Study on Operating Principle of Cockcroft-Walton Circuit to Produce Plasmas Using High-Voltage Discharge*)

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The basis of the operating principle of Cockcroft-Walton (CW) circuit, which produces a high-voltage, is studied using a simulator of an electrical circuit. The CW circuit experimentally contributed to the breakthrough of nuclear physics for the development of a particle accelerator. High-voltage discharges play an important role not only in producing plasmas but are also applicable in a wide variety of fields. However, studies on the operating principle of CW circuits are not sufficient, though improvement and application of it have been studied. Through our studies, we have found that a part of electrical charge stored in the capacitors remains, the output value of the voltage can be expressed in recurrence formula, and more time is needed for boosting circuits that have higher number of steps. Furthermore, we studied combinations between frequencies of alternating current and capacitance in a capacitor in order to boost effectively.

Keywords: high-voltage generator, booster circuit, boost effectively, Cockcroft-Walton circuit, atmospheric discharge

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1. Introduction

In 1919, H. Greinacher, a Swiss physicist, advocated a method to obtain double voltages compared to original voltages using an electrical circuit including capacitors and diodes. This circuit is called Greinacher circuit today, which is equal to the Cockcroft-Walton (CW) circuit of 1 step. J.D. Cockcroft and E.T.S. Walton invented this CW electrical circuit [1], which included more sets of capacitors and diodes modifying the Greinacher circuit in order to get a higher voltage. Operating principle of the CW circuit was applied to a nuclear accelerator, and voltage more than 800 kV was generated [2] through which recombination and disintegration of an atomic nucleus became possible. They succeeded in performing disintegration of an atomic nucleus by bombarding proton with high velocity to atoms of Li, F, Na, and K using CW circuit [2], and they experimentally contributed to the breakthrough of nuclear physics for development of particle accelerator.

Output voltage of the CW circuit increases as the steps increase. Output voltage V_{out} of the CW circuit with k steps can ideally show $V_{\text{out}} = 2kE_0$ when maximum value of the alternating current voltage is E_0. However, according to increasing the steps of circuit, impedance of the capacitors rises, and a voltage drop along with ripples at output voltage occur. When load voltage is V_{load}, output voltage is shown as $V_{\text{out}} = 2kE_0 - V_{\text{load}}$. This load voltage describes following relational expression:

$$V_{\text{load}} = (4k^3 + 3k^2 - k)I_{\text{load}}/(6fC).$$

Here, the load current is I_{load}, capacitance is C, alternating current frequency is f, and step is k. Since k^3 term influences V_{load} strongly, V_{load} becomes bigger in proportion as the steps increase. Ripple of output voltage E_{ripple} describes the following relational expression:

$$E_{\text{ripple}} = k(k + 1)I_{\text{load}}/(2fC).$$

Since k^2 term influences E_{ripple} strongly, E_{ripple} becomes bigger in proportion as the steps increase. An improved CW circuit is developed in which the load voltage and ripple in the output voltage both are reduced [3–7]. The CW circuit is applied to devices handling high-voltage and low current, such as nuclear accelerator, electron microscope, etc. [5, 6, 8]. Though the strength of output voltage attracts attention from a practical application perspective, it is hard to say that how to boost attracts attention. By using the CW circuit, we obtain the boosting estimation time, boost rates, and we study how to boost. Furthermore, it is necessary that appropriate combination between frequency f of the alternating current and capacitance C in the capacitor is selected in order to boost effectively.

Though CW circuits are being widely used since a long time to produce high-voltage as presented above, the operating principle still needs to be studied in detail. Therefore, we focus on the following: studying the boosting principle and obtaining the appropriate combination between f and C to boost effectively.

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2. Cockcroft-Walton (CW) Circuit

The CW circuit consists of an alternating current source and combination of two electrical elements that are capacitor and diode, and is able to generate high voltage through multistage connections. The CW circuit with k steps uses one alternating current source, $2k$ capacitors, and $2k$ diodes. Here, E : voltage of AC source, V_{D_l} : potential difference of lth diode, I_{D_l} : current of lth diode, V_{D_l} : potential difference of th capacitor, I_{C_l} : current of lth capacitor, and V_l : electrical potential. The voltage of AC source E can be described as $E = E_0 \sin \omega t$, where E varyies according to time. E_0 shows maximum voltage, ω shows angular frequency, and t shows time. In what follow, lth diode and lth capacitor are described as D_l and C_l, respectively. Here, E, V_{D_l}, I_{D_l}, V_{C_l}, I_{C_l} and V_l all depend on time. Displacement current of capacitors are shown as follow:

$$I_{\mathrm{C}_l} = C_l \frac{\mathrm{d}V_{\mathrm{C}_l}}{\mathrm{d}t} \quad (l = 1, 2, 3, \cdots, 2k).$$

Here, C_l shows the capacitance of lth capacitor. Current in the diode can be described using the following model equation of Shockley.

$$I_{\mathrm{D}_l} = I_{0_l} \left\{ \exp\left(\frac{qV_{\mathrm{D}_l}}{n_l k_{\mathrm{B}} T} \right) - 1 \right\} \quad (l = 1, 2, 3, \cdots, 2k).$$

Here, I_{0_l} : saturation current of lth diode, n_l : rising coefficient of current, k_{B} : Boltzmann constant, T : absolute temperature, and q : elementary charge. The output voltage is expressed in $2k$ times of the maximum voltage E_0 of the alternating current as follow:

$$V_{2k} = 2kE_0 = \sum_{i=1}^{k} V_{\mathrm{C}_{2i}}.$$

Here, we make three assumptions as follows: diode is a lossless ideal electrical element, internal resistance of alternating current source is sufficiently small, and the system discharges after the boosting is complete. The maximum output voltage of CW circuit with k steps becomes $2k$ times for input voltage. The voltage, current, and electric potential can be described using the first and second laws of Kirchhoff as follows, where $l = 1, 2, 3, \cdots, 2k$.

- Voltage

$$V_{\mathrm{C}_1} = E - V_{\mathrm{D}_1} \quad (\text{source} - \mathrm{D}_1 - \mathrm{C}_1 \text{ circuit}),$$
$$V_{\mathrm{C}_l} = -(V_{\mathrm{D}_{l-1}} + V_{\mathrm{D}_l}) \quad (\mathrm{D}_{l-1} - \mathrm{D}_l - \mathrm{C}_l \text{ circuit}).$$

- Current

$$I_{\mathrm{C}_l} = \sum_{i=l}^{2k} (-1)^{i+l} I_{\mathrm{D}_i}.$$

- Electrical potential

$$V_l = -\sum_{i=1}^{l} V_{\mathrm{D}_i}.$$

- Potential difference of diode

$$V_{\mathrm{D}_1} = V_1, \qquad V_{\mathrm{D}_l} = V_{l-1} - V_l.$$

- Potential difference of capacitor

$$V_{\mathrm{C}_1} = E + V_1, \qquad V_{\mathrm{C}_{2l-1}} = V_{2l-1} - V_{2l-3},$$
$$V_{\mathrm{C}_{2l}} = V_{2l} - V_{2l-2}.$$

In order to investigate the boosting process of CW circuit with k steps, in the next stage, general expression concerning potential difference of lth capacitor with k steps is shown as follows, where $l = 1, 2, 3, \cdots, 2k$.

$$\frac{\mathrm{d}V_{\mathrm{D}_l}}{\mathrm{d}t} = (-1)^{l+1} \frac{\mathrm{d}E}{\mathrm{d}t} + \sum_{i=1}^{l-1} \left\{ (-1)^{i+1} \left(\sum_{m=1}^{l-i} \frac{1}{C_m} I_{\mathrm{D}_{l-i}} \right) \right\} - \left(\sum_{m=1}^{l} \frac{1}{C_m} \right) \sum_{j=h}^{2k} (-1)^{h+k} I_{\mathrm{D}_j}.$$

When all of the capacitors are same, the expression can be changed as $C_m - C$ in the following equation.

$$\begin{aligned} \frac{\mathrm{d}V_{\mathrm{D}_l}}{\mathrm{d}t} &= (-1)^{l+1} \frac{\mathrm{d}E}{\mathrm{d}t} + \frac{1}{C} \sum_{i=1}^{l-1} \{ (-1)^{i+1} (l-i) I_{\mathrm{D}_{l-i}} \} \\ &\quad - \frac{1}{C} \sum_{j=l}^{2k} (-1)^{j+h} I_{\mathrm{D}_j} \\ &= (-1)^{l+1} \frac{\mathrm{d}E}{\mathrm{d}t} \\ &\quad + \frac{1}{C} \left[\sum_{i=1}^{i-1} \{ (-1)^{i+1} (l-i) I_{\mathrm{D}_{l-i}} \} - l \sum_{j=1}^{2k} (-1)^{j+l} I_{\mathrm{D}_j} \right]. \end{aligned}$$

3. Analysis of CW Circuit

LTspiceIV (the software that simulates analog behavior of an electronic circuit) developed by LINER TECNOLOGY is used for analysis of CW circuits. Using the software, we analyze the electric potential, potential difference of the diode, current in the diode, potential difference of the capacitor, and current in the capacitor. In addition, the boosting process of the voltage is studied. Clearly, the analyses become complicated as steps increase. Therefore, analyses start from the CW circuit with one step, and then, the steps increase gradually predicting the behaviors of the CW circuit. Here, we focus on the results analyzing the CW circuit with 3 steps, and we discuss the boosting process of voltage.

The CW circuit with k steps is shown in Fig. 1.

The conditions of AC source are as follows: maximum voltage $E_0 = 4500$ V, frequency $f = 35200$ Hz, and period $T = 1/f = 28.4$ μsec. We discuss the boosting process of voltage from the following 6 stages.

(1) $0 < t < T/4$

Current flows in the circuit $\mathrm{E}_1 \to \mathrm{D}_1 \to \mathrm{C}_1$, and capacitances $Q_1 = Q_0 = CE_0$ is charged in capacitor C_1.

(2) $T/4 \leq t < 3T/4$

Current flows in the circuit $\mathrm{E} \to \mathrm{C}_1 \to \mathrm{D}_2 \to \mathrm{C}_2$, and capacitances $Q_1 = 0$, $Q_2 = Q_0$ are charged in capacitors C_1, C_2, respectively.

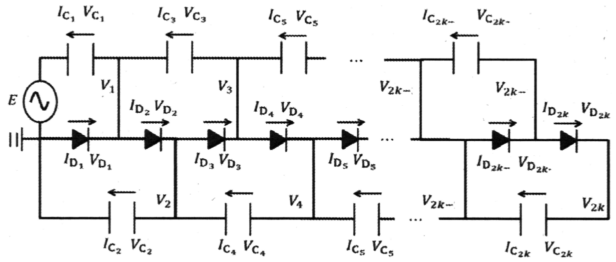

Fig. 1 CW circuit having k steps.

(3) $3T/4 \leq t < 5T/4$

Current flows in the circuit E $\rightarrow$ C_2 $\rightarrow$ D_3 $\rightarrow$ C_3 $\rightarrow$ C_1, and capacitances $Q_2 = Q_0/2$, $Q_3 = Q_0/2$ are equally charged in capacitors C_2, C_3, respectively. Furthermore, current flows in the circuit E $\rightarrow$ D_1 $\rightarrow$ C_1, and capacitance $Q_1 = Q_0$ is charged in capacitor C_1.

(4) $5T/4 \leq t < 7T/4$

Current flows in the circuit E $\rightarrow$ C_1 $\rightarrow$ C_3 $\rightarrow$ D_4 $\rightarrow$ C_4 $\rightarrow$ C_2, and capacitances $Q_3 = Q_0/4$, $Q_4 = Q_0/4$ are equally charged in capacitors C_3, C_4, respectively. Furthermore, the current flows in the circuit E $\rightarrow$ C_1 $\rightarrow$ D_2 $\rightarrow$ C_2, and capacitances $Q_1 = Q_0/4$, $Q_2 = 5Q_0/4$ are equally charged in capacitors C_1, C_2, respectively.

(5) $7T/4 \leq t < 9T/4$

Current flows in the circuit E $\rightarrow$ C_2 $\rightarrow$ C_4 $\rightarrow$ D_5 $\rightarrow$ C_5 $\rightarrow$$C_3$ $\rightarrow$ C_1, and capacitance $Q_4 = Q_0/8$, $Q_s = Q_0/8$ are equally charged in capacitors C_4, C_5, respectively. Next, the current flows in the circuit E $\rightarrow$ C_1 $\rightarrow$ C_3 $\rightarrow$ D_4 $\rightarrow$ C_4 $\rightarrow$ C_2, and capacitances $Q_3 = 3Q_0/4$, $Q_4 = 3Q_0/4$ are charged equally in capacitors C_3, C_4, respectively. Furthermore, current flows in the circuit E $\rightarrow$ D_1 $\rightarrow$ C_1, and capacitance $Q_1 = Q_0$ is charged in capacitor C_1.

(6) $9T4 \leq t < 11T4$

Current flows in the circuit E $\rightarrow$ C_1 $\rightarrow$ C_3 $\rightarrow$ C_5 $\rightarrow$ D_6 $\rightarrow$ C_6 $\rightarrow$ C_4 $\rightarrow$ C_2, and capacitances $Q_5 = Q_0 16$, $Q_6 = Q_0 16$ are equally charged in capacitors C_5, C_6, respectively. Next, current flows in the circuit E $\rightarrow$ C_1 $\rightarrow$ C_3 $\rightarrow$ D_4 $\rightarrow$ C_4 $\rightarrow$ C_2, and capacitances $Q_3 = 7Q_0 16$, $Q_4 = 7Q_0 16$ are charged equally in capacitors C_3, C_4, respectively. Furthermore, current flows in the circuit E $\rightarrow$ C_1 $\rightarrow$ D_2 $\rightarrow$ C_2, and capacitances $Q_1 = 3Q_0 8$, $Q_2 = 11Q_0 8$ are charged in capacitors C_1, C_2, respectively.

Charge stored in capacitors can be expressed in recurrence formula as follows:

$$
\begin{aligned}
&Q_1(\{n-3/4\}T) = Q_0,\\
&Q_2(\{n-3/4\}T) = Q_3(\{n-3/4\}T)\\
&\quad = \frac{1}{2}\{Q_2(\{n-5/4\}T) + Q_3(\{n-5/4\}T)\},\\
&Q_4(\{n-3/4\}T) = Q_5(\{n-3/4\}T)\\
&\quad = \frac{1}{2}\{Q_4(\{n-5/4\}T) + Q_5(\{n-5/4\}T)\},\\
&Q_6(\{n-3/4\}T) = Q_6(\{n-5/4\}T),\\
&Q_1(\{n-1/4\}T),\\
&\quad = \frac{1}{2}\{Q_1(\{n-3/4\}T) + Q_2(\{n-3/4\}T)\} - \frac{Q_0}{2},\\
&Q_2(\{n-1/4\}T)\\
&\quad = \frac{1}{2}\{Q_1(\{n-3/4\}T) + Q_2(\{n-3/4\}T)\} + \frac{Q_0}{2},\\
&Q_3(\{n-1/4\}T) = Q_4(\{n-1/4\}T)\\
&\quad = \frac{1}{2}\{Q_3(\{n-3/4\}T) + Q_4(\{n-3/4\}T)\},\\
&Q_5(\{n-1/4\}T) = Q_6(\{n-1/4\}T)\\
&\quad = \frac{1}{2}\{Q_5(\{n-3/4\}T) + Q_6(\{n-3/4\}T)\}.
\end{aligned}
$$

In Table 1, charge stored in capacitors that is obtained using above recurrence formula is shown. The unit is C (coulomb).

Capacitance Q_0 is once charged and Q_0, $Q_0/2$, $Q_0/4$, $Q_0/8$, $\cdots$, $Q_0/2^{n-1}$ are discharged at the time of T/4, 5T/4, 9T/4, 13T/4, $\cdots$, $\{n-3/4\}T$, respectively. Then, because the positive and negative of AC source changes, charged capacitors play a role of battery, and the charge transfers from a capacitor of lower step to the one of higher step. Boosting is done through such a process; therefore, all charge is not transferred immediately from the capacitor of lower step to the one of higher step in the boosting process; A part of the electrical charge stored in the capacitors remains.

4. Recurrence Expression of Output Voltage

Recurrence expression of output voltage V_{C_l} in CW circuits is obtained from results of analyses. Cycles and periods of oscillations of AC source are expressed as n and T, respectively. Required time to boost is shown as $t = nT$. $\Gamma'_{2l}(l = 1, 2, \cdots, k)$ is coefficient of charge stored in even-numbered capacitors at $t = (n-1/4)T$, when charge Q_0 is stored in the capacitor of the first step. Charge Q_{2l} stored in $2l$th capacitor is $Q_{21} = \Gamma'_{2l}Q_0$. When $t = (n-1/4)T$, the relational expression of the output voltage is described in the recurrence expression as follows:

$$
\frac{(V_2k')_n}{E_0} = \frac{1}{2}(\Gamma'_2)_{n-1} + \sum_{l=2}^{k}(\Gamma_2 l')_{n-1} + 1 \quad (k = 1, 2, 3, \cdots).
$$

Here,

$$
\begin{aligned}
&(\Gamma'_1)_n = \frac{1}{2}(\Gamma_2)_n, (\Gamma'_2)_n = \frac{1}{2}(\Gamma_2)_n + 1,\\
&(\Gamma'_{2l-1})_n = (\Gamma'_{2l})_n = \frac{1}{2}\{(\Gamma_{2l-1})_n + (\Gamma_{2l})_n\}\\
&\qquad (l = 2, 3, \cdots, k-1).
\end{aligned}
$$

When boost rate is τ, expression is described as follows:

$$
\begin{aligned}
\tau &= \frac{(V_{2k})'_n}{2kE_0} \times 100\\
&= \frac{1}{2k}\left[\frac{1}{2}(\Gamma'_2)_{n-1} + \sum_{l=2}^{k}(\Gamma'_{2l})_{n-1} + 1\right] \times 100.
\end{aligned}
$$

Table 1 Charge stored in capacitors.

n	t	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
1	$T/4$	Q_0	0	0	0	0	0
	$3T/4$	0	Q_0	0	0	0	0
2	$5T/4$	Q_0	$Q_0/2$	$Q_0/2$	0	0	0
	$7T/4$	$Q_0/4$	$5Q_0/4$	$Q_0/4$	$Q_0/4$	0	0
3	$9T/4$	Q_0	$3Q_0/4$	$3Q_0/4$	$Q_0/8$	$Q_0/8$	0
	$11T/4$	$3Q_0/8$	$11Q_0/8$	$7Q_0/16$	$7Q_0/16$	$Q_0/16$	$Q_0/16$
4	$13T/4$	Q_0	$29Q_0/32$	$29Q_0/32$	$Q_0/4$	$Q_0/4$	$Q_0/16$
	$15T/4$	$29Q_0/64$	$93Q_0/64$	$37Q_0/64$	$37Q_0/64$	$5Q_0/32$	$5Q_0/32$

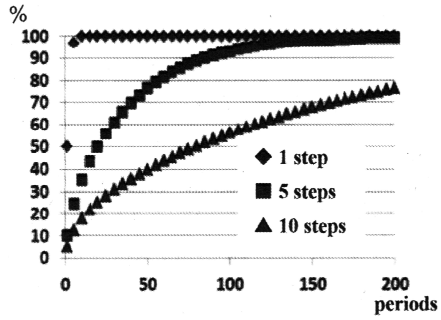

Fig. 2 Boost rate of CW circuit with 1, 5, and 10 steps.

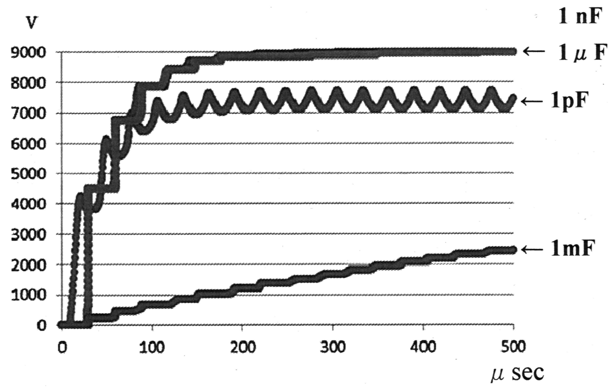

Fig. 3 Capacitance and boost voltage (E_0 = 4500 V, f = 35220 Hz, and k = 1).

Periods and boost rate of CW circuit with 1, 5, and 10 steps, which is calculated using obtained recurrence expression, are shown in Fig. 2. It is found that longer time is need until boost finishes according to the increasing steps.

5. Combination between Frequency *f* of AC Source and Capacitance *C* in Capacitor

In order to boost effectively, an appropriate combination between frequency f of AC source and capacitance C in the capacitor has to be selected. The parameters E_0 = 4500 V, f = 35220 Hz, and k = 1 are fixed, and boosting state of the circuit is shown in Fig. 3 by changing the capacitance of the capacitor as like C = 1 pF, 1 nF, 1 μF, and 1 mF.

When the frequency of the AC source is fixed, and the capacitor has one capacitance that is extremely small (C = 1 pF) and other extremely big (C = 1 mF), it is clear that boost is not done effectively from Fig. 3. As a result, appropriate combination between frequency of AC source and capacitance in the capacitor is studied in order to boost effectively.

If combinations of frequency and capacity enter a domain surrounding a plot, the CW circuit can work. Combinations between frequency f of AC source and capacitance C in capacitor with 1 step and 10 steps are shown in Fig. 4. Since Fig. 4 is shown in log plots, correspondence table concerning f and C is shown in Table 2 for convenience. It is clear that an appropriate combination between frequency f of AC source and capacitance C in the capacitor has to be selected in order to let CW circuit work effectively from Fig. 4. It is clear that the region of CW circuit with 10 steps moves to the region in which capacitance of the capacitors becomes smaller compared to the CW circuit with 1 step. Therefore, it is thought that the surrounding region moves to the region in which the capacitance of the capacitors becomes smaller with the increasing steps of the CW circuit.

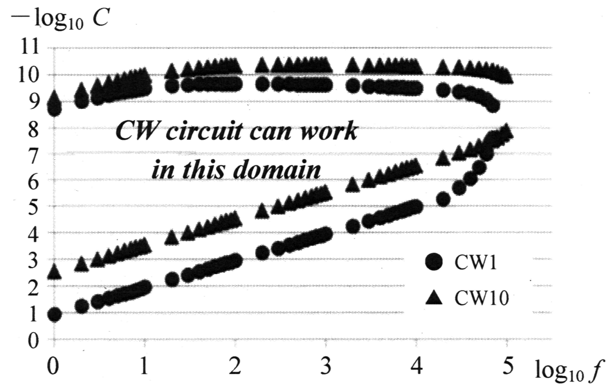

Fig. 4 Frequency of power source and capacity (If combinations of frequency and capacity enter the domain surrounding a plot, the CW circuit can work).

6. Conclusion

Obtained results through our study concerning CW circuit are concluded as follows.

Table 2 Correspondence table concerning f and C.

f	$\log_{10} f$	C	$-\log_{10} C$
1 Hz	0	1 F	0
10 Hz	1	100 mF	1
100 Hz	2	10 mF	2
1 kHz	3	1 mF	3
10 kHz	4	100 μF	4
100 kHz	5	10 μF	5
		1 μF	6
		100 nF	7
		10 nF	8
		1 nF	9
		100 pF	10

- Boosting is performed through the process that capacitance stored in capacitors transfer to other capacitors. Here, the capacitance stored in the capacitors is shared equally by the principle of conservation of charge, and part of the electrical charge stored in the capacitors remains.
- Output value of the voltage of the CW circuit with k steps can be expressed in recurrence formula; therefore, the value of the voltage and time required for boosting can be calculated if the physical quantity of the electric elements is detailed.
- More time is needed in order to boost the concerning circuits that have higher number of steps.
- A combination is studied in order to boost effectively between the frequency of the alternating current and the capacitance in the capacitor.

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