Quaternion Analysis of Transient Phenomena in Matrix Converter Based on Space-Vector Modulation*)

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The space vector of an output voltage can be rotated smoothly in a matrix converter. However, a zero-sequence component appears causing problems in the load, such as a motor. A quaternion is a four-dimensional hypercomplex number with an imaginary part that can simultaneously represent and deal with three-phase voltages. In addition, the quaternion is expressed in the exponential form; thus, it can easily represent the space vector rotation. Two zero configurations were used to optimize the ripple characteristics in fictitious pulse-width-modulated voltage-source inverter. The two zero configurations are used to remove the zero-sequence component from the matrix converter. The quaternion can be differentiated in time as well as rotate in space. Therefore, it is used to analyze transient phenomena in the matrix converter's rise-up and rise-down phases, and the switching's transition phase.

Keywords: quaternion analysis, annihilation of zero-sequence component, transient phenomena, direct matrix converter, space-vector modulation, three-phase to three-phase

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1. Introduction

A matrix converter can convert commercial AC voltage to another arbitrary-frequency AC voltage, as well as the power factor, with high efficiency [1]. Therefore, the converter can be used to the power supply for a resonant magnetic perturbation coil [2], increasing the efficiency of a fusion reactor.

The space vector of an output voltage can be rotated smoothly in a matrix converter [3]. Particularly, the space vector of a positive-sequence component draws a circle. However, a zero-sequence component appears, causing problems in the load, such as a motor. For example, the alternating neutral voltage may cause current leakage in the rotating machine structural case. A quaternion is a four-dimensional hypercomplex number with a three-dimensional vector part that can simultaneously represent and deal with three-phase voltages. In addition, the quaternion is expressed in the exponential form [4]; thus, it can easily represent the space-vector rotation as seen in computer graphics [5]. We analyze a matrix converter in detail using these quaternion capabilities.

In a conventional matrix converter, eighteen active configurations and three zero configurations are used among twenty-seven configurations composed of nine switches. All six active configurations and two zero configurations are used in a fictitious pulse-width-modulated voltage-source inverter, and the two zero configurations were used to optimize the ripple characteristics [6]. However, the two zero configurations are used to eliminate the zero-sequence component not only in an indirect matrix converter [7] but also in a direct matrix converter.

The quaternion can be differentiated in time as well as rotated in space. Therefore, it is used to analyze transient phenomena in matrix converter rise-up and rise-down, and commutation in switching.

2. Quaternion and the Differentiation

We introduce the quaternion (four-dimensional hypercomplex number), which is extended from a complex number [8] to express three-phase AC in three dimensions:

$$q = a + v = a + (iv_x + jv_y + kv_z), \tag{1}$$

$$i^2 = j^2 = k^2 = -1, \tag{2}$$

$$ij = -ji = k, jk = -kj = i, ki = -ik = j. \tag{3}$$

Quaternion is divided into a real part (scalar part) and an imaginary part (vector part). The imaginary part has a

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vector property, where the imaginary numbers behave as unit base vectors, but they also have hypercomplex number property. To assign three-phase AC voltages or currents to the vector part, let us consider the exponential representation of the quaternion:

$$q = a + \hat{n}\|v\| = \|q\|(\cos\theta + \hat{n}\sin\theta) = \|q\|\epsilon^{\hat{n}\theta}, \quad (4)$$
$$\hat{n} = (iv_x + jv_y + kv_z)/\|v\|, \quad (5)$$
$$\|q\|^2 = a^2 + \|v\|^2, \quad (6)$$
$$\|v\|^2 = (v_x)^2 + (v_y)^2 + (v_z)^2. \quad (7)$$

Let us assign three-phase AC phase (line-to-neutral) voltages to the vector part of the quaternion:

$$e = \sqrt{2}E\begin{Bmatrix} +i\cos(\omega t - 0\pi/3) + j\cos(\omega t - 2\pi/3) \\ +k\cos(\omega t - 4\pi/3) \end{Bmatrix},$$
$$= \epsilon^{\hat{n}\omega t}\sqrt{3}Ee_0, \quad (8)$$
$$\hat{n} = (+i + j + k)/\sqrt{3}, \quad (9)$$
$$e_0 = \begin{Bmatrix} +i\cos(-0\pi/3) + j\cos(-2\pi/3) \\ +k\cos(-4\pi/3) \end{Bmatrix} \Big/ \sqrt{\frac{3}{2}}. \quad (10)$$

It denotes that the initial three-phase (positive phase) AC voltage vector rotates counterclockwise with unit vector axis $\hat{n}$. In this case, the locus of the rotating vector is a circle on the plane, which is perpendicular to $\hat{n}$ and includes the origin.

Next, let us consider Ohm's law of three-phase AC circuit, where the load is inductive and the mutual inductances exist as follows:

$$\begin{bmatrix} e_\mathrm{a} \\ e_\mathrm{b} \\ e_\mathrm{c} \end{bmatrix} = \sqrt{2}E\begin{bmatrix} \cos(\omega t + \phi - 0\pi/3) \\ \cos(\omega t + \phi - 2\pi/3) \\ \cos(\omega t + \phi - 4\pi/3) \end{bmatrix},$$
$$= \begin{bmatrix} L + L_\mathrm{N} & M & M \\ M & L + L_\mathrm{N} & M \\ M & M & L + L_\mathrm{N} \end{bmatrix} p\sqrt{2}I\begin{bmatrix} \cos(\omega t - 0\pi/3) \\ \cos(\omega t - 2\pi/3) \\ \cos(\omega t - 4\pi/3) \end{bmatrix}, \quad (11)$$

where $p\,(= \mathrm{d}/\mathrm{d}t)$ is a differential operator. Because the neutral inductance (grounding inductance) does not affect symmetrical (positive phase) AC, the quaternion representation is as follows [9]:

$$e = \epsilon^{\hat{n}(\omega t+\phi)}\sqrt{3}Ee_0 = (L - M)p\,i,$$
$$= (L - M)p\,\epsilon^{\hat{n}\omega t}\sqrt{3}Ii_0 = \hat{n}\omega(L - M)\,i, \quad (12)$$

where the phase ϕ is $\pi/2$. Three-phase current can be expressed in the exponential form of a quaternion. The quaternion can be easily differentiated in time. Thus, the reactance of an inductance, L, can be simply expressed as $\hat{n}\omega L$.

Consequently, the quaternion representation of general Ohm's law can be presented as follows:

$$[R][i] + [L]\frac{\mathrm{d}}{\mathrm{d}t}[i] = [e], \quad (13)$$

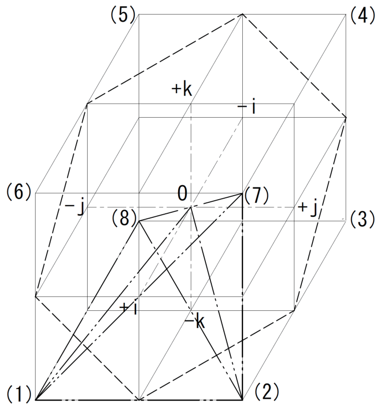

Fig. 1 Quaternion of three-phase AC phase voltage in a voltage-source inverter.

$$e = \epsilon^{\hat{n}\omega t}\sqrt{3}Ee_0 = \{R + (L - M)p\}\,i,$$
$$= \{R + (L - M)p\}\,\epsilon^{\hat{n}\omega t}\sqrt{3}Ii_0,$$
$$= \{R + \hat{n}\omega(L - M)\}\,i. \quad (14)$$

When the initial current quaternion is 0 (origin), the quaternion spirally approaches the final steady-state circular locus as follows:

$$i = \frac{1}{|Z|\epsilon^{\hat{n}\phi}}\left(\epsilon^{\hat{n}\omega t} - \epsilon^{-t/\tau}\right)\sqrt{3}Ee_0, \quad (15)$$
$$Z = R + \hat{n}\omega(L - M) = |Z|\epsilon^{\hat{n}\phi}, \quad (16)$$
$$\tau = (L - M)/R. \quad (17)$$

3. Zero-Sequence Component in Matrix Converter

First, let us consider an indirect-type matrix converter composed of a voltage-source rectifier and voltage-source inverter. In the voltage-source rectifier, the input current phase is controlled by a switching configuration. In the voltage-source inverter, the output voltage phase is controlled by a switching configuration. A quaternion vector (1) $[+1, -1, -1]$ is output by a switching configuration (Fig. 1).

In this quaternion vector space, the i-axis points to the near side, the j-axis is in the horizontal direction, and the k-axis is in the vertical direction. Each switching configuration generates the base active phase-voltage quaternions (1) to (6). By switching between (1) and (2) quaternions, arbitrary quaternion can be produced inside the first triangular sector O-(1)-(2). However, (1) is under the hexangular plane (broken line) perpendicular to the $[1, 1, 1]$ axis, whereas (2) is above it. Notably, the zero-sequence component appears in the output voltage quaternion. This appearance is a problem.

Alternatively, let us effectively consider zero-switching quaternions (7) and (8). Base active phase-voltage quaternions (1) to (8) are produced by each switching configuration. An arbitrary quaternion can be produced by switching between the (1), (2), (7), and (8) quaternions inside the first four-sided pyramid (1)-(2)-(7)-O-(8).

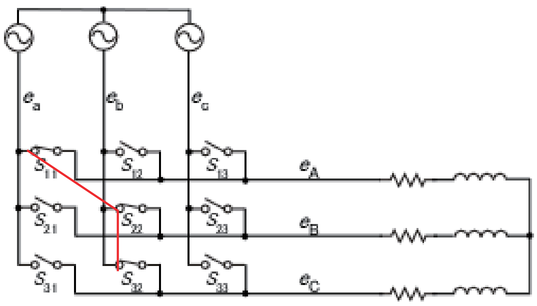

Fig. 2 Direct-type matrix converter. Switches S_{11}, S_{22} and S_{32} are on. The on switches are connected by the red line, corresponding to the switching configuration (+1) or (1, 2, 2).

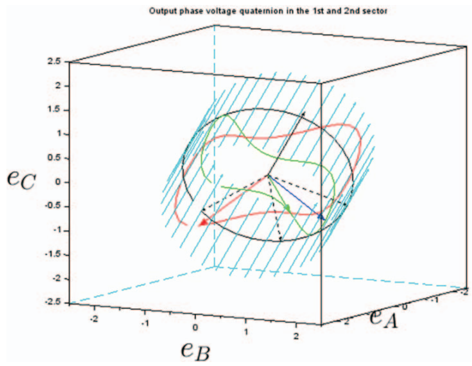

Fig. 3 Output phase voltage quaternion in the direct matrix converter. The output line voltage quaternion (a circle) is also displayed.

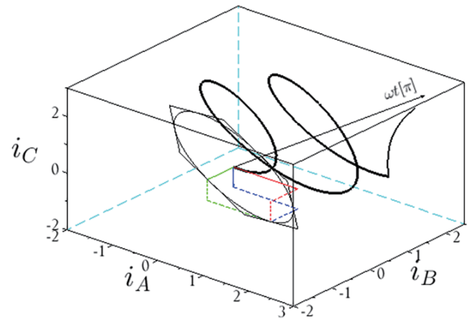

Fig. 4 Time evolution of three-phase AC output phase current quaternion in matrix converter.

Therefore, the zero-sequence component disappears from the output phase voltage quaternion. However, the voltage conversion ratio decreases by $\sqrt{3}/2$ times in exchange for eliminating the zero-sequence component.

Second, let us consider the direct matrix converter (Fig. 2). In a switching configuration (+1), switches S_{11}, S_{22}, and S_{32} are on. In (−3), S_{11}, S_{23}, and S_{33} are on. In (−7), S_{11}, S_{21}, and S_{32} are on. In (+9), S_{11}, S_{21}, and S_{33} are on. Switching configurations (+1) and (−3) constitute a quaternion on a plane containing $[2, -1, -1]$ and $[1, 1, 1]$ (Fig. 3). Further, switching configurations (−7) and (+9) constitute a quaternion on a plane containing $[1, 1, -2]$ and $[1, 1, 1]$. The duty factor ratio between (+1) and (−3), and that between (−7) and (+9) is determined to change the input current amplitude and phase. In addition, the duty factor ratio between (+1) and (−7), and that between (−3) and (+9) is determined to change the output voltage amplitude and phase.

Although the output voltage quaternion is adjusted on the cylindrical surface containing the circle, it cannot be adjusted on the circle [10]. In a zero switching configuration (0_1), switches S_{11}, S_{21}, and S_{31} are on. In (0_2), S_{12}, S_{22}, and S_{32} are on. In (0_3), S_{13}, S_{23}, and S_{33} are on. The duty factor ratio between these three zero configurations can be adjusted to annihilate the zero-sequence component. The switching configurations must be in the order $(0_3) - (-3) - (+9) - (0_1) - (-7) - (+1) - (0_2)$ or the inverse, so that just one switch commutates between adjacent configurations once.

4. Transient Phenomena in Matrix Converter

First, transient phenomena in the output current are governed by the differential Equation (13). At the beginning phase, the initial current quaternion is 0 (origin), and the quaternion spirally approaches the final steady-state circular locus as expressed by Equation (15). In the ending phase, the initial current quaternion is a steady-state, and the quaternion attenuates radially to the final current 0 (origin) (Fig. 4).

Second, let us consider a transition in the switching configuration (1, 2, 2), where outputs A, B, and C are connected to inputs a, b, and b (Fig. 2). Particularly, the B- and C-phase voltages are equal. In the transition phase, the final voltage quaternion includes negative- and zero-sequence components.

$$\begin{aligned} e &= \sqrt{2}E \begin{Bmatrix} +i\cos(\omega t - 0\pi/3) \\ +j\cos(\omega t - 2\pi/3) \\ +k\cos(\omega t - 2\pi/3) \end{Bmatrix} \\ &= e_0 + e_1 + e_2 \\ &= \sqrt{2}E_0\hat{n} + \epsilon^{+\hat{n}\omega t}\sqrt{3}E_1 e_0 + \epsilon^{-\hat{n}\omega t}\sqrt{3}E_2 e_0 \\ &= \cos(\omega t - \pi/2)\sqrt{2}E\hat{n} \\ &\quad + (1/\sqrt{3})\epsilon^{+\hat{n}\omega t}\sqrt{3}E\epsilon^{+\hat{n}\pi/6}e_0 \\ &\quad + (1/\sqrt{3})\epsilon^{-\hat{n}\omega t}\sqrt{3}E\epsilon^{-\hat{n}\pi/6}e_0. \end{aligned} \tag{18}$$

Zero-sequence current quaternion oscillates along the $[1, 1, 1]$ axis. Positive-sequence quaternion rotates counter-clockwise starting from the initial point, and negative-sequence quaternion rotates clockwise starting from the origin (Fig. 5).

The sum of the positive-sequence and negative-sequence quaternions oscillates almost linearly in the directions displayed by red lines. The sum of all quaternions rotates on the plane including the $[1, 1, 1]$ axis (Fig. 6).

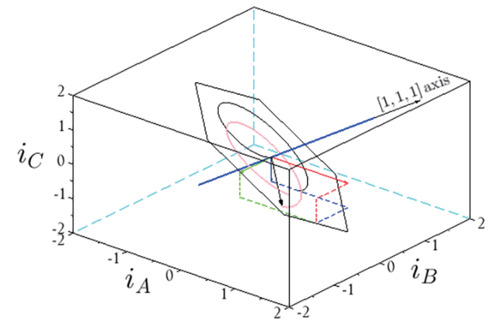

Fig. 5 Output phase current quaternion in a transition phase to switching configuration (+1) or (1, 2, 2). Blue line: zero-sequence current quaternion; red line: positive-sequence current quaternion; black curve: negative-sequence current quaternion.

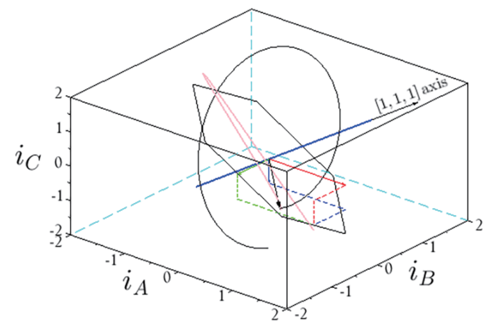

Fig. 6 Output phase current quaternion in a transition phase to switching configuration (+1) or (1, 2, 2). Blue line: zero-sequence current quaternion; red line: positive- and negative-sequence current quaternion; black curve: the sum of zero-, positive-, and negative-sequence current quaternion.

5. Summary

A quaternion is a four-dimensional hypercomplex number with a three-dimensional vector part that can simultaneously represent and deal with three-phase voltages. In addition, the quaternion is expressed in the exponential form. Therefore the quaternion can represent the rotation of the space vector easily. We analyzed the zero-sequence component of the output voltage and the transient phenomena of a matrix converter in detail using these quaternion capabilities.

In a conventional matrix converter, eighteen active configurations are used to adjust the output voltage and input current space vectors. Four active configurations are switched in one phase sector to simultaneously modify the output voltage and input current space vectors. The four duty ratios cannot eliminate the zero-sequence component. The two zero configurations were used to eliminate the zero-sequence component in fictitious pulse-width-modulated voltage-source inverter of the indirect matrix converter. However, the voltage conversion ratio decreases by $\sqrt{3}/2$ times compared with the optimized matrix converter. In the case of a direct matrix converter, the three zero configurations could be used to eliminlate the zero-sequence component. However, the three duty ratios are not equal and must be controlled according to the corresponding switching sector.

The quaternion can be rotated in space and differentiated in time. Therefore, it was used to analyze transient phenomena in the matrix converter's rise-up and rise-down phases and the switching's commutation transition phase directly without any transformation.

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