Spontaneous Ion Temperature Gradient in Inhomogeneous Magnetic Fields and Its Effect on the Parallel Heat Transport[*)]

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A new closure model for the parallel conductive heat flux of the perpendicular component of ion energy ($q_{\mathrm{i},\perp}$) is proposed which considers the effect of the spontaneous parallel gradient of the perpendicular ion temperature in inhomogeneous magnetic fields. Profiles of plasma parameters and the particle confinement efficiency are compared between the new $q_{\mathrm{i},\perp}$ model and a conventional one in a simple mirror system. It is found that the conservation of the magnetic moment is reproduced with the new $q_{\mathrm{i},\perp}$ model. Comparisons of ion power flux profiles show that the new $q_{\mathrm{i},\perp}$ model changes the direction of $q_{\mathrm{i},\perp}$ keeping the spontaneous parallel gradient of the perpendicular ion temperature. Almost linear relations between the particle confinement time and the ion-ion Coulomb collision time are also obtained with both $q_{\mathrm{i},\perp}$ models.

Keywords: plasma fluid model, closure model, conductive heat flux, ion temperature anisotropy, inhomogeneous magnetic field, mirror effect, magnetic moment

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1. Introduction

Several code packages have been developed for simulations of scrape-off layer (SOL)-divertor plasmas, such as SOLPS [1], SONIC [2, 3], UEDGE [4] and EMC3-EIRENE [5], in which plasmas are modeled by the Braginskii's plasma fluid model [6]. The same plasma fluid model is also applied to open-field devices [7–9] in order to analyze details of experimental results. Because of the necessity for applying it to a wide range of collisionality, kinetic corrections to the closure models are indispensable. In a usual way, the parallel conductive heat fluxes q are modeled by so-called the Fick's law (i.e. $q = -\kappa\nabla_{\parallel}T$) and the heat conductivities κ are corrected based on results of kinetic simulations [10, 11].

In plasma confinement systems, however, there exists inhomogeneity of magnetic fields, and a spontaneous parallel gradient of the temperature ($\nabla_{\parallel}T$) is generated without q due to the mirror effect. It indicates that the above-mentioned Fick's-law model for q is no longer appropriate in inhomogeneous magnetic fields.

We have been developing a one-dimensional (1D) plasma fluid model incorporating the anisotropic ion temperature (or pressure) and applying it to homogeneous magnetic fields [12–14]. Recently, we have been extending it to inhomogeneous magnetic fields based on generalized plasma fluid models [15, 16] and applying it to some simple configurations without the parallel conductive heat fluxes of ion q_{i} [17] or with the Fick's-law q_{i} model [18]. In this paper, we propose a generalized model for the parallel conductive heat flux of the perpendicular component of ion energy $q_{\mathrm{i},\perp}$ in inhomogeneous magnetic fields. We also demonstrate the performance of our new $q_{\mathrm{i},\perp}$ model by comparing the profiles of plasma parameters and the particle confinement efficiency between our new $q_{\mathrm{i},\perp}$ model and the conventional Fick's-law one in a simple mirror system.

2. Closure Model Appropriate in Inhomogeneous Magnetic Fields

2.1 Spontaneous parallel gradient of the perpendicular ion temperature

First, we discuss the parallel gradient of the perpendicular ion temperature ($T_{\mathrm{i},\perp}$) which spontaneously appears in inhomogeneous magnetic fields by using our 1D plasma fluid model based on the anisotropic ion temperature [17]. By combining Eqs. (1) and (4) in Ref. [17] (i.e., the equations of continuity of ion and perpendicular component of ion energy) with an assumption of the steady state, no volumetric sources, no Coulomb collisions and no conductive heat flux ($q_{\mathrm{i},\perp}$), following spontaneous parallel gradient of $T_{\mathrm{i},\perp}$ caused by the inhomogeneity of the

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magnetic field is obtained;

$$\left.\frac{\partial T_{\mathrm{i},\perp}}{\partial s}\right|_{\mathrm{sp}} = \frac{T_{\mathrm{i},\perp}}{B}\frac{\partial B}{\partial s}. \tag{1}$$

Here, notations are the same as Ref. [17] and the subscript "sp" denotes the spontaneous parallel gradient. If we continue to apply the Fick's law to $q_{\mathrm{i},\perp}$ (i.e., $q_{\mathrm{i},\perp} \propto -\kappa_{\mathrm{i}}(\partial T_{\mathrm{i},\perp}/\partial s)$), no parallel gradient of $T_{\mathrm{i},\perp}$ is allowed to keep $q_{\mathrm{i},\perp} = 0$, which contradicts Eq. (1). It is, thus, necessary to introduce an appropriate closure model for $q_{\mathrm{i},\perp}$ in inhomogeneous magnetic fields.

2.2 New closure model for $q_{\mathrm{i},\perp}$

We propose a new $q_{\mathrm{i},\perp}$ model which considers the spontaneous parallel gradient of $T_{\mathrm{i},\perp}$ caused by the inhomogeneity of the magnetic field. It is assumed in this model that finite $q_{\mathrm{i},\perp}$ arises when the parallel gradient of $T_{\mathrm{i},\perp}$ deviates from the spontaneous one given by Eq. (1) as follows;

$$\begin{aligned} q_{\mathrm{i},\perp} &\propto -\kappa_{\mathrm{i}}\left[\frac{\partial T_{\mathrm{i},\perp}}{\partial s} - \left.\frac{\partial T_{\mathrm{i},\perp}}{\partial s}\right|_{\mathrm{sp}}\right] \\ &= -\kappa_{\mathrm{i}} B \frac{\partial}{\partial s}\left(\frac{T_{\mathrm{i},\perp}}{B}\right). \end{aligned} \tag{2}$$

It is worth noting that Eq. (2) coincides with the Fick's law in homogeneous magnetic fields.

We also interpret this new $q_{\mathrm{i},\perp}$ model from a simple kinetic picture. Let us consider a net heat flux generated by one-way ion fluxes passing through a certain position s_0 together with the presence of a finite parallel gradient of the macroscopic magnetic moment $\mu_{\mathrm{m}}(s) \equiv T_{\mathrm{i},\perp}/B$. Due to a finite mean free path of ion-ion Coulomb collisions λ_{mfp}, ions keep the local μ_{m} at the positions of their last Coulomb collisions which are assumed to be deviated from s_0 by a half of λ_{mfp}. From this simple kinetic picture, too, we can derive the same $q_{\mathrm{i},\perp}$ model as Eq. (2) as follows;

$$\begin{aligned} q_{\mathrm{i},\perp} &\sim n\left\langle\left|v_{\parallel}\right|\right\rangle B(s_0)\,\mu_{\mathrm{m}}\left(s_0 - \frac{\lambda_{\mathrm{mfp}}}{2}\right) \\ &\quad - n\left\langle\left|v_{\parallel}\right|\right\rangle B(s_0)\,\mu_{\mathrm{m}}\left(s_0 + \frac{\lambda_{\mathrm{mfp}}}{2}\right) \\ &\approx -n\left\langle\left|v_{\parallel}\right|\right\rangle \lambda_{\mathrm{mfp}} B \frac{\partial}{\partial s}\left(\frac{T_{\mathrm{i},\perp}}{B}\right) \\ &\approx -\kappa_{\mathrm{i}} B \frac{\partial}{\partial s}\left(\frac{T_{\mathrm{i},\perp}}{B}\right). \end{aligned} \tag{3}$$

Here, $\left\langle\left|v_{\parallel}\right|\right\rangle$ stands for the average of the absolute values of the parallel random velocities of ions. This fact indicates that our new $q_{\mathrm{i},\perp}$ model involves the effect of conservation of the magnetic moment.

Because $q_{\mathrm{i},\perp}$ is defined by $q_{\mathrm{i},\perp} \equiv m_{\mathrm{i}} n \left\langle v_{\parallel} v_{\perp}^2 \right\rangle/2$ [16] ($v_{\perp}$ is the perpendicular random velocities of ions), it is necessary in a rigorous treatment to consider a synergetic effect of the gradients of $T_{\mathrm{i},\parallel}$ and $T_{\mathrm{i},\perp}$. In this study, however, we assume that the effect of the gradient of $T_{\mathrm{i},\parallel}$, which corresponds to the difference in $\left\langle\left|v_{\parallel}\right|\right\rangle$ of the one-way heat fluxes in Eq. (3), is small enough compared to that of the gradient of $T_{\mathrm{i},\perp}$. This assumption is also equivalent to Eq. (2) in which $q_{\mathrm{i},\perp}$ is assumed to be dependent only on the gradient of $T_{\mathrm{i},\perp}$. In this sense, it is necessary to validate our new $q_{\mathrm{i},\perp}$ model by comparing with a rigorous analytical model or kinetic simulations.

3. Numerical Model

3.1 Plasma fluid model

The performance of our new $q_{\mathrm{i},\perp}$ model is investigated by introducing it into our 1D plasma fluid model [17] and applying this to a simple mirror system. The basic equations for plasmas and notations are the same as Ref. [17]. We tentatively assume that the parallel conductive heat flux of the parallel component of ion energy, $q_{\mathrm{i},\parallel}$, is 0 although its Fick's-law form is given by $q_{\mathrm{i},\parallel} = -(1/3)\,\kappa_{\mathrm{i},\parallel}(\partial T_{\mathrm{i},\parallel}/\partial s)$. Meanwhile, we apply the Fick's law to the conductive heat flux of electron like $q_{\mathrm{e}} = -\kappa_{\mathrm{e}}(\partial T_{\mathrm{e}}/\partial s)$ for simplicity. Here, the heat-flux-limiting factor of electron $\alpha_{\mathrm{e}} = 0.5$ is introduced in the same way as Ref. [17].

As for $q_{\mathrm{i},\perp}$, we apply our new model;

$$q_{\mathrm{i},\perp} = -\frac{2}{3}\kappa_{\mathrm{i},\perp} B \frac{\partial}{\partial s}\left(\frac{T_{\mathrm{i},\perp}}{B}\right), \tag{4}$$

as well as the conventional Fick's law model;

$$q_{\mathrm{i},\perp} = -\frac{2}{3}\kappa_{\mathrm{i},\perp}\frac{\partial T_{\mathrm{i},\perp}}{\partial s}. \tag{5}$$

We call the latter, Eq. (5), "the normal $q_{\mathrm{i},\perp}$ model" hereafter. The coefficients "2/3" comes from the degrees of freedom of the Larmor motion. The heat-flux-limiting factor of the perpendicular component of ion $\alpha_{\mathrm{i},\perp} = 0.5$ is also introduced by $q_{\mathrm{i},\perp} = \left(1/q_{\mathrm{i},\perp}^{\mathrm{SH}} + 1/\alpha_{\mathrm{i},\perp} q_{\mathrm{i},\perp}^{\mathrm{FS}}\right)^{-1}$ in which the Spitzer-Härm heat conduction $q_{\mathrm{i},\perp}^{\mathrm{SH}} = -(2/3)\kappa_{\mathrm{i}}^{\mathrm{SH}} B\partial(T_{\mathrm{i},\perp}/B)/\partial s$ for the new $q_{\mathrm{i},\perp}$ model and $q_{\mathrm{i},\perp}^{\mathrm{SH}} = -(2/3)\kappa_{\mathrm{i}}^{\mathrm{SH}}\partial T_{\mathrm{i},\perp}/\partial s$ for the normal one, respectively, and the free-streaming heat flux $q_{\mathrm{i},\perp}^{\mathrm{FS}} = (2/3)\,nT_{\mathrm{i},\perp}\sqrt{T_{\mathrm{i},\parallel}/m_{\mathrm{i}}}$.

The ion-ion Coulomb collision time, which is estimated by using the effective isotropic ion temperature $T_{\mathrm{i}} \equiv (T_{\mathrm{i},\parallel} + 2T_{\mathrm{i},\perp})/3$, is given by $\tau_{\mathrm{i}} = \alpha_\tau 12\pi^{3/2}\epsilon_0^2\sqrt{m_{\mathrm{i}}}T_{\mathrm{i}}^{3/2}/\left(e^4 n\ln\Lambda\right)$ with $\alpha_\tau = 1$ [6] except for Sec. 4.3. For Sec. 4.3, α_τ is chosen from $0.3 < \alpha_\tau < 5$ in order to change the ion collisionality.

3.2 Calculation conditions

Figure 1 (a) shows the parallel-to-$\boldsymbol{B}$ profiles of the magnetic field strength B and corresponding cross-sectional area of the flux tube A used in this study. The dimensions of this system are chosen to be similar to those of the anchor cell of the tandem mirror device GAMMA 10/PDX [19]. The parallel lengths of the system and between the local maxima of B are set to be $L = 3.27$ m and $L_{\mathrm{m}} = 2.41$ m, respectively. A Gaussian-shape particle source is given by $S = S_0\exp[-\alpha_S(s/L)^2]$ as shown

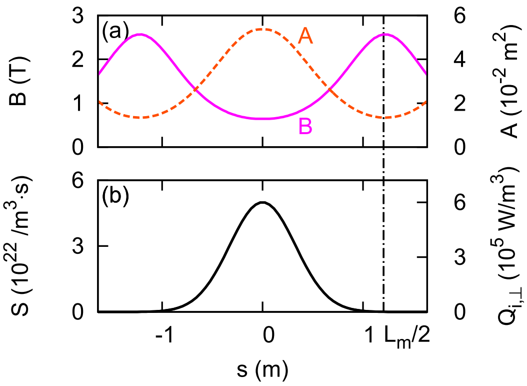

Fig. 1 Parallel-to-$\boldsymbol{B}$ profiles of (a) B (solid line), A (broken line), (b) S and $Q_{i,\perp}$. The vertical chain line represents the position of the local maximum of B ($s = L_m/2$).

in Fig. 1 (b) in which $S_0 = 5 \times 10^{22}$ /m^3·s and $\alpha_S = 50$ are used. The momentum source is $M_m = 0$. The ion heat sources are given like $Q_{i,\parallel} = 0$ and $Q_{i,\perp} = T_{in,i,\perp} S$, respectively. Thus, ions initiate from the outside of the loss cone. The electron heat sources is given like $Q_e = (3/2) T_{in,e} S$. In order to keep T_e much lower than $T_{i,\parallel}$ and exclude the effect of the electrostatic potential, the source temperatures are set to be $T_{in,i,\perp} = 75$ eV and $T_{in,e} = 0.5$ eV, and the equipartition terms are turned off. The local sound speed is, therefore, $c_s \equiv \sqrt{(T_{i,\parallel} + T_e)/m_i} \approx \sqrt{T_{i,\parallel}/m_i}$. The Mach number is evaluated by $M \equiv V/c_s$. Sheath boundary conditions are imposed at both boundaries (i.e. $s = \pm L/2$) by using a virtual divertor model [13].

4. Results and Discussions

4.1 Comparison of plasma profiles

Figure 2 shows a direct comparison of plasma profiles between two $q_{i,\perp}$ models. When focusing on the confined region (i.e. $-L_m/2 < s < L_m/2$), n becomes higher, V becomes slower, $T_{i,\parallel}$ and $T_{i,\perp}$ become lower with the new $q_{i,\perp}$ model than the normal one. As shown in Fig. 2 (c), the plasma flow is almost suppressed in the confined region but becomes supersonic in the downstream diverging B regions with both $q_{i,\perp}$ models. Note also that $T_{i,\perp}$ is almost proportional to B with the new $q_{i,\perp}$ model while it is almost flat with the normal one as shown in Fig. 2 (e). It is, therefore, demonstrated that conservation of the magnetic moment is reproduced with the new $q_{i,\perp}$ model.

4.2 Ion power flux profiles

Parallel-to-$\boldsymbol{B}$ profiles of ion power fluxes are shown in Fig. 3. Ion power fluxes by the conduction ("cond") and convection ("conv") of the perpendicular component, the convection of the internal ("int") and flow energy of the parallel component are denoted by $P_{i,\perp}^{cond} \equiv q_{i,\perp} A$, $P_{i,\perp}^{conv} \equiv n T_{i,\perp} V A$, $P_{i,\parallel}^{conv,\,int} \equiv (3/2)\, n T_{i,\parallel} V A$ and $P_{i,\parallel}^{conv,\,flow} \equiv (1/2)\, m_i n V^3 A$, respectively. When focusing on the confined region, a negative $P_{i,\perp}^{cond}$ occurs with the normal $q_{i,\perp}$

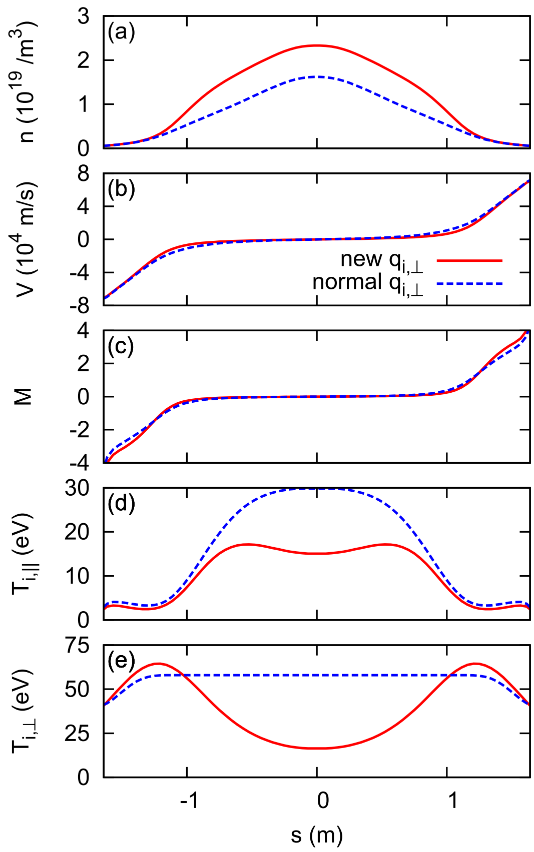

Fig. 2 Parallel-to-$\boldsymbol{B}$ profiles of plasma parameters between the new $q_{i,\perp}$ model (solid lines) and the normal one (broken lines); (a) n, (b) V, (c) M (d) $T_{i,\parallel}$ and (e) $T_{i,\perp}$.

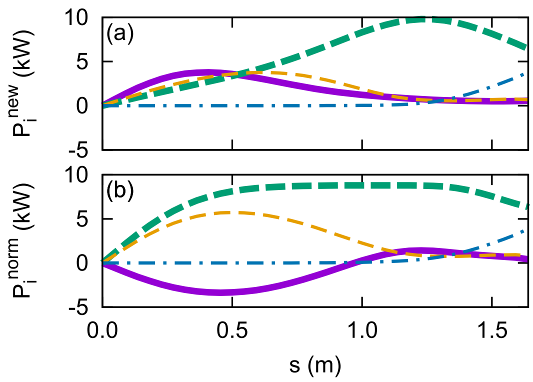

Fig. 3 Power fluxes, $P_{i,\perp}^{cond}$ (thick solid), $P_{i,\perp}^{conv}$ (thick broken), $P_{i,\parallel}^{conv,\,int}$ (thin broken) and $P_{i,\parallel}^{conv,\,flow}$ (thin chain lines), with (a) the new $q_{i,\perp}$ model and (b) the normal one.

model which flattens $T_{i,\perp}$ profile (but still slightly hollow) as shown in Fig. 2 (e). With the new $q_{i,\perp}$ model, on the other hand, a positive $P_{i,\perp}^{cond}$ occurs keeping the spontaneous parallel gradient of $T_{i,\perp}$. It is also shown from Fig. 3 that $P_{i,\parallel}^{conv,\,int}$ decreases by half in $0 \le s \le 0.5$ m with the new $q_{i,\perp}$ model. In order to identify the reason for this, we also investigated detailed ion power fluxes including the energy transfer between parallel and perpendicu-

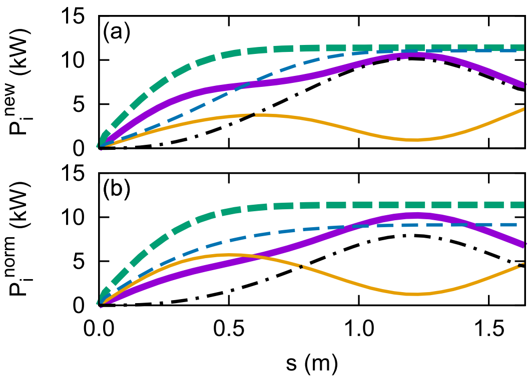

Fig. 4 Power fluxes, $P^{\rm tot}_{\rm i,\perp}$ (thick solid), $P^{\rm tot}_{\rm i,\parallel}$ (thin solid), $P^{Q}_{\rm i,\perp}$ (thick broken), $P^{\perp\to\parallel}_{\rm i,\,rlx}$ (thin broken) and $P^{\parallel\to\perp}_{\rm i,\,mirr}$ (thin chain lines), with (a) the new $q_{\rm i,\perp}$ model and (b) the normal one.

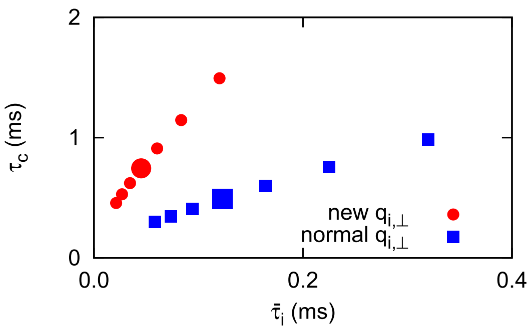

Fig. 5 Particle confinement time $\tau_{\rm c}$ as a function of $\bar{\tau}_{\rm i}$; the new $q_{\rm i,\perp}$ model (circles) and the normal one (squares). The larger symbols represent the cases with $\alpha_\tau = 1$.

lar components. Here, ion power fluxes by $Q_{\rm i,\perp}$, the collisional relaxation of the ion temperature anisotropy from perpendicular to parallel components and the mirror effect from parallel to perpendicular components are denoted by $P^{Q}_{\rm i,\perp} = \int_0^s Q_{\rm i,\perp} A ds'$, $P^{\perp\to\parallel}_{\rm i,rlx} = \int_0^s [n(T_{\rm i,\perp} - T_{\rm i,\parallel})/\tau_{\rm rlx}] A ds'$ and $P^{\parallel\to\perp}_{\rm i,mirr} = \int_0^s [(nT_{\rm i,\perp}V + q_{\rm i,\perp})/B](\partial B/\partial s') A ds'$, respectively. These ion power fluxes are shown in Fig. 4 with the total ion power fluxes of perpendicular and parallel components defined by $P^{\rm tot}_{\rm i,\perp} \equiv P^{\rm cond}_{\rm i,\perp} + P^{\rm conv}_{\rm i,\perp}$ and $P^{\rm tot}_{\rm i,\parallel} \equiv P^{\rm conv,\,int}_{\rm i,\parallel} + P^{\rm conv,\,flow}_{\rm i,\parallel}$, respectively. Due to the energy conservation, $P^{\rm tot}_{\rm i,\perp} \simeq P^{Q}_{\rm i,\perp} - P^{\perp\to\parallel}_{\rm i,rlx} + P^{\parallel\to\perp}_{\rm i,mirr}$ and $P^{\rm tot}_{\rm i,\parallel} \simeq P^{\perp\to\parallel}_{\rm i,rlx} - P^{\parallel\to\perp}_{\rm i,mirr}$ hold. In $0 \le s \le 0.5$ m, $P^{\perp\to\parallel}_{\rm i,rlx}$ with the new $q_{\rm i,\perp}$ model is smaller than that with the normal one while $P^{\parallel\to\perp}_{\rm i,mirr}$ is almost the same between two $q_{\rm i,\perp}$ models. That decreases $P^{\rm conv,\,int}_{\rm i,\parallel}$ and $T_{\rm i,\parallel}$ by half with the new $q_{\rm i,\perp}$ model in this region as shown in Fig. 2 (d) under the assumption of $q_{\rm i,\parallel} = 0$. We will develop an appropriate model for $q_{\rm i,\parallel}$ near future.

4.3 Particle confinement time

The particle confinement time $\tau_{\rm c}$ of this single mirror system, which is defined by $\tau_{\rm c} \equiv \int_{-L_{\rm m}/2}^{L_{\rm m}/2} nA ds / \int_{-L_{\rm m}/2}^{L_{\rm m}/2} SA ds$, is also investigated and compared between two $q_{\rm i,\perp}$ models under the assumption of $q_{\rm i,\parallel} = 0$. Figure 5 shows $\tau_{\rm c}$ as a function of $\bar{\tau}_{\rm i}$ which is defined by $\bar{\tau}_{\rm i} \equiv \int_{-L_{\rm m}/2}^{L_{\rm m}/2} \tau_{\rm i} A ds / \int_{-L_{\rm m}/2}^{L_{\rm m}/2} A ds$. Almost linear relations between $\tau_{\rm c}$ and $\bar{\tau}_{\rm i}$ are observed with both $q_{\rm i,\perp}$ models. Meanwhile, the efficiency $\tau_{\rm c}/\bar{\tau}_{\rm i}$ is higher with the new $q_{\rm i,\perp}$ model than the normal one by a factor of ~ 3, which is also indicated by the increase in n as shown in Fig. 2 (a). We will investigate the relationship between $\tau_{\rm c}$ and the mirror ratio after introducing an appropriate model of $q_{\rm i,\parallel}$ near future.

5. Summary

The Fick's-law modeling of the parallel conductive heat fluxes q (i.e. $q = -\kappa\nabla_\parallel T$) in the widely-used Braginskii's plasma fluid model [6] is no longer appropriate in inhomogeneous magnetic fields due to the spontaneous parallel gradient of the ion temperature. In this paper, we propose a new closure model for the parallel conductive heat flux of the perpendicular component of ion energy ($q_{\rm i,\perp}$) which considers the spontaneous parallel gradient of the perpendicular ion temperature in inhomogeneous magnetic fields.

We also introduce the new $q_{\rm i,\perp}$ model into our 1D plasma fluid model based on the anisotropic ion temperature [17] and compare the profiles of plasma parameters and the particle confinement efficiency with a conventional (i.e. Fick's-law) $q_{\rm i,\perp}$ model in a simple mirror system under an assumption of the parallel conductive heat flux of the parallel component of ion energy $q_{\rm i,\parallel} = 0$. It is found that the conservation of the magnetic moment is reproduced with the new $q_{\rm i,\perp}$ model while the profile of the perpendicular ion temperature becomes flat with the conventional one. Comparisons of ion power flux profiles show that the new $q_{\rm i,\perp}$ model changes the direction of $q_{\rm i,\perp}$ keeping the spontaneous parallel gradient of the perpendicular ion temperature. It is also shown that the collisional relaxation of the ion temperature anisotropy becomes weaker making the parallel ion temperature lower with the new $q_{\rm i,\perp}$ model under the assumption of $q_{\rm i,\parallel} = 0$. We also obtain almost linear relations between the particle confinement time and the ion-ion Coulomb collision time with both $q_{\rm i,\perp}$ models.

As our future works, an appropriate model for $q_{\rm i,\parallel}$ in inhomogeneous magnetic fields will be introduced. With this new $q_{\rm i,\parallel}$ model, the relationship between the particle confinement time and the mirror ratio will be investigated. We also have a plan to compare our numerical results with experimental ones of GAMMA 10/PDX, which is characterized by the high and anisotropic ion temperature [20].

[1] R. Schneider *et al*., Contrib. Plasma Phys. **46**, 3 (2006).
[2] H. Kawashima *et al*., Plasma Fusion Res. **1**, 031 (2006).
[3] K. Shimizu *et al*., Nucl. Fusion **49**, 065028 (2009).
[4] T.D. Rognlien *et al*., J. Nucl. Mater. **196-198**, 347 (1992).
[5] Y. Feng *et al*., Plasma Phys. Control. Fusion **44**, 611 (2002).

[6] S.I. Braginskii, *Reviews of Plasma Physics*, vol. **1** (Consultants Bureau, New York, 1965), p.205.
[7] H. Kastelewicz and G. Fussmann, Contrib. Plasma Phys. **44**, 352 (2004).
[8] N. Ohno *et al*., Contrib. Plasma Phys. **36**, 339 (1996).
[9] M.S. Islam *et al*., Plasma Phys. Control. Fusion **59**, 125010 (2017).
[10] A. Froese *et al*., Plasma Fusion Res. **5**, 026 (2010).
[11] D. Tskhakaya *et al*., Contrib. Plasma Phys. **48**, 89 (2008).
[12] S. Togo *et al*., J. Nucl. Mater. **463**, 502 (2015).
[13] S. Togo *et al*., J. Comput. Phys. **310**, 109 (2016).
[14] S. Togo *et al*., Contrib. Plasma Phys. **56**, 729 (2016).
[15] E. Zawaideh *et al*., Phys. Fluids **29**, 463 (1986).
[16] W. Fundamenski, Plasma Phys. Control. Fusion **47**, R163 (2005).
[17] S. Togo *et al*., Contrib. Plasma Phys. **58**, 556 (2018).
[18] S. Togo *et al*., Plasma Fusion Res. **13**, 3403022 (2018).
[19] Y. Nakashima *et al*., Nucl. Fusion **57**, 116033 (2017).
[20] N. Ezumi *et al*., AIP Conf. Proc. **1771**, 060002 (2016).