Spatiotemporal Structure of Ionization Waves in a Glow Discharge Plasma^{*)}

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Spatiotemporal structures formed in ionization waves are experimentally investigated in this study. A system involving ionization waves in a discharge tube has a few degrees of freedom in time and space. In our experiment, neon plasma is produced in a glass tube by a glow discharge between electrodes after the tube is evacuated to form a high vacuum. Spatiotemporal signals for the analysis are sampled as fluctuations in the light intensity by using a line-scan camera and photodiodes. The largest Lyapunov exponents are calculated from the time-series data sampled from the line-scan camera and photodiodes. Reconnection in the spatiotemporal structure is observed, which is caused by Eckhaus instability. Topological defects in the spatiotemporal structure are observed, which result in the appearance of spatiotemporal chaos; this leads to an order structure when an electric pulse is applied to the system as an external force or when coupled oscillators are synchronized.

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1. Introduction

In recent years, studies on nonlinear dynamics, such as chaos, fractals, and pattern formation, in dissipative systems have gained attention. Such studies are also important in plasma science because plasma is a typical medium, which includes nonlinearity [1-5]. In this paper, we focus on the dynamic behaviors of ionization waves [6,7] in glow discharge plasma. Ionization waves show various kinds of nonlinear behaviors, including chaos, as a function of the discharge current. Furthermore, spatiotemporal structures can be observed using a simple detector such as a photodiode and line-scan camera. Thus, ionization waves can be considered a suitable medium to study the universal properties of nonlinearity. The dynamics of ionization waves has been thoroughly investigated experimentally and theoretically [8–13]. The mechanism of spatiotemporal structures in ionization waves has been discussed from the viewpoint of Eckhaus instability as well [14, 15].

In this paper, in Sec. 2, three types of experimental configurations are described. In Sec. 3, the results obtained from the three experiments on spatiotemporal structures in ionization waves are discussed, and the obversations of the study are summarized in Sec. 4.

2. Experimental Setup

The configurations of the experimental setups are shown in Fig. 1. In this study, three types of experiments,

as shown in Figs. 1 (a) - (c) are performed using a glass tube with a diameter and length of 0.02 m and 0.75 m, respectively. Neon gas is introduced into the tube at a pressure of about 478 Pa after evacuating the tube to a high vacuum. In all our experiments, neon plasma is produced by glow discharge between the electrodes when dc electric field is applied to the electrodes. Typical electron and ion temperatures are about 10 eV and 0.025 eV, respectively. The discharge current is a typical control parameter that governs the system.

Figure 1 (a) shows the experimental configuration using one discharge tube. Figure 1 (b) shows an experimental configuration using one discharge probe by applying an electric pulse as an external force. Figure 1 (c) shows an experimental configuration to study dynamical behaviors of two coupled nonlinear oscillators by using two discharge tubes. We sampled 52 data points in Figs. 1 (a) and (b) and 16 data points in Fig. 1 (c), all sampled at every 1 cm interval. Light fluctuations along the central axis are observed using a line-scan camera. The positive column in ionization waves shows various kinds of dynamic behavior, including chaos and solitons [16], when varying the discharge current as the control parameter of the system. Herein, in most of the discharge current situations, group velocity propagates from the cathode to the anode as a traveling wave; in contrast, phase velocity propagates from anode to cathode, i.e., the ionization waves show features of the backward wave.

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Fig. 1 Experimental setup of three experimental configurations. The pressure in the tubes is fixed at about 478 Pa, and the discharge current is varied as a control parameter. The time-series signals are obtained from fluctuations in the light intensity by using photodiodes and a line-scan camera. (a) Experimental configuration using single discharge tubes, (b) experimental configuration using a single discharge tube by applying an electric pulse as an external force, and (c) experimental configuration to study dynamical behaviors of two coupled nonlinear oscillators by using two discharge tubes.

3. Results and Discussion

A system involving ionization waves in a discharge tube has degree of freedom in time and space, and time-



Fig. 2 Spatiotemporal structures: (a) typical order structure (25.8 mA) and (b) typical chaotic structure (24.8 mA) are shown.

series signals are obtained from fluctuations in the light intensity by using photodiodes and a line-scan camera. In our previous works [13, 17], only time-series data for the analysis were sampled using a photodiode. Moreover, in this study, data corresponding to the spatial position is sampled simultaneously using a line-scan camera, where time-series signals are obtained every $35.0 \,\mu s$ and the spatial data are obtained as the fluctuations in the light intensity.

First, the results corresponding to the experiment shown in Fig. 1 (a) are described. When the discharge current is varied as a control parameter, the system shows a variety of behaviors, as mentioned before. Figure 2 shows spatiotemporal structures: (a) typical order structure (25.8 mA) and (b) typical chaotic structure (24.8 mA).

We now discuss the mechanism through which spatiotemporal structure is formed in ionization waves. In a spatiotemporal structure, reconnections are caused by slip between two wave numbers: in the cathode side (right side of the figure) and anode side (left side of the figure). These are chimera states of two waves, and it can be explained that these states are caused by the Eckhaus instability [15], which causes a bifurcation of the wave number.

If the wave number bifurcates in orderliness, the spatiotemporal structure shows a regular pattern and not a chaotic one. In contrast, in the case that the wave number bifurcates in disorderliness, the spatiotemporal structure shows a chaotic pattern. In Fig. 2, the spatiotemporal



Fig. 3 Spatiotemporal structures (a) before and (b) under electric pulse application of a chaotic system, are shown. The electric pulse has a frequency of 1.8 kHz and the discharge current is 29.0 mA. (a) Chaotic and (b) order structure is shown, respectively.

structure shows chaotic dynamics, with several topological defects [18, 19] observed in the structure (Fig. 2 (b)). Discontinuous reconnection of two wave numbers in the positive column of ionization waves caused topological defects.

The results corresponding to the laboratory experiment shown in Fig. 1 (b). The effect of electric pulse application on the nonlinear dynamic systems is investigated using one discharge tube.

For chaotic analysis, Lyapunov exponents are calculated to examine the effect of the electric pulse application. The Lyapunov exponents are calculated from the time-series signals of waves sampled from the line-scan camera on a tube, based on the algorithm proposed in Ref. 20 and the time-series data obtained in our experiments. The largest Lyapunov exponent shows a positive value for chaotic oscillations; this value is higher for a more chaotic state and approaches zero for a system with periodic oscillations.

Figure 3 shows spatiotemporal structures before (Fig. 3 (a)) and during electric pulse application (Fig. 3 (b)), when the original system shows spatiotemporal chaos, i.e., under applied electric pulse to a chaotic system. The electric pulse has a frequency of 1.8 kHz, which is almost half the fundamental frequency of the original system. Here, the discharge current as a control parameter of the system is 29.0 mA. The chaotic system changes to a periodic one under this applied electric pulse.

The transformer incorporated in the circuit of

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- Fig. 4 Spatiotemporal structures (a) before and (b) under electric pulse application of a periodic system, are shown.The electric pulse has a frequency of 1.6 kHz and the discharge current is 26.0 mA. (a) Order and (b) chaotic structure are shown, respectively.
- Table 1Values of the largest Lyapunov exponents are shown for
(a) pulse application to a periodic system and (b) pulse
applied to a chaotic system.

(a)

	λ_{MAX} (anode side)	λ_{MAX} (cathode side)
Before	0.566 ± 0.013	0.460 ± 0.013
Under	0.394 ± 0.012	0.422 ± 0.001

(b)

	λ_{MAX} (anode side)	λ_{MAX} (cathode side)
Before	0.366 ± 0.017	0.263 ± 0.009
Under	0.586 ± 0.019	0.262 ± 0.020

Fig. 1 (b) has a resistance that it has an effect on the dynamic behaviors of discharge current values, which made it differ from one in Fig. 1 (a).

Figures 4 (a) and 4 (b) show spatiotemporal structures before and during the electric pulse application, to a periodic system. The electric pulse has a frequency of 1.6 kHz, which is almost half the fundamental frequency of the original system. Here, the discharge current as a control parameter of the system is 26.0 mA. In contrast to results shown in Fig. 3, the electric pulse application, changes the periodic system to a chaotic system.

Table 1 lists the values of the largest Lyapunov exponents for: (a) pulse applied to a periodic system, and (b)



Fig. 5 Spatiotemporal structures (a) before and (b) under coupling, when two original oscillators show spatiotemporal chaos, are shown. The discharge currents of wave 1 and 2 are 15.80 mA and 24.20 mA, respectively. (a) Chaotic and (b) order structure is shown, respectively. Horizontal and vertical ranges indicate 10 cm and 0.25 ms, respectively.

pulse applied to a chaotic system. Error bars are calculated using standard deviating several measurements. As mentioned before, the anode side (left side of the tube) and cathode side (right side of the tube) have different wave numbers through reconnection; therefore, the largest Lyapunov exponents for both sides are calculated independently.

From these results, the structures on the cathode side do not change significantly with the pulse application; in contrast, the structures on the anode side are affected by pulse application. Pulse application to a chaotic system result in chaos suppression because the value of the largest Lyapunov exponents becomes smaller; in contrast, the pulse applied to a periodic system causes chaos excitation because the value of the largest Lyapunov exponents becomes larger.

Finally, the results corresponding to the experiment shown in Fig. 1 (c) is described. The spatiotemporal dynamic behaviors of two coupled nonlinear oscillators are studied here using two discharge tubes.

Figures 5 (a) and 5 (b) show the spatiotemporal structures before and under coupling when the two original oscillators show spatiotemporal chaos. The discharge currents of wave 1 and 2 are 15.80 mA and 24.20 mA, respectively. It means that the spatiotemporal structure changes from chaos to order because of the coupling interaction.

Figure 6 shows the spatiotemporal structures (a) before and (b) during coupling when the two original oscil-





- Fig. 6 Spatiotemporal structures (a) before and (b) under coupling, when two original oscillators show spatiotemporal periodic states, are shown. The discharge currents of wave 1 and 2 are 20.30 mA and 13.95 mA, respectively.
 (a) Order and (b) chaotic structure are shown, respectively. Horizontal and vertical ranges indicate 10 cm and 0.25 ms, respectively.
- Table 2Total values of the largest Lyapunov exponents of two
oscillators are shown for (a) coupling interaction of two
chaotic systems and (b) coupling interaction of two pe-
riodic systems.

$\langle \rangle$			
(a)		λ_{MAX} (anode side)	λ_{MAX} (cathode side)
	Before	0.271 ± 0.013	0.080 ± 0.026
	Under	0.212 ± 0.033	0.099 ± 0.008
(b)		λ_{MAX} (anode side)	λ_{MAX} (cathode side)
	Before	0.139 ± 0.006	0.094 ± 0.017
	Under	0.199 ± 0.023	0.068 ± 0.012

lators show spatiotemporal periodic states. The discharge currents of wave 1 and 2 are 20.30 mA and 13.95 mA. It suggests the spatiotemporal structure changes from order to chaos are caused by the coupling interaction.

Table 2 and Fig. 7 list the total values of the largest Lyapunov exponents of the two oscillators (a) before coupling and (b) in coupling. The Error bars are calculated using standard deviating many measurements. The largest Lyapunov exponents for both sides (right and left sides) are calculated independently.

From these results, the structures on the cathode side do not transform by coupling; in contrast, the structures on the anode side are affected by coupling interaction. Coupling interaction of two chaotic systems results in chaos suppression because the value of the largest Lyapunov ex¥

anode side

¥

anode side

(a)

(b)

Largest Lyapunov Exp.[a.u.]

Largest Lyapunov Exp.[a.u.]

0.3 -

0.2

0.1

0.0

0.3 -

0.2

0.1

0.0



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cathode side

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cathode side

X Under

Before

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ponents becomes smaller; in contrast, coupling interaction of two periodic systems causes chaos excitation because the value of the largest Lyapunov exponents becomes larger.

The value of discharge current as the control parameter of systems is selected in such a way that dynamic behaviors of two oscillators change by coupling, therefore, a detail study on the relationship between the value of discharge current and the change of dynamic behavior by coupling is considered as future work.

4. Conclusions

Experimental study on spatiotemporal structures formed in ionization waves is summarized as follows.

• Reconnections and topological defects in the spatiotemporal structure in ionization waves are observed, and spatiotemporal chaos appears because of topological defect.