Reduction Rate of the MHD Potential Energy by a Toroidal Plasma Current Drive during Tokamak Plasma Formation with DC_{CS} *)

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The reduction rate of the magnetohydrodynamic (MHD) potential energy δW_{M} is defined as the energy difference between the vacuum magnetic field energy before a tokamak plasma formation and the MHD potential energy of an equilibrium tokamak plasma. Stationary direct current in a central solenoidal coil (DC_{CS}) can reduce δW_{M}, which means a reduction of the required heat energy for the tokamak plasma formation. The increase of the hole component ration in a toroidal plasma current distribution increases the toroidal plasma current and the stored thermal energy without increase of the plasma size. For tokamak plasmas with DC_{CS} = 100 A, δW_{M} has a local minimum with respect to the toroidal plasma current distribution. In this state, a perturbation of the toroidal plasma current is energetically inhibited.

Keywords: DC_{CS}, expulsion of magnetic field energy, reduction rate of MHD potential energy

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1. Introduction

A temporally changing current is applied to the central solenoidal coil (CS coil) in the central axis of a usual tokamak, generating a temporally changing magnetic field energy in order to drive a toroidal plasma current for Joule heating in induction heating experiments. On the other hand, no current is applied to the CS coil for non-induction heating experiments because a tokamak plasma formation and its sustainment by non-induction heating have been investigated for steady state operation at a constant equilibrium state and for the elimination of the CS coil [1–6]. The CS coil current has little influence on the equilibrium configuration of tokamak plasmas because a solenoidal coil ideally only generates a small leakage magnetic field. However, the magnetohydrodynamic (MHD) potential energy of the equilibrium tokamak plasma in non-induction heating experiment also depends on the CS coil current.

The CS coil current is an important factor for the stability of the tokamak plasma even if the current is a stationary direct current that has been applied before the start of heating [7]. Similar to the MHD potential energy, the transition energy of the tokamak plasma formation depends on both the CS current and the time evolution of the toroidal plasma current. During the temporal change of the toroidal plasma current by non-induction heating, the magnetic field energy in the inner region of the CS coil (CS region) changes and influences the plasma in the outer region of the CS coil through the temporal change of its vector potential; this process is similar to Joule heating. Since a stationary direct current in the CS coil (DC_{CS}) can enhance the vertical magnetic field only only in the CS region, a reverse of the vertical magnetic field in the CS region at the tokamak plasma formation can be changed by DC_{CS} to a decrease of the vertical magnetic field [7].

Based on the difference of the transition energies of the tokamak plasma formation, the effect of the CS current on the tokamak plasma formation by non-induction heating is investigated. The reduction rate of the MHD potential energy δW_{M} is defined as the energy difference between the vacuum magnetic field energy before the tokamak plasma formation W_{M0} and the MHD potential energy of an equilibrium tokamak plasma W_{M}. δW_{M} is normalized to W_{M0}. The reduction rates are compared based on the distributions of the toroidal plasma current and DC_{CS}.

In this paper, section 2 and 3 describe the calculation method and calculation results, respectively. Conclusions and discussions are presented in section 4. A summary is given in section 5.

2. Calculation Method

The force balance in the direction of the magnetic field at an equilibrium configuration for a steady state tokamak plasma is derived from the ideal MHD equations; it depends on the plasma pressure. A toroidal plasma current has no driving term except the pressure gradient driving term. The functional form of the plasma pressure P in eq. (1) has two components; Gaussian (n_1, σ_1) and hole (n_2, σ_2) components of the toroidal plasma current distribution.

$$P(\Psi) = 2T_0 \left[n_1 \exp\left\{ \frac{1}{\sigma_1} \left(\frac{\Psi - \Psi_{\mathrm{ax}}}{\Psi_{\mathrm{ax}} - \Psi_{\mathrm{sep}}} \right) \right\} \right.$$

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$$+n_2 \exp\left\{-\frac{1}{\sigma_2}\left(\frac{\Psi-\Psi_{\mathrm{ax}}}{\Psi_{\mathrm{ax}}-\Psi_{\mathrm{sep}}}\right)^2\right\}\Bigg]. \qquad (1)$$

Here, n_1 and n_2 are plasma densities. $\sigma_1 = 0.28$ and $\sigma_2 = 0.48$ define plasma pressures at a mid-plane of an inboard limiter. T_0 is the plasma temperature. Ψ is the magnetic flux surface function, and Ψ_{ax} and Ψ_{sep} are the magnetic flux surface functions at the magnetic axis and X-points, respectively. The magnetic flux surface function is selected, after repeated equilibrium calculations in order to minimize the MHD potential energy W_{M}:

$$W_{\mathrm{M}} = \int \mathrm{d}V\left[\frac{P}{\gamma-1} + \frac{1}{2\mu_0}\boldsymbol{B}^2\right] \equiv W_{\mathrm{TH}} + W_{\mathrm{B}}, \qquad (2)$$

where W_{TH} and W_{B} are the stored thermal energy and the total magnetic field energy of the equilibrium tokamak plasma, respectively. μ_0 and $\gamma = 5/3$ are the vacuum permeability and the ratio of the specific heats, respectively.

The reduction rate δW_{M} of the MHD potential energy is defined as follows.

$$\delta W_{\mathrm{M}} = \frac{W_{\mathrm{M}} - W_{\mathrm{M0}}}{W_{\mathrm{M0}}} \times 100, \quad W_{\mathrm{M0}} = \int \mathrm{d}V \frac{\boldsymbol{B}_0^2}{2\mu_0}. \quad (3)$$

Here, $\boldsymbol{B}_0$ is the vacuum magnetic field without the tokamak plasma. δW_{M} is normalized to the total of the vacuum magnetic field energy W_{M0}, which is the energy state before the tokamak plasma formation. The same size plasma with different toroidal plasma current has different W_{M0}, because the poloidal field (PF) coils current is varied to converge to the local minimum energy states while sustaining the specified positions [8]. W_{M0} depends on not only the plasma size but also the plasma pressure profile ratio $n_1/(n_1+n_2)$.

A coil configuration of TST2 [5,6] is utilized; the configuration comprises a CS coil with 239 coil turns and three pairs of PF coils with 9 (PF1), 6 (PF2), and 40 (PF3) coils turns in one-side which are numbered beginning at the top of the device, as shown in Figs. 1 (A-1) and (C-1). Since positions and currents of the coils are symmetric with respect to the mid-plane, a magnetic axis must exist at $Z = 0$. In addition, the position of the two X-points is the same except for the sign of the height Z. Therefore, the specified positions have three degrees of freedom that can be independently turned by the adjustment of the currents applied to the three PF coils.

For all calculations, a plasma temperature of $T_0 = 100$ eV and a density of $n_1+n_2 = 3\times10^{16}\ \mathrm{m}^{-3}$ are used. The radii of the inboard and outer limiters are 0.13 and 0.63 m, respectively. The height of the plasma region is limited to $z = \pm\, 0.5$ m. It is assumed that there is no plasma outside the region of the limiters and the last closed flux surface (LCFS).

3. Calculation Result

In the relaxation method, all tokamak plasma equilibria in this study converge to a local minimum of the MHD

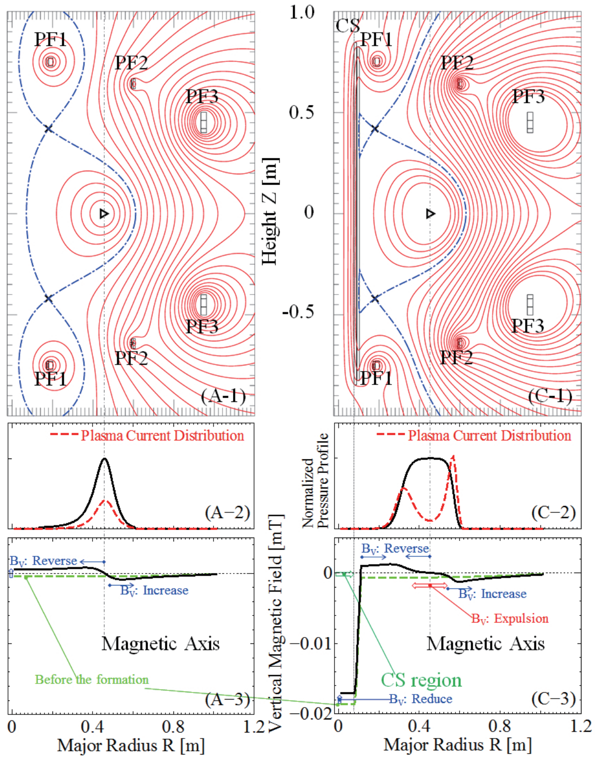

Fig. 1 Comparison of equilibrium tokamak plasma shapes. Figure series (A-) and (C-) show situations without $\mathrm{DC_{CS}}$ and with $\mathrm{DC_{CS}} = -100$ A, respectively. Magnetic flux surfaces and last closed flux surfaces are indicated by solid and dashed lines in Figs. 1 (A-1) and (C-1), respectively. Triangle and cross marks indicate the magnetic axes and X-points, respectively. The PF and CS coil blocks are indicated by squares. The plasma pressures and toroidal plasma current distributions are indicated by solid and dashed lines in Figs. 1 (A-2) and (C-2). In Figures 1 (A-3) and (C-3), the vertical magnetic fields B_{V} before and after the tokamak plasma formations are indicated by dashed and solid lines, respectively. Horizontal axes are in radial direction and are common for all figures.

potential energy by adjusting a plasma pressure profile at each toroidal plasma current distribution ratio $n_2/(n_1+n_2)$. In parallel, the current applied to the three PF coils is adjusted in order to sustain a specified magnetic axis and two X-points which determine the shape of the LCFS. In the calculations of the tokamak plasma equilibrium, in order to sustain the specified positions corresponding to the three degrees of freedom, the three currents applied to the PF coils are adjusted, as indicated by the dashed lines in Figs. 2 (A,B,C-4).

Both dependencies of the hole current profile component n_2 on the stored thermal energy W_{TH} and on the toroidal plasma current for the equilibrium tokamak plasma are independent of $\mathrm{DC_{CS}}$. Judging from the LCFSs indicated by the chained lines in Figs. 1 (A-1) with ($\mathrm{DC_{CS}} = 0$ A, $n_2/(n_1+n_2) = 0$) and (C-1) with ($\mathrm{DC_{CS}} = 100$ A, $n_2/(n_1+n_2) = 0.9$), the volumes of the tokamak plasma are almost the same, even if both toroidal plasma current distributions and $\mathrm{DC_{CS}}$ are different. Therefore,

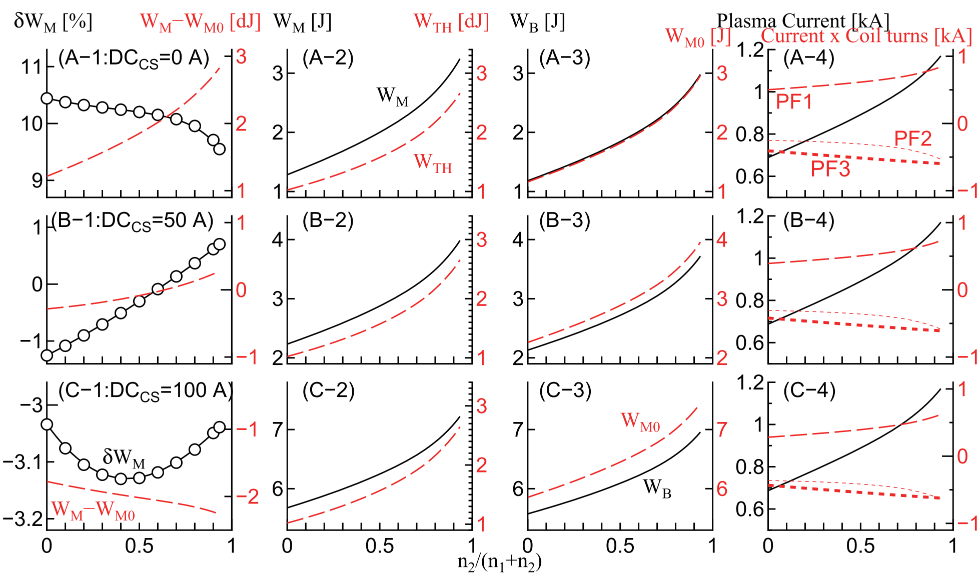

Fig. 2 Dependencies of energies and currents on the ratio of Gaussian n_1 to hole n_2 components of the toroidal plasma current distribution. Solid and other lines correspond to the left and right of the vertical axes, respectively. The magnification ratios of the vertical axes of the respective left and right parts are the same for (A-), (B-), and (C-), except for Fig. 2(C-1). The (A-), (B-), and (C-) series correspond to DC_{CS} = 0, −50 and −100 A cases, respectively. The reduction rate of the MHD potential energy δW_M is indicated by a combination of circle marks and solid lines. The difference of the magnetic field energy $W_B - W_{M0}$ is indicated by dashed lines in Figs. 2 (A,B,C-1). The MHD potential energies W_M and their stored thermal energies W_{TH} are plotted in Figs. 2 (A,B,C-2). Figures 2 (A,B,C-3) indicate the magnetic field energies after (W_B) and before (W_{M0}) the tokamak plasma formation. In Figures 2 (A,B,C-4), the toroidal plasma current hardly depends on DC_{CS}, and the PF coils currents PF1, PF2 and PF3 are indicated by dashed, dotted and short dashed lines, respectively. Here, the current of each PF coil is the product of the coil current and the coil turns.

W_{TH} indicated by the dashed line in Figs. 2 (A,B,C-2) and the toroidal plasma current indicated by the solid line in Figs. 2 (A,B,C-4) hardly depend on the DC_{CS}.

A comparison of Figs. 1 (A-2) and (C-2) shows that n_2 expands the plasma pressure profile around the magnetic axis indicated by solid lines. Therefore, W_{TH} and the toroidal plasma current are also increased by n_2, as shown in Figs. 2 (A,B,C-2,4). To sustain the increased W_{TH}, the vacuum magnetic field energy W_{M0} generated by the coils current is increased as the dashed lines in Figs. 2 (A,B,C-3). As a result, the total magnetic field energy including the toroidal plasma current W_B and MHD potential energy $W_M = W_B + W_{TH}$ are increased by n_2.

To reduce the MHD potential energy W_M, the tokamak plasma has two magnetic energy expulsion mechanisms due to the toroidal plasma current drive. Figures 1 (A,C-3) show that the vacuum vertical magnetic field outside (inside) the region of the magnetic axis indicated by the dashed lines increases (reverses), and changes into the solid line profile due to the toroidal plasma current drive at the tokamak plasma formation. In Fig. 1 (C-3), the vacuum vertical magnetic field in the CS region is enhanced by $DC_{CS} = -100$ A and is reduced by the toroidal plasma current drive. Therefore, in all n_2 cases, δW_M is reduced by DC_{CS} as shown in Fig. 3. In addition, the toroidal plasma current expels the vertical magnetic field not only in the CS region but also around the magnetic axis, as shown in Fig. 1 (C-3).

δW_M with $DC_{CS} = -100$ A has a local minimum with respect to the toroidal plasma current at $n_2/(n_1 + n_2) = 0.4$, as shown in Fig. 2 (C-1). On the other hand, δW_M with DC_{CS} = 0 and −50 A decreases and increases with increasing n_2, respectively, as shown in Figs. 2(A,B-1). δW_M with DC_{CS} = −50 A at the center of the horizontal axis in Fig. 3 (A) is indicated in Figs. 2 (B-1). Similarly, δW_M with DC_{CS} = 0 and −100 A at the left and right limits of the horizontal axis in Figs. 3 (B) and (C) are indicated in Figs. 2 (A-1) and (C-1), respectively. Since δW_M is normalized to DC_{CS}, the effect of the magnetic field energy

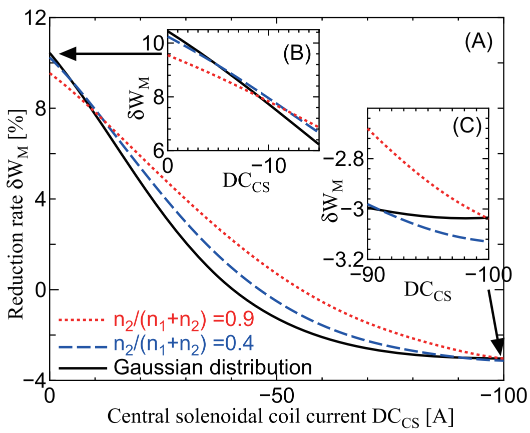

Fig. 3 The dependence of the reduction rate on the DC_{CS}. (B) and (C) are extended figures of (A) at DC_{CS} = 0 and −100 A, respectively. δW_M for current distributions $n_2/(n_1 + n_2)$ = 0, 0.5 and 1 are indicated by solid, dashed and dotted lines, respectively.

decrease in the CS region is relatively decreased, as shown in Fig. 3.

4. Conclusions and Discussions

The reduction rate of the MHD potential energy δW_M is defined by normalizing the energy necessary for equilibrium tokamak plasma formation $W_M - W_{M0}$ to the vacuum magnetic field energy W_{M0} for the tokamak plasma confinement, as expressed by eq. 3. The tokamak plasma equilibrium converges to a local minimum of the MHD potential energy W_M by applying the relaxation method for the magnetic flux surface function Ψ, as defined in eq. 1. The plasma pressure profile P in eq. 1 is given by a Gaussian n_1 and hole n_2 current component. Simultaneously, the magnetic axis and two X-points are sustained together by the adjustment of the three PF coil currents in order to form a uniform volume of an equilibrium tokamak plasma. The PF1 coil current is also modified to cancel the leakage magnetic field of the DC_{CS}.

DC_{CS} reduces δW_M, as shown in Fig. 3. Since an expulsion of the magnetic field energy by the toroidal plasma current is amplified by DC_{CS}, as shown in Fig. 1 (C-3), DC_{CS} hardly influences the equilibrium of the tokamak plasma. This is because ideally there is no leakage of magnetic field due to the CS coil at the plasma region. Therefore, without modification of the tokamak plasma equilibrium, DC_{CS} reduces the required external heat energy for the tokamak plasma formation.

An increase of the hole current component n_2 increases the capacity for the storing thermal energy W_{TH} in a given tokamak plasma volume, as shown in Figs. 2 (A,B,C-2), and expands the expulsion area of the magnetic field energy around the magnetic axis, as shown in Fig. 1 (C-3). Therefore, in the case of $DC_{CS} = 0$ A, δW_M decreases as n_2 increases, as shown in Fig. 2 (A-1). Since the vertical magnetic field in the CS region is reversed, the magnetic field energy decrease in the CS region is small. The magnetic field energy efficiency of the tokamak plasma confinement is improved increasing n_2, but the equilibrium tokamak plasma can energetically return to the vacuum state solely by consuming W_{TH}.

In the case of $DC_{CS} = -50$ A case, the required energy reduction rate is negative below the toroidal plasma current of 0.9 kA at $n_2/(n_1+n_2) = 0.6$, as shown in Figs. 2 (B-1,4). Below the toroidal plasma current, an external energy injection is necessary to return to the vacuum state from the tokamak plasma equilibrium state. Conversely, the equilibrium tokamak plasma can return to the vacuum state by only consuming the stored thermal energy, if the toroidal plasma current exceeds 0.9 kA.

Although the volumes of the equilibria tokamak plasmas in the calculations are made uniform by the specification of the magnetic axis and two X-points, the toroidal plasma current increases with increase in the hole current component n_2 due to the rise of the stored thermal energy W_{TH}. For the case $n_2/(n_1+n_2) = 0.4$ with $DC_{CS} = -100$ A, as in Fig. 2 (C-1), a perturbation of the toroidal plasma current is accompanied by an increase of W_M/W_{M0} because $\delta W_M \propto W_M/W_{M0} - 1$. If the toroidal plasma current increases due to the perturbation, the MHD potential energy W_M also increases. Therefore, the toroidal plasma current can return to $n_2/(n_1+n_2) = 0.4$ state by rejecting the extra perturbation energy. In contrast, when the toroidal plasma current decreases due to the perturbation, W_M/W_{M0} increases, even if all coils currents are constant. In that case, W_M-W_{M0} in Fig. 2 (C-1) further increases with decreasing toroidal plasma current. As a result, at $n_2/(n_1+n_2) = 0.4$, the toroidal plasma current can be energetically recovered from both an increase and a decrease of toroidal plasma current.

During an experiment, if all coil currents are constant, a tokamak plasma can absorb thermal energy with not only a shift of plasma positions but also modifications of the plasma pressure profile. The shift of plasma positions means a shift of the magnetic axis and the X-points with a change of the plasma volume. In any plasma volume, DC_{CS} can improve the stability of the tokamak plasma equilibrium by preparing a local minimum state of δW_M, which depends on the non-induction heating power.

5. Summary

The reduction rate of the MHD potential energy is introduced as an energy index of tokamak plasma formation and its stability. The capacity of the stored thermal energy of the equilibrium tokamak plasma can be increased by increasing the hole current component. DC_{CS} can results in a local minimum of the reduction rate of the MHD potential energy δW_M with respect to the toroidal plasma current distribution compared with equilibrium tokamak plasmas of the same volume. This state cannot change rapidly beyond the non-induction heating energy per time because both increase and decrease of the toroidal plasma current require the increase of the MHD potential energy if the coil current is constant.

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