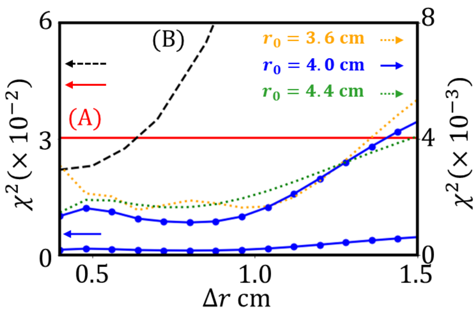

Fig. 2 The residuals as a function of the width for the three cases of the position r_0 = 3.6, 4.0 and 4.4 cm, with the residuals of (A) FBF and (B) simply tanh-multiplied FBF.

16 cm × 16 cm. Here, the image is assumed to have no emission outside of the plasma since no significant emission is observed there in standard operations. The other figures show the differences between the original and the fitted images using both 43 bases of FBFs and M-FBFs : the number of the bases is selected to give the minimum to the Akaike information criterion [3, 4]. The figures show that the difference of M-FBF is much smaller than that of FBF, and the ghost values in FBF are eliminated in the M-FBF. In addition, the oscillations seen in plasma edge of FBF case disappear and turn smooth in the case of M-FBF.

In the presented M-FBF fitting in Fig. 1, the modification parameters are selected to minimize the residual, Eq. (4): the parameters are r_0 = 4.0 cm and Δr = 0.8 cm. Figure 2 shows the residuals as a function of the width for three positions, r_0 = 3.6, 4.0 and 4.4 cm. The comparison indicates that the residuals are much smaller than those of FBF. For reference, the dashed line shows the residual between the original and the FBF image simply multiplied by

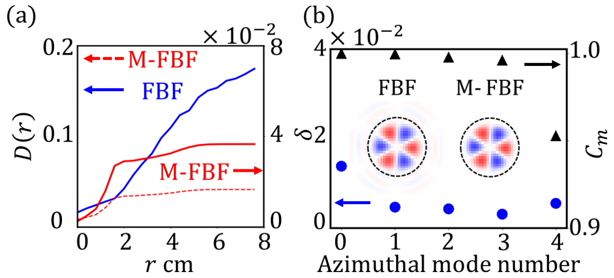

Fig. 3 (a) Comparison in radial integrated difference, (b) difference and similarity of azimuthal modal structure obtained with FBF and M-FBF, and the inset shows the reconstructed azimuthal patterns of m = 3 with FBFs and M-FBFs.

$M(r)$ with the position set as r_0 = 5.0 cm. Even in this case, the residual is smaller than the original when the width is sufficiently small (Δr < 0.65 cm) since it eliminates the ghost values outside.

Figure 3 shows the details of the radially integrated difference between the original and the fitting profiles using the FBF and M-FBF. Here, the radially integrated difference is defined as

$$D(r) = \sqrt{\int \{\epsilon_{obs}(r,\theta) - \epsilon_{FIT}(r,\theta)\}^2 \, dS}\,. \tag{5}$$

It is obvious that the radially integrated difference for M-FBF is roughly a few times smaller than that of FBF, even inside of the plasma. Although it is a matter of course, in addition, the difference increases monotonically in the FBF case due to the ghost values, while that in the M-FBF becomes constant outside the plasma. On the other hand, Fig. 3 (b) shows the modal difference and the pattern similarity between FBF and M-FBF for each azimuthal mode. The modal difference δ, and similarity C_m, are defined as

$$\delta = \sqrt{\frac{\int_{inside} \{\epsilon_{FBF}(r,\theta) - \epsilon_{M-FBF}(r,\theta)\}^2 \, dS}{\int_{inside} \epsilon_{FBF}^2(r,\theta) dS}}, \tag{6}$$

$$C_m = \frac{\int \epsilon_{FBF}(r,\theta)\epsilon_{M-FBF}(r,\theta) dS}{\sqrt{\int \epsilon_{FBF}^2(r,\theta) dS}\,\sqrt{\int \epsilon_{M-FBF}^2(r,\theta) dS}}. \tag{7}$$

As shown in Fig. 3 (b), the azimuthal mode patterns are almost the same; the modal difference is less than 1.4 % to the maximum values of the modes until m = 3 and the similarity is almost 1 for every mode. Thus, in the analysis of the modal pattern, both expansions should give the same results within experimental errors.

Finally, a method is proposed to analyze plasma images using M-FBFs. The results demonstrate the advantages of the method: the residual of M-FBF case is improved considerably to that of FBF, owing to giving a better fitting inside the plasma and eliminating the ghost values outside the plasma, although both methods provide the same capability for structure and modal pattern analysis.

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