Development of the Grating Mirror for the High Power Transmission System and Its General Theory*)

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In the transmission system of the high power Electron Cyclotron Heating (ECH), many mirrors have been installed in the injection antenna section in particular. In some cases, it is difficult to design a normal mirror due to the restriction of the boundary condition. A grating mirror is one of the solutions in such cases. Moreover, controlled power splitting or combining at high power can be realized by a grating mirror. Diffraction angle, efficiency, and distortion/conversion of the polarization can be controlled by the grating shape design. In this study, we developed a grating mirror to efficiently separate the second harmonics component from the fundamental high power beam at 82.7 GHz and higher harmonics. The performance of this grating is confirmed experimentally for the use of vortex beam formation. During this development, the design procedure of a grating mirror is established and generalized to apply for various purposes.

Keywords: grating mirror, ECH, ECE, 2nd harmonics, optical vortex

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1. Introduction

In the magnetic confinement fusion device, the high power and high frequency microwave is used for generating the high temperature high density plasma. This device is the Electron Cyclotron Heating (ECH), and it is used not only for heating but also for plasma start up and current drive. This microwave is generated by a cyclotron maser action of a relativistic hollow electron beam injected in a cylindrical cavity under a resonance magnetic field, which is called a gyrotron. In the transmission system of the high power ECH, many mirrors have been installed in the injection antenna section, in particular. In some cases, it is difficult to design a normal mirror due to the restriction of the boundary condition. A grating mirror is one of the solutions in such cases. Moreover, controlled power splitting or combining at high power can be realized by the grating mirror. Diffraction angle, efficiency, and distortion/conversion of the polarization can be controlled by the grating shape design. This design can be decided by considering the grating condition and the local wave-number on the mirror surface. In the case of the normal mirror, the mirror surface must satisfy the law of reflection by considering the local wave-number on the mirror surface. In the geometrical optics limit, the mirror surface is expressed as part of the ellipsoid, which satisfies a constant distance from the focal points of the input beam and the output beam. In the case of Gaussian optics, the constant phase distance surface defines the quasi-optical mirror [1]. This constant phase distance satisfies a local reflection condition between the wave-numbers of the input beam, the output beam and the normal vector of the mirror surface. By considering the reflection direction on the grating and the diffraction direction of the desired diffraction order, we can design the grating on a given mirror surface so that the diffracted wave forms a desired beam satisfying a grating condition. Here, we have developed a grating mirror to efficiently separate the second harmonics component from the fundamental high power beam at 82.7 GHz and higher harmonics. The performance of this grating is confirmed experimentally for the use of vortex beam formation. During this development, design procedure is established and generalized to apply for various purposes.

As one of the applications of the diffraction grating, the optimized grating mirror designed for the generation experiment of the Electron Cyclotron Emission (ECE) with vortex property plays an important role. Recently, it is theoretically demonstrated that the radiation from an electron in spiral motion has a vortex property [2]. We are developing a method to generate the vortex radiation directly from the electrons in cyclotron motion with controlled gyro-phase by Right Hand Circular Polarization (RHCP) wave as shown by the experimental device in Fig. 1. Electrons are injected from the electron gun installed behind the reflecting mirror through a hole along the magnetic field. The RHCP wave into the vacuum vessel is converted into the Gaussian beam with a focal point at the center of the superconducting magnet and propagated along the field line. The group of electrons resonantly accelerated by the RHCP wave field is expected to emit second or higher har-

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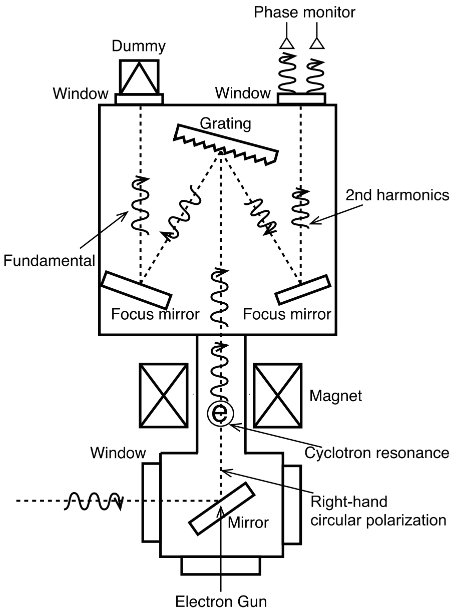

Fig. 1 Experimental set up of the ECE measurement with vortex properties.

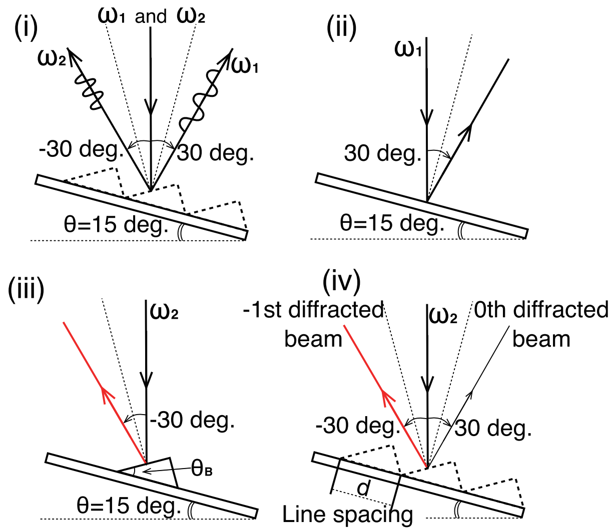

Fig. 2 (i) Required grating mirror performance. Fundamental high power beam is to be reflected only toward +30 deg. Main power of the 2nd harmonics is to be diffracted toward −30 deg. (ii) The radiation of the frequency ω_1 is only reflected or 0th order diffracted toward +30 deg. direction without higher order diffraction by setting $d < \lambda/2$ or $k_g > 2k_0$ on the grating mirror. (iii) The main power of the radiation of the frequency ω_2 is diffracted toward −30 deg. by the slope with the blaze angle θ_B on the mirror. (iv) Many slopes are placed on the mirror in order to make the grating. Determine the line spacing so that the −1th beam can be directed toward −30 deg.

monics radiation with the vortex feature. Here, the grating mirror is designed to separate the second harmonics from the fundamental and higher harmonics. The second harmonics (165.4 GHz) is separated using the grating mirror. In this case, the grating mirror is very effective.

This paper is composed of four sections. In section 2, development of the plane grating mirror to separate the second harmonics from the high power fundamental and other higher harmonics radiation is described. And Results of the low power experiment of the grating mirror also are briefly described. In section 3, the general theory of the development of the grating mirror is discussed. The summary of this paper is provided in section 4.

2. Development of the Grating Mirror

In the analysis of radiation with many frequency spectral peaks, the grating mirror plays an important role in separating these frequencies toward its specific angle. Although the diffraction beam is usually a weak signal, we can optimize the grating parameters to maximize the diffraction efficiency by designing the grating with an adequate blaze angle. In this case, the two frequencies ω_1 = 82.7 GHz and ω_2 = 165.4 GHz are considered. The grating mirror is designed as shown in Fig. 2 (i). That is, ω_1 is reflected toward +30 deg. and ω_2 is reflected and diffracted toward −30 deg. The grating mirror is designed by the order shown from Fig. 2 (ii) to Fig. 2 (iv). (ii): Note that the ω_1 is only satisfied with the law of reflection (only 0th order diffraction). That is, the injection angle is 15 deg. with respect to the base mirror surface. (iii): To reflect toward −30 deg. with respect to the ω_2, the slope shaped saw-tooth is added on the base mirror surface. The angle of the slope is reflected toward −30 deg. with respect to this slope surface for ω_2. That is, the angle of the slope is 30 deg. This is the blaze angle. (iv): Considering the case in which many slopes are placed regularly, this corresponds to the grating with line spacing d. The direction of the diffracted beam is determined by d. Here, let us decide that the line spacing can be only propagated to −30 deg. for the diffracted beam of the ω_2. A well known diffraction condition is

$$\sin\theta_{\rm in} + \sin\theta_{m,{\rm out}} = \frac{mk_g}{nk_0}, \tag{1}$$

where m is the diffraction order, n is the harmonics number, $\theta_{\rm in}$ is the injection angle, $\theta_{m,{\rm out}}$ is the diffraction angle of the mth order, k_g is the wave-number of the grating which is defined by $k_g = 2\pi/d$, and k_0 is the wave-number of the nth harmonics beam. Figure 3 shows the relationship between the line spacing and the direction of the diffracted beam. As can be seen from this figure, the line spacing d should be set at 1.878 mm in order to propagate in only the 2nd diffracted beam to −30 deg. direction. Therefore, although the $n = 0$ component, which is only a reflection component on the mirror surface, cannot be ignored, the main diffracted beam can be propagated −30 deg. Fundamental radiation ω_1 can be only reflected to +30 deg. (0th order) without generating the extra diffracted beam by choosing the proper line spacing. Note that the surface roughness of the grating mirror we developed is too small for ω_1 (the wave length λ_1 of the ω_1 (λ_1 = 3.628 mm) is much greater than the height of the slope which is 0.813 mm.) and that the surface is recognized as a plane mirror. That is, ω_1 is

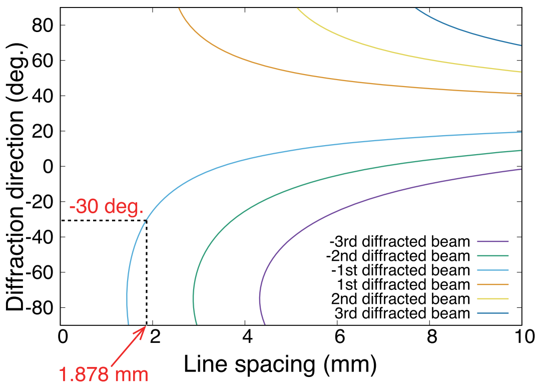

Fig. 3 Relationship between the line spacing and the direction of the diffracted beam.

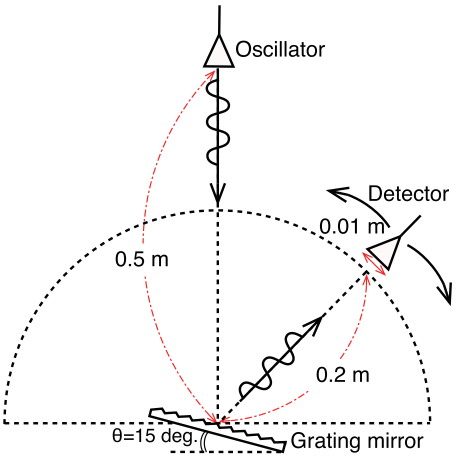

Fig. 4 Low power experimental set up: The beams which are emitted by the oscillator are reflected or diffracted by the grating mirror. These beam radiation patterns are measured by the angle scanning receiving antenna by 1 deg. Since the experimental setup is arranged so that the both long side of the antenna area and moving distance of the antenna per deg. are almost the same, the angular resolution is best.

neither refracted nor diffracted on the added slopes. Therefore, ω_1 is reflected toward +30 deg. and ω_2 is almost reflected and diffracted toward −30 deg. by the many slopes on the mirror. This corresponds to the Littrow mount condition.

In order to confirm the efficiency of the grating mirror we developed, low power experiments are carried out. Figure 4 shows the experimental set up. The 84 GHz and the 165.4 GHz beam sources are used. These beams are propagated to the grating mirror individually and the reflected or/and the diffracted beam are detected by the horn antenna. Figure 5 shows the experimental results. In the 84 GHz, it is found that the beam was propagated to the direction designed precisely. In the 165.4 GHz, the reflected beam and the diffracted beam were distributed mainly in the direction of −30 deg. Some diffracted beams are propagated in the direction of 30 deg. However, it was clearly demonstrated that the radiation at the ω_1 and the ω_2 can be

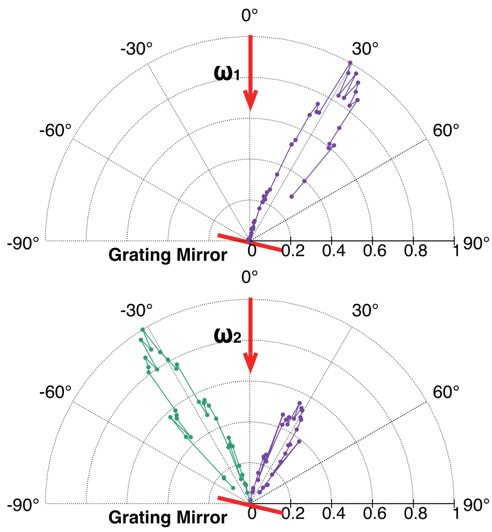

Fig. 5 Angular distribution of the ω_1 and the ω_2: Intensity is represented by the radial direction and it is normalized by the maximum. Propagation directions of the reflection or diffraction beams are represented by the angle direction.

separated by this grating mirror as designed.

3. General Theory of the Grating Mirror Development

The grating mirror which was developed here is plane-shaped because machining or development was easier. However, this design method can be generalized to the grating mirror which has an arbitrary base mirror shape such as a concave mirror.

3.1 Definition of the mirror surface

The local wave-number $\boldsymbol{k}$ of the Gaussian beam which has finite spatial spread is defined by a gradient of the equiphase front Φ [3] as follows,

$$\boldsymbol{k} = \nabla \Phi. \tag{2}$$

Once the input and the output of the Gaussian beam parameters are defined, local wave numbers $\boldsymbol{k}_{\rm in} = \nabla \Phi_{\rm in}$ and $\boldsymbol{k}_{\rm out} = \nabla \Phi_{\rm out}$ on a mirror surface should satisfy the equation below,

$$\boldsymbol{k}_{\rm in} - \boldsymbol{k}_{\rm out} = \nabla S_{\rm mirror} \parallel \boldsymbol{N}. \tag{3}$$

The $S_{\rm mirror}$ is the mirror surface function defined by $S_{\rm mirror} = \Phi_{\rm in} - \Phi_{\rm out}$. The $\boldsymbol{N}$ is the normal vector of the mirror surface. By integrating eq. (3), the mirror surface function is determined as follows,

$$S_{\rm mirror} = \Phi_{\rm in} - \Phi_{\rm out} = {\rm const.} \tag{4}$$

In other words, the mirror surface is defined as the constant surface of the difference between $\Phi_{\rm in}$ and $\Phi_{\rm out}$. This is a phase matching condition or a constant phase distance condition of between input and output beam on the mirror, and can be interpreted as a generalization of the elliptical mirror that satisfies a constant distance between focal points for a geometrical optics [1]. It should be noted that the input beam and the output beam are Gaussian beams. The beam parameter, waist size, and waist position should be matched not only to the phase but also to the intensity on the mirror surface in 0th order. This gives the condition of input and output Gaussian beam parameters.

3.2 Relationship between grating condition and mirror surface

The grating condition for the vector representation is

$$\boldsymbol{k}_{\mathrm{out},m} \cdot \nabla G = \boldsymbol{k}_{\mathrm{out}} \cdot \nabla G + m\boldsymbol{k}_G \cdot \nabla G, \tag{5}$$

where $\boldsymbol{k}_{\mathrm{out},m}$ is wave-number vector of the mth order diffracted beam and $\boldsymbol{k}_G$ is the wave-number of the grating. G is the grating function which is defined by $\boldsymbol{k}_G = \nabla G$. Here, the direction of the $\boldsymbol{k}_G$ is periodic direction of the grating. The orthogonal condition between the grating surface and the mirror surface is

$$\nabla S_{\mathrm{mirror}} \cdot \nabla G = 0. \tag{6}$$

Although the relationship which determines the mirror surface function is given by eq. (3), the grating mirror is considered as a form in which the grating function is incorporated into this mirror surface function. These relationships are represented as follows,

$$\nabla \Phi_{\mathrm{in},\omega_1} - \nabla \Phi_{\mathrm{out},\omega_1} = \nabla S_{\omega_1}, \tag{7}$$

$$\nabla \Phi_{\mathrm{in},\omega_2} - \nabla \Phi_{\mathrm{out},\omega_2} = \nabla S_{\omega_2}, \tag{8}$$

$$\nabla \Phi_{\mathrm{out},\omega_2} \cdot \nabla G + m\nabla G \cdot \nabla G = \nabla \Phi_{\mathrm{out},\omega_2,m} \cdot \nabla G, \tag{9}$$

where $\Phi_{\mathrm{in},\omega_1}$, $\Phi_{\mathrm{out},\omega_1}$, and S_{ω_1} are equiphase front of the injection beam, the reflection beam, and the mirror surface function of the these beams, respectively. $\Phi_{\mathrm{in},\omega_2}$, $\Phi_{\mathrm{out},\omega_2}$, and S_{ω_2} are equiphase front of the injection beam, the refraction beam, and the mirror surface function of these beams, respectively. Eq. (9) is replaced by substituting eq. (2) for eq. (5). $\Phi_{\mathrm{out},\omega_2,m}$ is equiphase front of the mth order diffraction beam. Multiplying eq. (8) by ∇G and using the orthogonal condition eq. (6), we can obtain the following equation

$$\nabla \Phi_{\mathrm{in},\omega_2} \cdot \nabla G + m\nabla G \cdot \nabla G = \nabla \Phi_{\mathrm{out},\omega_2,m} \cdot \nabla G. \tag{10}$$

That is,

$$\nabla(\Phi_{\mathrm{in},\omega_2} - \Phi_{\mathrm{out},\omega_2,m} + mG) \cdot \boldsymbol{N}_G = 0, \tag{11}$$

where $\boldsymbol{N}_G$ is the normal vector on the grating function, which is defined by $\boldsymbol{N}_G = \nabla G/|\nabla G|$. If the equiphase of the ω_2 and the equiphase of the mth order diffracted beam are known, the grating function G is determined. Therefore, the grating function G is defined on the mirror surface function S_{ω_1}.

Furthermore, the grating efficiency can be optimized by defining grating plane by eq. (8). This is equivalent to optimizing blaze angle in order to coincide with the directions of diffraction and reflection on each grating plane. This is a generalization of the method to design a grating on an elliptical mirror support proposed by Tran [4]. So far, only the matching condition of the phase front are discussed. In addition to this phase matching, it is also important to make the intensity profiles match on the mirror/grating surface in order to maintain the purity of the Gaussian beam property. Detailed discussion will be given elsewhere.

4. Summary

In this research, development of the grating mirror for the high power transmission system was carried out and its general theory was derived. The development of the plane grating mirror was carried out in order to separate the fundamental beam and the 2nd harmonics beam. This grating mirror can concentrate the reflection beam and the mth order diffraction beam to the same direction because the shape of the grating has saw-tooth surface. In the results of the low power experiment, although the diffraction beam of the 0th could not be vanished, the fundamental beam and the 2nd harmonics could be separated precisely.

The general theory of the grating mirror was derived. Although the grating mirror is usually plane-shaped because machining or development is easier, it is possible to develop a grating mirror which has arbitrary mirror shape by defining the grating function on the base mirror surface. That is, when Gaussian beam parameters of the injection beam and desired mth order diffraction beam are given, the grating mirror which satisfies the Littrow mount condition can be designed. The grating described in section 2 was a simple plane grating. We can apply this design method to upgrade this plane grating mirror. Fundamental high power beam can be efficiently focused into a dummy load by designing optimized base mirror. The 2nd harmonics beam can be separated efficiently toward -1st order diffraction direction to form designed Gaussian beam parameter that simplifies the measurement of vortex feature at the 2nd harmonics frequency. Such an idea of designing general Littrow mount grating can be applicable to various situations in which normal mirror design is difficult due to limited space or boundary conditions even under high power.

Acknowledgments

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[1] S. Kubo *et al.*, Fusion Eng. Des. **26**, 319 (1995).
[2] M. Katoh *et al.*, Phys. Rev. Lett. **118**, 094801 (2017).
[3] L.D. Landau and E.N. Lifshitz, *The Classical Theory of Fields*, 4th Rev. English Ed. (Butterworth-Heinemann, 1980).
[4] M.Q. Tran *et al.*, J. Appl. Phys. **73**, 2089 (1993).