Sensing a Change in Size of a Circular Tokamak Plasma Using a Single Magnetic Probe: a Theoretical Approach[*]

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A tokamak is a toroidal device in which the donut shaped plasma is confined by means of external magnetic field. The externally measured magnetic field due to the plasma column carries several information about the plasma and these include total plasma current, the position of plasma column centroid, the shape of plasma etc. To diagnose these mentioned parameters, several magnetic diagnostics are used and this requires adequate data processing, which is not always very straight forward. In contrary, present theoretical study focus on a rather simple approach of estimating magnetic field at a given location with respect to circular plasma to figure out the change in size of the plasma column. Moreover, this study reveals that the required estimation is very much sensitive to the choice of location for measurement of magnetic field and completely depends on the geometry of the tokamak, and hence tokamak machine-specific. Finally, a subtle approach is explored to make these observations more generalized and hence the usefulness of this approach for any other tokamak-like machine is established.

Keywords: tokamak, circular plasma, magnetic field, major radius, minor radius

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1. Introduction

Tokamak is aimed for fusion by means of generating and confining plasma with the help of external magnetic field. The size of a tokamak plasma with circular cross-section is specified by means major radius R and minor radius a [1]. The movement of the circular toroidal plasma, both in horizontal and vertical directions, results in a change of both R and a, and hence in size. For the estimation of such changes in size of plasma column, several techniques, like estimation of positional shifts [2], equilibrium reconstructions [3] etc., are adapted. These techniques require data input from a number of magnetic diagnostics and adequate data processing. In contrary to these well-established methods, this theoretical approach starts with a very basic question: is a single magnetic probe capable of capturing the change in size of a tokamak plasma? The answer is reached through a rigorous study of the change in magnetic field profile due to toroidal plasma column with the change in its size and finally a number of interesting outcomes are achieved, addressing a positive answer to the above mentioned question.

The paper is arranged in the following manner. Section 2 gives the details about the theoretical approach that is adapted for the analysis. Several impactful outcomes, as a result of this analysis, is then revealed in the next section and hence the idea of point of invariance (PoI) is introduced. PoI is further discussed in section 4 and hence a tokamak-geometry independent nature of PoI is achieved. Finally, the importance of these consequences as well as the scope of future works for the experimental validation as well as practical implementations of these outcomes is discussed in section 5.

2. Theoretical Analysis

The basis of the entire theoretical study stands on finding the impact of size of a toroidal non-filamentary current of circular cross-section on the corresponding magnetic field in its vicinity. The magnetic field for a source current is found using a numerical tool, Poisson Superfish [4, 5], which solves the magneto-static Poisson's equation by successive point over-relaxation method. In the present approach few assumptions are taken into account for the simplicity of understanding as well as analysis. Firstly, the plasma column is assumed to consist of a uniform current density and the density profile is exempted from the scope of this work. Secondly, the change in size of plasma column is taken in such a way that only there is a change in a, though R is unaltered, until and unless it is specified separately. Thirdly, the current sources are static and time variation of the same is not taken into account. Finally, the source current is supposed to have a toroidal symmetry. The static uniform current density in a toroidal geometry is numerically produced and the corresponding magnetic

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fields at all required locations with respect to the torus are found using this numerical tool.

Figure 1 gives a schematic where plasma column is presented by the thick black circle; outer circle has a radius of R; N, P, Q are points on that circle at three different locations; O is the center of the torus and C is the center of the column. Firstly, magnetic fields (B) for 1 kA of toroidal current is calculated at two diametrically opposite points O and Q for different a, due to a fixed value of R and this is given in Figs. 2 and 3, respectively. Interestingly, the changes in a have a significant impact on B at both O and Q location, though in an opposite way. This observation answers positively to the question raised in introduction, as this clearly shows that the detection of size can be successfully captured by the estimation of corresponding magnetic field at a single location.

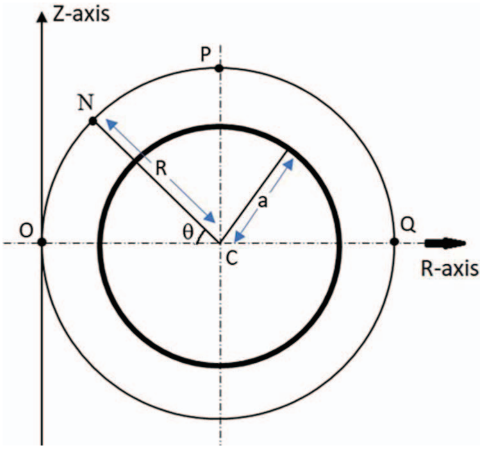

Fig. 1 Poloidal cross-section of a tokamak plasma, with R and a as major and minor radii respectively.

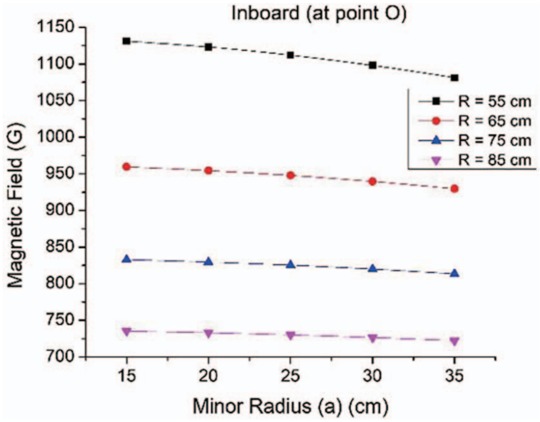

Fig. 2 The variations of magnetic field at the center of torus 'O' (due to 1 kA of current) with minor radii for a set of R are presented.

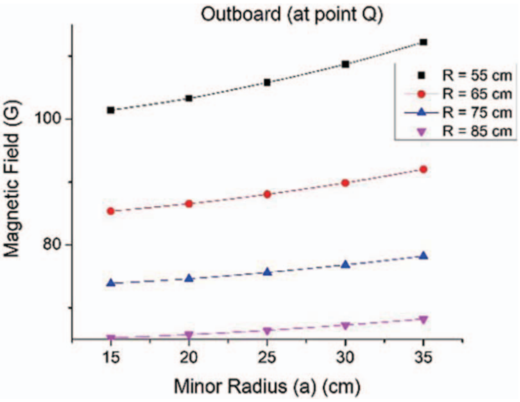

Fig. 3 The variations of magnetic field (due to 1 kA of current) with minor radii for a set of R are presented at an outboard point Q.

3. Idea of Invariance

Though the detection of change in size of a circular plasma by using a single magnetic probe is possible, the location for measurement plays a crucial role as, the nature of change in B differs with locations (e.g., at O and Q). To find this appropriate location for measurement, the change in B with a at a number of loci between O and Q is calculated. Figure 4 presents a similar plot at P and the trend of the curves repeats to that at Q, though the values are higher. Thus, the difference in the trend of change in B with a between P and O indicates that there must be a point in between P and O where, the change in a has no effect on the corresponding B. The point on that circle is N, say, which is named as point of invariance (PoI) and shown in Fig 1. PoI is specified by an angle θ, made with respect to the major axis (R) in the toroidal geometry. If another concentric circle, having center at C, but with radius different from R is imagined, a different angle of invariance (θ) or AoI is obtained and this is true for any other concentric circle. Hence, it is to be emphasized here that AoI (θ) is not unique until and unless the radius of the circle is specified. Thus, different concentric circles, having centers at C, correspond to different PoI and thus a specific value for AoI. This is further discussed in the next section.

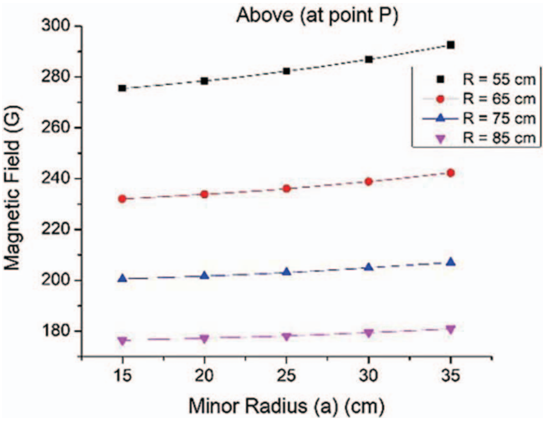

Fig. 4 The variations of magnetic field (due to 1 kA of current) with minor radii for a set of R are presented at P.

4. Unification of PoI

A number of circles, having center at C and radii smaller than R, are imagined and the variations of B due to the change in a are studied along each of the arcs. For every circle, a PoI is found and the corresponding AoI (θ) is observed to rise as the radius of the circle, say r, approaches from R to a and this is obvious from Fig 5, in which AoI (θ) is plotted with r for different R values. The linearity with a negative gradient gives a similarity in the nature of AoI irrespective of R, though the values of gradient are not equal and vary with R. To remove this R dependency of AoI (θ) a subtle way is adapted, as follows. Figure 5 is re-plotted with the abscissa r scaled by R and provided in Fig. 6. Interestingly, the linear drop of θ with (r/R) gives a generalized trend for all toroidal geometry of current and finally AoI is presented in a R independent way, which follows a universal linear drop with gradient and intersection −29.25° and 71.21°, respectively.

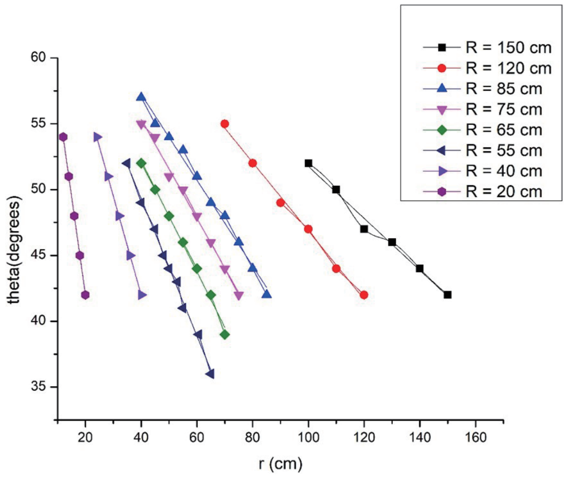

Fig. 5 Plots for angle of invariance (θ) with radial distance r from 'C'.

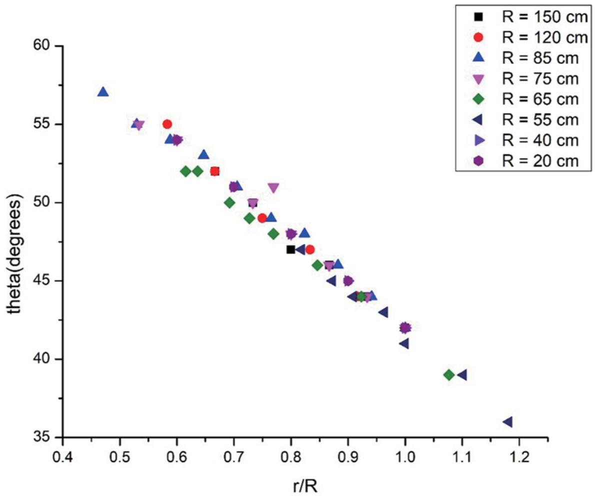

Fig. 6 The change in angle of invariance (θ) with relative radial distance (r/R) from the center of plasma column are plotted for different major radii.

Figure 6 is obtained for a number of tori with different major radii R and for individual plot, AoI is found separately using Poisson-Superfish, which estimates the magnetic field B numerically. For a given R, B is found at a number of coordinate points in the vicinity of the torus and, hence the change in B due to a change in a at every location of interest is found. Finally, in the entire estimation process, a maximum uncertainty of 0.1% is left in the calculated B, which leads to a spread of final curve resulting in a maximum uncertainty of 1° in AoI and this is reflected in Fig. 6.

5. Discussions and Conclusions

This paper summarizes the method of sensing the change in the size of a toroidal conductor with circular cross-section by measuring corresponding magnetic field with the help of a single magnetic probe. For such a technique, the importance of the proper choice of location with respect to the toroidal geometry for magnetic measurement is further explored and consequently this method is shown to be useful for any toroidal current, like tokamaks or tokamak-like machines.

The analysis restricts the regime of application of these consequences to idealistic current sources only due to the underlying basic assumptions. To implement the idea of this approach in case of real-life toroidal current sources like, tokamaks etc., the effect of discarding these assumptions on final consequences needs to be studied in a thorough manner. For example, the change in minor radius a of a plasma column in a tokamak-like machine essentially incorporates a change in major radius R and so the effect of a simultaneous change in R and a on the consequences is required to be studied. Moreover, the Grad-Shafranov shift in the horizontal direction also needs to be considered, as this is obvious in case of tokamak-like plasma [1]. Also, the current density profile of plasma is inherent for such a tokamak-like plasma and so a profiled current is supposed to be investigated, though the corresponding effects are not supposed to affect these consequences until and unless quantitative analysis is taken into account. Actually, current density profile, $J(r)$, does not affect the analysis in a very strong manner, as point of invariance PoI and hence choice of appropriate location are independent of minor radius. Profile only redistribute the current by keeping the centroid of the current column at the same location for a constant major radius R and it becomes very crucial only when the analysis deals with the analytical values of the corresponding magnetic field, and hence quantitative consequences are inferred.

This paper aims only to explore the possibilities for detection of change in size of plasma column using a single probe and so the entire approach is taken in a very idealistic situation with the underlying assumptions. A practical implementation of this approach is being investigated and is a subject to future scope of publications.

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