

A Calibration of Setting of Mach Probes by Observing GAM Oscillations

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(Received 10 December 2015 / Accepted 25 December 2015)

The influence of relative displacement of Mach probe (which is placed near the top of magnetic surface) on the interference of signals is discussed. An error can arise in measured value of poloidal electric field. The Mach number perturbation at the GAM frequency has an interference from the density perturbation. The interference from the density perturbation can propagate to all of Mach number measurement. By observing the signals associated with GAM oscillations, the error in setting the probe arrays can be detected. This result can be applied to correct the positioning of probes.

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Keywords: geodesic acoustic mode, probe measurement, Mach probe, probe alignment

DOI: 10.1585/pfr.11.1402002

1. Introduction

In order to study the transport of toroidal momentum, fluctuation of plasma flow along the magnetic field line was measured by use of Mach probes (See, e.g., [1–4]). Mach probe was recently applied to the study of geodesic acoustic mode (GAM) [5]. The measurement of transport fluxes requires the phase relation between several quantities [6–8], so that the cross talk between the flow along the field line and other flow components or the density perturbation might cause the systematic error in the study of transport characteristics.

One of the origins of incorrect observation is an error of the setting of the probe with respect to the magnetic field lines and/or magnetic surfaces [9]; If a Mach probe is not completely parallel to the magnetic field line, the deduced fluctuation of Mach number can include influences of perpendicular velocities. The influence of other quantities (like density perturbation) is possible to occur. We here discuss a possible method to calibrate the position of the Mach probe in the edge of tokamak plasmas. By observing the signals associated with GAM oscillations, the error of placing the probe arrays can be detected. This result can be applied to calibrate the position of probes. The calibration method depends on what must be measured correctly: e.g., the accurate measurement of parallel flow for drift wave fluctuations may require different principle of calibration from that for GAM oscillations. In this note

we study the case where the flows in GAM oscillations are measured.

Possible applications of GAMs to advanced diagnostics were proposed. For instance, GAM spectroscopy has been proposed to measure the isotope ratio of fuel ions [10]. A method to identify the plasma surface by observing GAM oscillation was proposed [11]. This note discusses an additional application of GAMs for detailed analysis of diagnostics.

2. Geometry

An example of an array of Langmuir probes is illustrated in Fig. 1. A pair of probes 9 and 10 (or those of 11 and 12) constitutes a Mach probe [5]. Local coordinates on the probe array are defined as is illustrated. The x - and y -coordinates are (nearly) in the radial and poloidal directions. The z -coordinate is designed to take the direction of magnetic field line so as to measure the plasma flow velocity along the field line. The setting of the probe array is illustrated in Fig. 2. The Fig. 2 (a) shows the poloidal cross-section. The major radius of the probe position may be different from that of the center of the magnetic surface. The Fig. 2 (b) shows the top view. The Mach probe may not be tangent to the magnetic field line and to the magnetic surface. Two tips of the Mach probe may be on different magnetic surface. For the illustration of the problem, we discuss here in the case of a circular magnetic surface in a high-aspect-ratio tokamak.

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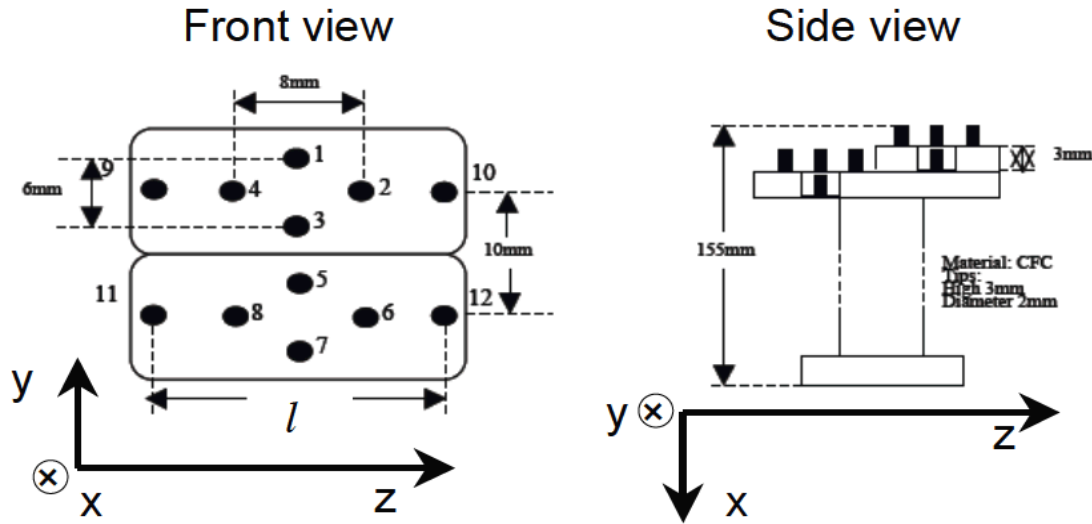


Fig. 1 Front-view and side view of probe array are illustrated. The local (x, y, z) coordinates on the probe are also denoted.

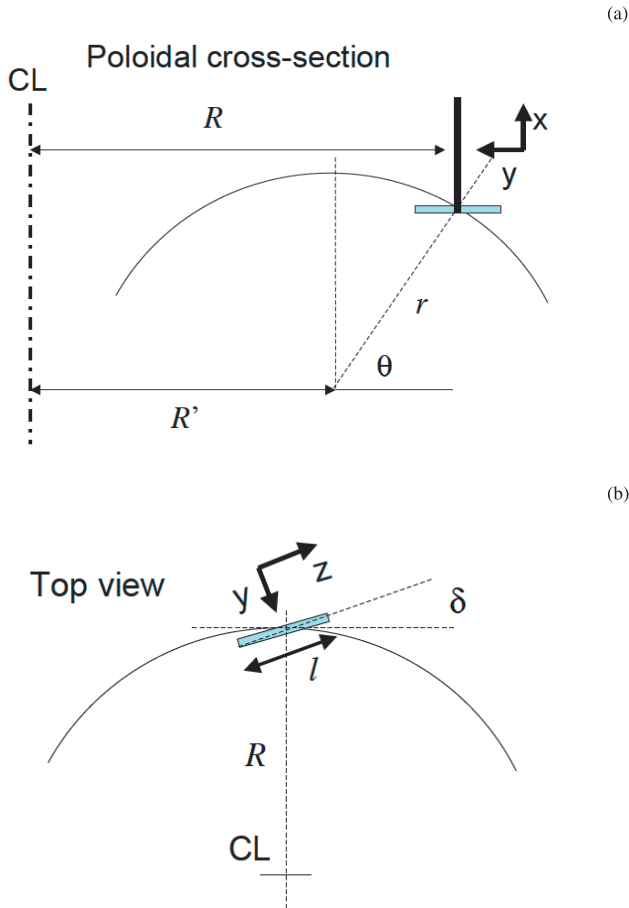


Fig. 2 Poloidal cross-section of the magnetic surface and the probe array, (a). The local coordinates indicate the direction of the probe. The top view of the probes (b). The circle indicates the major radius R . Inclination of the probe (relative rotation angle of the probe around the probe stem) is indicated by the angle δ .

3. Flow of GAM Oscillation and Calibration

The GAM oscillation has a particular feature of toroidal symmetry, and the associated electric field is almost in the minor radius direction. The eigenfunction is explained in [12]. Ratio of the toroidal component of velocity to the poloidal one was given as $-2q \cos \theta$ for low frequency zonal flows, (where q is the safety factor and θ is the poloidal angle,) and that for GAMs is $q^{-1} \cos \theta^\dagger$. The eigenfunction of GAMs is given as

$$\mathbf{V} = -\frac{E_r}{B} \begin{pmatrix} 0 \\ 1 \\ \frac{1}{q} \cos \theta \end{pmatrix}. \quad (1)$$

The electric field is mainly in the radial direction, and those projected on the x - y coordinates are given as

$$\begin{aligned} E_x &= E_r \sin \theta, \\ E_y &= -E_r \cos \theta. \end{aligned} \quad (2)$$

That is,

$$\theta = \arctan(-E_x/E_y). \quad (3)$$

Thus one can estimate the shift of probe position R with respect to the top of the magnetic surface R' , by measuring the ratio between E_x and E_y on the probe arrays as

$$R - R' = r \cos \left\{ \arctan(-E_x/E_y) \right\}. \quad (4)$$

In interpreting the data E_y in terms of ' E_θ ', two probes are assumed to be on the same magnetic surface. If two

[†]It is well known that the impact, which induces a poloidal flow V_0 (with no toroidal flow), results in the low frequency zonal flow with the amplitude of $(1 + 2q^2)^{-1} V_0$ after the decay of GAM oscillations, as is explained in [12]. This is because the eigenfunction of GAM is given as Eq. (1) and the initial poloidal flow (with no toroidal flow) is a combination of zonal flow and GAM, the poloidal velocities of which are given as $(1 + 2q^2)^{-1} V_0$ and $q^2 (1 + 2q^2)^{-1} V_0$, respectively.

probes (which measure the ‘poloidal’ electric field) are not parallel to the magnetic surface, a small portion (that is proportional to E_r) is included in the interpreted ‘ E_θ ’. For GAM, the ratio between E_θ and E_r is known, and is zero at top ($\theta = \pi/2$). Thus, one can evaluate a small angle between the two probes and the magnetic surface.

We next consider possible inclination of the Mach probe with respect to the magnetic field line. The Mach number M is estimated by comparing the ion saturation currents at two probes

$$M = 0.4 \ln(I_{s1}/I_{s2}), \quad (5)$$

where I_{sj} ($j = 1, 2$) is the ion saturation current at probe j . The GAM eigenmode (1) has small parallel velocity near the top of the magnetic surface as

$$V_{\text{tor}}/V_\theta \sim q^{-1} \cos \theta. \quad (6a)$$

Combining Eq. (3) and Eq. (6a), one has

$$V_{\text{tor}}/V_\theta \sim q^{-1} \cos \left\{ \arctan(-E_x/E_y) \right\}. \quad (6b)$$

Thus, if the fluctuating Mach number at GAM frequency, being coherent with potential perturbations of GAMs, shows the value much larger than Eq. (6a), it suggests that the Mach probe may not be tangent to the magnetic surface. One possibility of the measurement error is that the obtained value of Mach number Eq. (5) is influenced by the density perturbation. Consider the case where the Mach probe is inclined by an angle δ as is illustrated in Fig. 2 (b) (top view). The distance in between the two probes, which is projected in the direction of the minor radius, Δ , is given as

$$\Delta = l \sin \delta \cos \theta, \quad (7)$$

where l is the distance of two probes in a Mach probe. Because GAMs have a large wave number in the radial direction, k_r , this distance Δ introduces a difference of fluctuating density at probes 1 and 2 (the probe 1 is chosen as one with the smaller z position) as

$$n_1 - n_2 = -ik_r \Delta \tilde{n}, \quad (8)$$

where \tilde{n} is the amplitude of density perturbation of GAM oscillation. Since the ion saturation current is proportional to the product of density and ion sound speed, $I_s = nc_s$, the ratio I_{s1}/I_{s2} is approximately evaluated as

$$I_{s1}/I_{s2} = 1 + (n_1 - n_2)/n_0 = 1 - i(k_r \Delta) \tilde{n}/n_0. \quad (9)$$

Here, n_0 is the mean density, and we simply assume that the relative temperature fluctuation of GAM is smaller than that of density change at GAM frequency. Substituting Eq. (9) into Eq. (5), one comes to the estimate that the probe provides the signal

$$M_{\text{int}} = -0.4i(k_r l) \tilde{n} n_0^{-1}, \quad (10a)$$

or

$$M_{\text{int}} = -0.4i(k_r l) \tilde{n} n_0^{-1} \sin \delta \cos \theta, \quad (10b)$$

where the suffix ‘int’ indicates the contribution of interference. In this process of interference, apparent Mach number M_{int} and the density perturbation have the phase difference with $3\pi/2$. Because the GAMs have short radial wavelength (of the order of $10\rho_s$), the coefficient 0.4 ($k_r l$) can be of the order unity. The relative density perturbation is estimated as

$$\frac{\tilde{n}}{n_0} = - \left(\sqrt{2} k_r \rho_s \frac{e\tilde{\phi}}{T_e} \right) \sin \theta. \quad (11)$$

Equation (10b) shows that the apparent value of M , combined with the density perturbation Eq. (11) and Eq. (3), gives an evaluation of the angle δ .

An evaluation of tolerance in the misalignments may be deduced from Eq. (10). Combining Eq. (1) and Eq. (11), the Mach number for GAM eigenmode is given as

$$M_{\text{GAM}} = -i \frac{1}{q \sqrt{2}} \frac{\tilde{n}}{n_0} \frac{\cos \theta}{\sin \theta}. \quad (12)$$

From Eqs. (10b) and (12), the ratio of M_{int} (apparent value by misalignment) to M_{GAM} is given as

$$\frac{M_{\text{int}}}{M_{\text{GAM}}} = 0.4 \sqrt{2} (k_r l) q \sin \delta \sin \theta. \quad (13)$$

Note these two quantities have the same phase. The condition, that the apparent contribution M_{int} by error in placing is much smaller than the real value for GAM, is rewritten as

$$|\sin \delta| \ll \frac{2.5}{\sqrt{2} (k_r l) q |\sin \theta|}. \quad (14)$$

4. Discussion on Mach Probe Measurement

In measuring the flow velocity that is parallel to the magnetic field line, it is assumed that two probes in a pair for a Mach probe is on the same magnetic field line. In other words, the vector between the two probes (the z -axis in Fig. 1) must be (almost) 90-degree to the major-radius-direction. If this angle is misaligned, the measurement of Mach number includes an error. In the case of GAM, the toroidal mode number is zero, so that the error in the angle between major-radius-direction and probes-vector alone, does not cause an error when the probe is correctly placed at the top. If the deviation of the probe position from the top of magnetic surface exists simultaneously, the density perturbation at one probe tip is different from the density perturbation at the other tip. Thus, Mach number perturbation at GAM frequency has an interference from the density perturbation. The interference from the density perturbation can propagate to all of Mach number measurement.

This hypothesis can be tested as follows.

- (a) The relative distance $R-R'$ can be confirmed by MHD equilibrium reconstruction. One can further confirm Eq. (3) by observing the change of ratio between E_θ and E_r at GAM frequency under the condition of moving the toroidal axis of plasma (for fixed probe position).
- (b) In this process (of moving magnetic axis), the change of observed Mach number perturbation at GAM frequency is observed. One may also rotate the probe (so as to change the angle δ between the major-radius direction and the vector of two probes), and the Mach number perturbation at GAM frequency is observed.

If the hypothesis applies, “the Mach number perturbation at GAM frequency” is a unique indicator to confirm the relevance of measurements.

The argument in this note can be extended for the case of drift waves. An order of magnitude estimate is briefly discussed here. In drift waves, the perturbation of the ion parallel flow velocity has an order of magnitude of $v_\parallel \sim k_\parallel e\phi/(m_i\omega)$ [13]. That is, the perturbation in Mach number is

$$M_{\text{drift}} \sim \frac{k_\parallel c_s}{\omega} \frac{e\phi}{T_e}. \quad (15)$$

Putting an estimate $\omega \sim k_\theta \rho_s c_s / L_n$, where L_n is the density gradient scale length,

$$M_{\text{drift}} \sim \frac{k_\parallel L_n}{k_\theta \rho_s} \frac{e\phi}{T_e} \sim \frac{k_\parallel L_n}{k_\theta \rho_s} \frac{\tilde{n}}{n_0}. \quad (16)$$

For the simplicity of the argument, we take the case when the probe is at top of the magnetic surface ($R = R'$) and that it has a small inclination angle δ . The poloidal wave number is finite for drift waves, and the difference of density perturbation at the two tips of Mach probe is

$$n_1 - n_2 = -ik_\theta l \sin \delta \tilde{n}. \quad (17)$$

From Eq. (10a), one sees that an error in the Mach number is induced by this inclination as

$$M_{\text{int}} \sim -0.4ik_\theta l \sin \delta \frac{\tilde{n}}{n_0}. \quad (18)$$

Thus, the condition that the error in the evaluation in the velocity fluctuation is much smaller than the real fluctuation of velocity for drift waves is evaluated as

$$|\sin \delta| \ll 2.5 (k_\parallel L_n / k_\theta \rho_s) (k_\theta l)^{-1}. \quad (19a)$$

If one puts further estimate, $k_\parallel \sim (Rq)^{-1}$ and $k_\theta \rho_s \sim 1$, Eq. (19a) takes a form

$$|\sin \delta| \ll 2.5 (L_n / qR) (\rho_s / l). \quad (19b)$$

This condition is usually more stringent than that for the case of GAM oscillations (Eq. (14)). It is also noted that the error in M fluctuation by the inclination has a phase difference by the amount of $\pi/2$, compared to the density perturbation (or potential perturbation). This phase difference must also be taken care of when one evaluates the turbulence-driven flux.

Acknowledgments

This note is an outcome from the joint-data-analysis workshop (November 12 to November 20, 2015 at Kyushu University), which was organized in the framework of the international collaboration programme of RIAM Kyushu University. The study is partly supported by a Grant-in-Aid for Scientific Research of JSPS, Japan (23244113, 15H02155, 15K14282, and 15H02335), Asada Science Foundation, National Science Foundation of China (11375054), and National Magnetic Confinement Fusion Science Program of China (2013GB111000, and 2013GB112008).

- [1] C. Hidalgo *et al.*, Phys. Rev. Lett. **91**, 065001 (2003).
- [2] B. Labit *et al.*, Phys. Plasmas **18**, 032308 (2011).
- [3] S. Oldenburger *et al.*, Plasma Fusion Res. **7**, 2401146 (2012).
- [4] S. Inagaki, submitted to Sci. Rep. (2015).
- [5] K.J. Zhao, presented at the Japan-China Joint-Data-Analysis Workshop (November 2015, at Kyushu University).
- [6] S.-I. Itoh, Phys. Fluids B **4**, 796 (1992).
- [7] P.H. Diamond *et al.*, Nucl. Fusion **53**, 104019 (2015).
- [8] Y. Kosuga, S.-I. Itoh and K. Itoh, Plasma Fusion Res. **10**, 3401024 (2015).
- [9] J.P. Gunn *et al.*, Phys. Plasmas **8**, 1995 (2001).
- [10] S.-I. Itoh, K. Itoh, M. Sasaki, A. Fujisawa, T. Ido and Y. Nagashima, Plasma Phys. Control. Fusion **49**, L7-L10 (2007).
- [11] Y. Nagashima, K. Itoh, A. Fujisawa, K. Shinohara, S.-I. Itoh, T. Ido, M. Yagi, K. Hoshino, A. Ejiri, Y. Takase, K. Uehara, Y. Miura, Plasma Phys. Control. Fusion **51**, 065019 (2009).
- [12] P.H. Diamond, S.-I. Itoh, K. Itoh and T.S. Hahm, Plasma Phys. Control. Fusion **47**, R35 (2005).
- [13] B.B. Kadomtsev, *Plasma Turbulence* (Academic Press Inc., New York, 1965).