

Surface Current Approximation of Magnetization Currents of Magnetic Materials in Axisymmetric Tokamak Devices for the Correction of their Effects on Magnetic Diagnostics^{*)}

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Magnetic materials such as ferritic steel are planned to be installed in fusion DEMO reactors to support blankets and in-vessel components with low activation. Since a magnetic material disturbs magnetic measurement, it is difficult to perform equilibrium control when plasma and sensors are placed between magnetic materials. To calculate the position of plasma current centroid with the disturbed magnetic measurement, we devised a simple scheme to correct their effects on magnetic diagnostics by approximating the magnetization currents as surface currents. Estimated errors of the position by this scheme, especially in the R position, are about 1%. Those with positionally dependent surface currents become smaller by relocating the filament positions for plasma current. Additionally, flux surfaces reconstructed by this correction show agreement with the true flux surfaces calculated by finite element method.

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1. Introduction

Magnetometry for position and cross-sectional shape measurement of the main plasma is essential for plasma control and MHD analysis in tokamak devices. However, it can be disturbed by the usage of magnetic materials. In the case of an iron core transformer, the magnetic materials which are located on the outside of the poloidal region that is encircled by magnetic sensors have already been considered and have been applied to real-time control such as Boundary Element Method [1] and Integral Equations Method [2].

Support structures made of low activation ferritic steel F82H [3] will be installed in the vacuum vessel of DEMO reactors from the perspective of heat load and neutron bombardment [4]. Since the F82H is a magnetic material, there are adverse effects on magnetometry and plasma control. Concerning this problem, correction effectiveness was confirmed in the experiment on JT-60U where magnetic tiles were employed for toroidal magnetic field ripple reduction [5]. The measured data affected by thin ferritic steel tiles, which were installed near sensors, were corrected under an assumption of magnetic saturation by a strong toroidal magnetic field which is characteristic to tokamak devices [6]. However, such assumption is not applicable in DEMO reactors since bulky magnetic materials

are planned to be installed as support structures of blankets.

The objective of this study is to calculate plasma current centroid position which is surrounded by magnetic materials. The magnetic materials are installed axisymmetrically. To correct the adverse effect of the magnetic materials, we propose a simple improved scheme to the filament current approximation method [7], which has been conventionally used in plasma position and cross-sectional shape determination. The correction was based on the hypothesis that the magnetization currents, which flow in each magnetic material block, are considered as one toroidal ring. We put the ring current as a surface current, inspired by the Cauchy-Condition Surface (CCS) Method which calculates the surface current distribution on a Cauchy-condition surface [8]. Then, the magnetization currents approximated as surface currents are calculated with mutual inductances between sensors and surface currents which act as filament currents in the filament current approximation method. The advantage of this scheme is the unnecessary of explicitly calculating relative permeability. In this scheme, magnetization currents are calculated in the same way as the filament currents by solving an inverse problem. Therefore, the plasma position can be calculated even in a system which includes magnetic materials whose relative permeability changes. In Sec. 2, the calculation scheme will be described. The results and dis-

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cussions will be shown in Sec. 3, and the findings will be summarized in Sec. 4.

2. Model Analysis with Magnetic Materials

To investigate the improved scheme effectiveness, we use a simplified two-dimensional axisymmetric model to calculate the data sets of poloidal magnetic flux as experimental inputs. By using these individual data sets, the position of plasma current centroids and the magnetic surfaces are evaluated by the filament current approximation method.

2.1 Calculation model

In this study, the dataset calculation is performed with a finite element method (FEM) code in the COMSOL Multiphysics software. The simple model is as follows: poloidal magnetic field coils (PFC) and plasma current which consist of filaments which generate poloidal magnetic fluxes ϕ_p and ϕ_{plasma} , respectively, and block structures made of magnetic material. Vacuum vessel is not included because this study is performed in a static condition which induces no eddy currents. This model is simplified from three-dimensional one as shown in Fig. 1. The electrical conductivity of magnetic materials is set to be zero to reflect a toroidal cut. The relative permeability is fixed at from 1 (unity) to 1000.

2.2 Calculation procedure

The calculation procedure is comprised of three steps. First, the magnetic vector potential $\mathbf{A}$ is calculated with the above mentioned FEM condition. The plasma position modeled with a filament is moved around a region surrounded by the magnetic material to obtain each data set. Each location is used as a reference for position estimation. Then, the poloidal magnetic flux values are calculated with the equation $\phi_{\text{FL},i} = \oint_{L_i} \mathbf{A} \cdot d\mathbf{r}$, where L_i is a closed curve along i -th flux loop whose radius is R_i . In the axisymmetric condition, $\phi_{\text{FL},i}$ equals to $2\pi R_i A_\varphi$. Finally, the current centroid position ($R_{\text{centroid}}, Z_{\text{centroid}}$) and magnetic flux surfaces are reconstructed by improved filament-current approximation using the sampled magnetic flux vector (ϕ_{FL}).

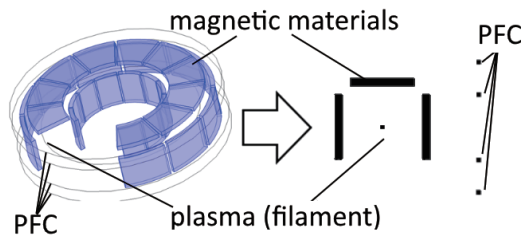


Fig. 1 Illustration of simplified 2D axisymmetric model. There are no magnetic blocks in lower side to test up-down asymmetric configurations.

The filament-current approximation method is based on least-squares fit using equation $\phi = \mathbf{M}\mathbf{I}$ under axisymmetry assumption, where $\mathbf{M}$ is the mutual inductance matrix between filaments and flux loops, and $\mathbf{I}$ is the approximated filament current vector. The mutual inductance between the j -th filament at (R_j, Z_j) and the i -th flux loop at $(R_{\text{FL},i}, Z_{\text{FL},i})$ is calculated in Eq. (2) as magnetic flux ϕ_i when 1 A current flows in the j -th filament.

$$\begin{aligned} M_{ij} &= 2\pi R_i A_\varphi|_{I_j=1} \\ &= 2\pi R_i \\ &\quad \times \frac{\mu_0 I_j}{4\pi} \int_0^{2\pi} \frac{R_j \cos\theta d\theta}{\sqrt{R_j^2 + R_{\text{FL},i}^2 - 2R_{\text{FL},i}R_j \cos\theta + (Z_j - Z_{\text{FL},i})^2}} \Big|_{I_j=1} \\ &= \frac{\mu_0 R_i}{2} \int_0^{2\pi} \frac{R_j \cos\theta d\theta}{\sqrt{R_j^2 + R_{\text{FL},i}^2 - 2R_{\text{FL},i}R_j \cos\theta + (Z_j - Z_{\text{FL},i})^2}}. \end{aligned} \quad (1)$$

$$(2)$$

Under a circumstance which includes poloidal magnetic flux ϕ_p from poloidal field coils, ϕ_{plasma} can be obtained by subtracting ϕ_{FL} with ϕ_p .

When the filament current vector $\mathbf{I}$ is obtained by solving the inverse problem, the position of current centroid ($r_{\text{centroid}}, z_{\text{centroid}}$) and poloidal magnetic flux function ψ are calculated as follows,

$$\begin{cases} R_{\text{centroid}} = \frac{\sum R_j \cdot i_j}{\sum i_j} \\ Z_{\text{centroid}} = \frac{\sum Z_j \cdot i_j}{\sum i_j}, \end{cases} \quad (3)$$

$$\psi(R, Z) = \frac{1}{2\pi} \iint_{S(R,Z)} \mathbf{B} \cdot d\mathbf{S} \quad (4)$$

$$= \frac{1}{2\pi} \mathbf{M}_{R,Z} \mathbf{I}. \quad (5)$$

The parameter i_j is the value of j -th filament current. $\mathbf{M}_{R,Z}$ is a mutual inductance vector between loop on (R, Z) and filaments.

2.3 Simply-improved method

In fusion DEMO reactors, these magnetic components are installed discretely but very closely. Consequently, the magnetization current which flows through each block can be considered as a combined-large toroidal ring as shown by broken curves in Fig. 2. Hence, the magnetization currents can be modeled as surface currents flowing on magnetic materials. In Fig. 3, magnetization surface currents are shown by straight lines. Due to the fact that the current flows within each block, the summation of currents of combined toroidal loops must be zero. Thus, the surface current of the n -th magnetic material is defined as a pair of position-dependent surface current $\pm k_n$ [A/m] expressed by equation $k_n = \sum_m c_m f_m(x)$ where x is the coordinate of the position along the surface of the cross-section.

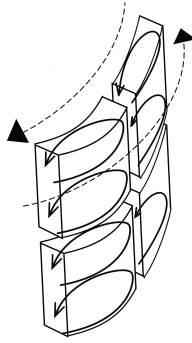


Fig. 2 The magnetization current flows in the magnetic blocks (solid lines) and can be considered as large-combined toroidal ring (broken lines).

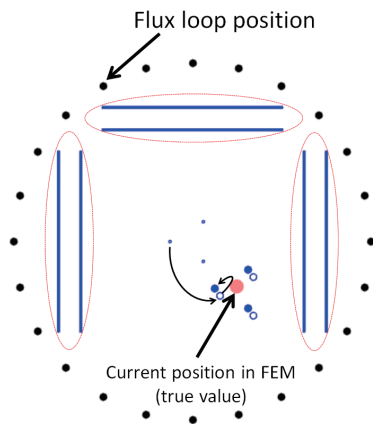


Fig. 3 The magnetization currents which flow on the surface of magnetic blocks as surface currents are shown by straight lines enclosed by broken lines. The curved arrows indicate the relocation of three current filaments to surround the previously computed current centroid for the reconstruction of nested magnetic surfaces.

In this paper, the surface current density distributions $\pm k_n$ [A/m] was defined as uniform function (i.e. $k_n = c_0$) and linear function (i.e. $k_n = c_1 x + c_0$). Based on this approximation, the mutual inductance between flux loops and pairs of surfaces (M_{surface}), can be obtained by integrating the function which assigns k_n for I of Eq.(1) in the x -direction. Then, we combined this M_{surface} and M_{filament} into M_{all} . With this mutual inductance M_{all} , the filament currents and surface currents were computed by least-squares fit as in the case of conventional filament approximation method.

In the case of MHD equilibrium analysis, the convergent calculation can be performed. Carved arrows in Fig. 3 show the iteration steps of filament current position relocation.

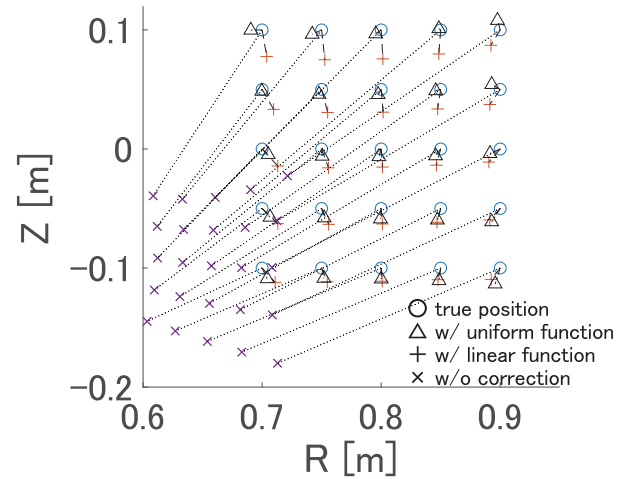


Fig. 4 True positions and calculated positions of plasma current centroid. Circles show the true positions. Plus, triangle and cross symbols shows the positions computed with uniform function, with linear function and without correction, respectively.

3. Result of Filament-Surface Approximation

Figure 4 shows an example of a comparison of computed plasma current centroid position under the effects of magnetic blocks with and without the correction. The computed positions of plasma centroid without the correction indicated by cross symbols are shifted to the inner side of the torus and lower than the true positions which are shown by circles. Those indicated by plus and triangle symbols are nearby the true ones after the using surface current correction.

As a result of the improved filament approximation, which approximates the magnetization current as surface current with linear positional dependence, root mean square errors (RMSE) are summarized in Tables 1-3. RMSE is defined as root mean square of errors shown by Eq. (6). The error is defined as the difference between computed value ($R_{\text{calc}}, Z_{\text{calc}}$) and true value ($R_{\text{true}}, Z_{\text{true}}$).

$$\begin{cases} (\text{RMSE})_R = \sqrt{\frac{1}{n} \sum_{n=1}^n (R_{\text{calc}} - R_{\text{true}})^2} \\ (\text{RMSE})_Z = \sqrt{\frac{1}{n} \sum_{n=1}^n (Z_{\text{calc}} - Z_{\text{true}})^2} \\ (\text{RMSE}) = \sqrt{\frac{1}{n} \sum_{n=1}^n \{(R_{\text{calc}} - R_{\text{true}})^2 + (Z_{\text{calc}} - Z_{\text{true}})^2\}}. \end{cases} \quad (6)$$

In Tables 1-3, the first row shows relative permeabilities at different orders of magnitude and the second row shows the iteration number of plasma filament relocation. Rows “uniform” and “linear” express the function types of surface current density distributions, and “w/o” means the case without correction. These tables show that the posi-

Table 1 RMSE of R position [mm].

μ_r	1		10		100		1000	
steps	1	3	1	3	1	3	1	3
uniform	6	1	6	7	7	6	8	7
linear	12	1	12	6	12	5	12	5
w/o	8	-	78	-	135	-	145	-

Table 2 RMSE of Z position [mm].

μ_r	1		10		100		1000	
steps	1	3	1	3	1	3	1	3
uniform	16	0.2	19	11	20	15	20	16
linear	10	0.3	14	9	13	7	13	7
w/o	10	-	45	-	92	-	102	-

Table 3 RMSE [mm].

μ_r	1		10		100		1000	
steps	1	3	1	3	1	3	1	3
uniform	17	1	20	13	21	16	22	17
linear	16	1	19	11	18	9	18	8
w/o	13	-	90	-	164	-	177	-

tion of the current centroid surrounded by magnetic blocks is computed much better with surface currents and it is best estimated with surface currents fitted with linear functions by three iterations in all cases of thousand-fold different permeabilities. Besides, in the case of $\mu_r = 1$ (i.e. without magnetic material blocks), these tables show that surface currents do not adversely affect the position estimation. It is interesting that the errors in the R position of the current centroid without iteration are smaller with uniform surface currents. Accordingly, the uniform surface current approximation might be able to be used in the real-time control of plasma position in tokamaks with magnetic materials inside with about 1% error.

Figure 5 shows some examples of magnetic flux surfaces comparison between the input data from the FEM analysis and the reconstructed ones. The flux surfaces by the FEM analysis shown by the broken dot curves were drawn with circulation integral of A_ϕ calculated by FEM, and the reconstructed ones indicated by solid curves were obtained with currents calculated by the filament-surface approximation. In Eq. (5), I is expressed by these currents. These results and figures demonstrate the potential of the simple correction scheme to evaluate the centroid position of plasma current and plasma boundary.

4. Summary

In a two-dimensional axisymmetric model, with a surface current approximation on magnetic materials, the cur-

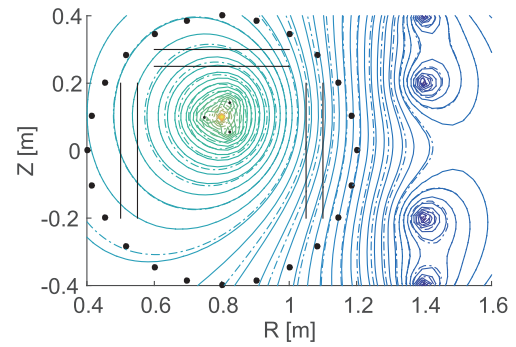
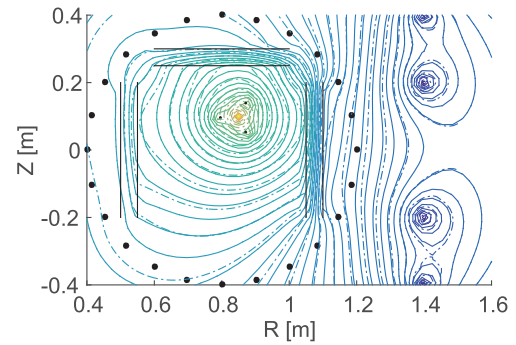
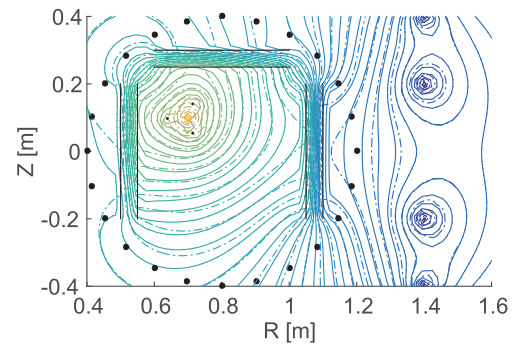
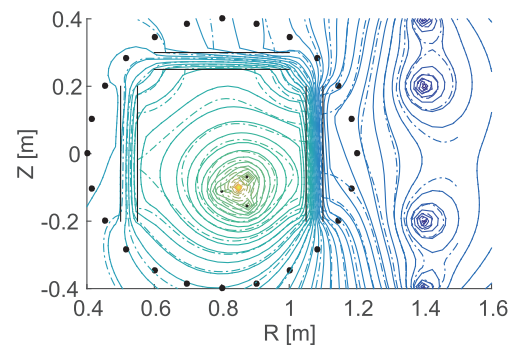
(a) $\mu_r = 1, R_{\text{true}} = 0.8, Z_{\text{true}} = 0.1$ (b) $\mu_r = 10, R_{\text{true}} = 0.85, Z_{\text{true}} = 0.1$ (c) $\mu_r = 100, R_{\text{true}} = 0.7, Z_{\text{true}} = 0.1$ (d) $\mu_r = 1000, R_{\text{true}} = 0.85, Z_{\text{true}} = -0.1$

Fig. 5 Magnetic flux surfaces comparison. The solid curves show reconstructed magnetic flux surfaces with approximated magnetization currents with different relative permeabilities, whereas the broken dot curves show the original ones calculated with COMSOL.

rent centroid position, especially in the R position, was calculated with an error of 1% and flux surfaces reconstructed by this correction show agreement with the true flux surfaces. Therefore in future work, the cross-sectional shape will be calculated in the same situation, and the position and the shape will be calculated in three-dimensional rotational symmetry model.

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