Density Peaking by Parallel Flow Shear Driven Instability*)

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A theory to describe coupled dynamics of drift waves and D'Angelo modes is presented. The coupled dynamics is formulated by calculating fluctuation energy evolution. When drift waves dominate, turbulence production is due to release of free energy in density profile. Drift waves in turn exert Reynolds stress to drive secondary axial flows. When parallel flow shear is strong, D'Angelo modes dominate. Turbulent production occurs from release of free energy in parallel flow shear. D'Angelo modes can generate a secondary structure in density profile and can peak density profile. It is shown that when D'Angelo modes are unstable, they necessarily contribute to an inward particle flux, that compete against an outward, down-gradient flux. Net inward, up-gradient particle flux can result for strong flow shear, which can lead to density peaking in plasmas. Application to laboratory and astrophysical plasmas is discussed.

Keywords: drift wave, D'Angelo mode, transport, up-gradient transport, residual stress

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1. Introduction

Structural formation in turbulent plasmas is one of fundamental problems in plasma physics [1, 2]. A well-known example is the formation of zonal flows [3]. Primarily, turbulence is driven by free energy stored in density or pressure gradient [4]. Once excited, drift wave turbulence generate zonal flow by exerting Reynolds stress. More recently, it is discussed that a different type of secondary flow structure, i.e. flows along magnetic fields, can emerge from drift wave turbulence [5]. In this case, drift wave turbulence with broken parallel symmetry can exert residual stress to drive flows [6]. Flows in turn impact dynamics of turbulence. For example, flows can stabilize the background turbulence. Flows themselves also can serve as a source for turbulence. As a consequence, turbulent plasmas are characterized by multiple driving free energy and by multiple secondary structures. Their feedback and cross coupling are key to understand dynamics of turbulent plasmas.

As a specific example of turbulent plasmas with coupled free energy source and secondary structures, here we focus on the coupling of gradient in density and flows along magnetic field lines. Indeed, flows along magnetic field is ubiquitous and can be found in many systems. For example, there are toroidal flows in magnetically confined plasmas [5]. The toroidal rotation can be driven either by external torque exerted from neutral beam injection or by intrinsic torque exerted by drift wave turbulence with broken parallel symmetry. Another example of flows along magnetic field can be found in the earth's magnetosphere [7]. In this case, flows along the dipole magnetic filed of the earth can drive several phenomena. In the polar magnetic cusp region, the parallel flows drive turbulent fluctuation and can impact anomalous electron pitch angle scattering [7]. In addition to these, astrophysical jets can also flows along magnetic field. Magnetic field aligned with flows themselves play an important role in collimating the jet structure [8]. Thus, flows along magnetic fields play a key role to understand phenomenology in several systems, including laboratory, space, and astrophysical plasmas.

Impact of parallel flows on turbulence dynamics has been studied in basic experiments. In the presence of parallel flow shear, parallel compression for ion acoustic waves increases and the phase velocity of ion acoustic increases. As a consequence, the shear modified ion acoustic waves (SMIA) are less Landau-damped and are easier to be destabilized [9–11]. The coupling of SMIA to drift wave type instability has also been studied [12]. When the parallel flow shear increases further, the parallel compression can be negative to drive fluid like, Kelvin-Helmholtz (KH) type instability. The instability is called a D'Angelo mode [14] and the recent experiment [13] reveals the coupling of D'Angelo modes and drift waves. Importantly, that experiment measures transport flux and identifies coupled density and parallel flow transport. In short, it was reported that: i.) density gradient driven turbulence leads to outward particle flux and exert Reynolds forcing to drive

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inversion of axial flows. ii.) parallel flow shear can also act as a source for turbulence. When turbulence is driven by parallel flow shear, particle flux can be up-gradient to peak the density profile.

From theoretical perspective, coupling of drift waves and parallel flow shear driven KH was studied by D'Angelo [14]. In that study, a detailed feature of instability was discussed. Later the effect of magnetic field shear was addressed [15]. Stabilizing effect of parallel flows on underlying turbulence is tested numerically [16–18]. For the impact on transport, application to toroidal momentum transport and heat transport was studied [19]. However, the effect of parallel flow shear driven instability on particle transport has not been understood well. For example, the observed density peaking behavior cannot be addressed by existing theories.

In this paper, we present a theory to describe the coupled dynamics of drift waves and D'Angelo modes. We use a simplified fluid model for collisional drift wave instability and parallel flow shear driven D'Angelo mode. The coupled dynamics of drift waves and D'Angelo modes is formulated in terms of fluctuation energy balance evolution. The fluctuation energy evolution is determined by the balance between collisional dissipation and fluctuation production. When drift waves dominate, the production of fluctuation energy is due to the release of free energy in density gradient. Flows along magnetic field can be driven as a secondary structure. When D'Angelo modes dominate, fluctuation energy is produced by releasing free energy in parallel flow shear. As a secondary structure, peaked density profile can be generated. A simplified quasilinear calculation implies that when D'Angelo modes are unstable, inward particle flux is possible.

The remaining of the paper is organized as follows. In section 2, a simplified fluid model for collisional drift waves and D'Angelo modes is presented. In section 3, feature of linear instability is summarized. Instability diagram is presented. The coupled dynamics of drift waves and D'Angelo modes is analyzed in terms of fluctuation energy balance in section 4. In section 5, the form of transport flux is discussed by using quasilinear calculation. Section 6 is summary and discussion.

2. Model

Here we describe a model used to analyze coupled dynamics of drift waves and D'Angelo modes. The model is given by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_\mathrm{s}^2\nabla_\perp^2\frac{e\phi}{T_\mathrm{e}} = -D_\parallel\nabla_\parallel^2\left(\frac{e\phi}{T_\mathrm{e}} - \frac{n_\mathrm{e}}{n_0}\right), \tag{1a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{n_\mathrm{e}}{n_0} = -D_\parallel\nabla_\parallel^2\left(\frac{e\phi}{T_\mathrm{e}} - \frac{n_\mathrm{e}}{n_0}\right) - \nabla_\parallel v_\parallel, \tag{1b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}v_\parallel = -c_\mathrm{s}^2\nabla_\parallel\frac{n_\mathrm{e}}{n_0}. \tag{1c}$$

Here, $\mathrm{d}/\mathrm{d}t = \partial_\mathrm{t} + (c/B)\hat{z}\times\nabla\phi$ is the total time derivative with $E\times B$ advection, ρ_s is the ion sound Larmor radius, ϕ is electrostatic potential, $D_\parallel = v_\mathrm{the}^2/\nu_\mathrm{e}$ is the parallel diffusivity of electrons, v_the is electron thermal velocity, ν_e is electron collision frequency, n_e is the electron density, n_0 is a reference density, $v_\parallel$ is the parallel velocity of ions. For notation, $\parallel$ and $\perp$ denote the direction parallel and perpendicular to the magnetic field, respectively. The correspondence $\parallel\leftrightarrow z$ and $\perp\leftrightarrow(r,\theta)\leftrightarrow(x,y)$ is understood hereafter. The model describes coupled dynamics of drift waves and D'Angelo modes. Up to coupling to ion parallel flows, the model reduces to Hasegawa-Wakatani model for collisional drift wave turbulence [20]. The phase shift for electron density is caused by electron collision. By including coupling to ion parallel flows, we introduce another source of free energy, i.e. parallel flow shear.

3. Linear Analysis

Here we describe linear mode analysis to elucidate the instability dynamics described by the model. By linearizing the equations and by Fourier analyzing, we obtain dispersion relation as:

$$\frac{\rho_\mathrm{s}^2k_\perp^2}{ik_\parallel^2D_\parallel}\omega = \frac{-(\omega-\omega_{*\mathrm{e}})\omega + (c_\mathrm{s}^2k_\parallel^2 - c_\mathrm{s}k_\parallel\rho_\mathrm{s}k_y\langle v_z\rangle')}{(\omega + ik_\parallel^2D_\parallel)\omega - c_\mathrm{s}^2k_\parallel^2}. \tag{2}$$

Here $\omega_{*\mathrm{e}} \equiv \rho_\mathrm{s}k_y(c_\mathrm{s}/L_n)$ is the electron drift frequency and $L_n^{-1} = -\langle n\rangle'/n_0$ is the electron density scale length. The bracket $\langle ...\rangle$ is used to denote mean quantities.

In order to elucidate instability caused by parallel flow shear, here we focus on adiabatic electron limit $k_\parallel^2D_\parallel \gg \omega$. In this limit, the dispersion relation reduces to:

$$(1+k_\perp^2\rho_\mathrm{s}^2)\omega^2 - \omega_{*\mathrm{e}}\omega - k_\parallel^2c_\mathrm{s}^2\left(1 - \frac{k_y}{k_\parallel}\frac{\langle v_z\rangle'}{\omega_\mathrm{ci}}\right) = 0. \tag{3}$$

From this relation, we can see that parallel flow shear modifies parallel compressibility of ion acoustic waves. When $k_yk_\parallel\langle v_z\rangle' < 0$, the parallel flow shear enhances parallel compression. The phase velocity in the parallel direction becomes faster and as a result, ion acoustic waves are less Landau-damped. This shear modified ion acoustic waves (SMIA) are thus easier to be destabilized [9]. For example, this effect can be at work for current driven ion acoustic waves, where modes are destabilized by electron inverse Landau damping. The same mechanism is effective for electron drift waves, which are driven by inverse electron Landau-damping and damped by ion Landau-damping [12]. Here we note that in these cases, parallel sheared flows are *not* acting as a direct free energy source. Rather, it mediates other modes to access to free energy, such as electron current for ion acoustic waves or density profile for drift waves. On the other hand, when $k_yk_\parallel\langle v_z\rangle' > 0$, the parallel compressibility can be 'negative'. In these cases, parallel flow shear act as free energy source for driving instability. Strong fluid like instability can arise to release

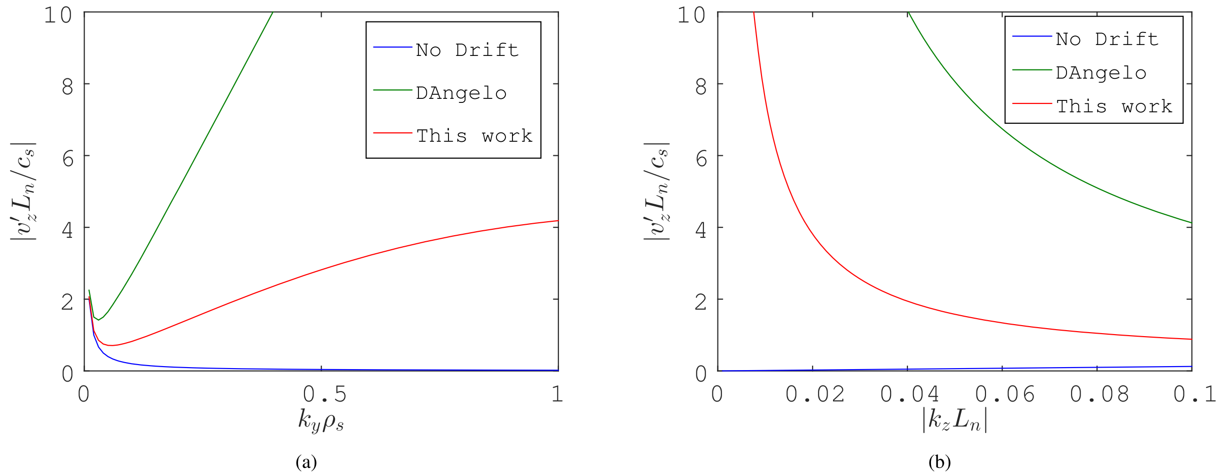

Fig. 1 Instability diagram. (a) Instability diagram with $k_{\|} L_n = 1/50$ and (b) Instability diagram with $k_y \rho_s = 0.8$. In both cases, blue is without coupling to drift waves, green is original work by D'Angelo, and red is this work. Above the red curve, D'Angelo modes can be unstable and fluctuations are supported by D'Angelo modes. Below the curve, D'Angelo modes are stable and drift waves are supporting fluctuation.

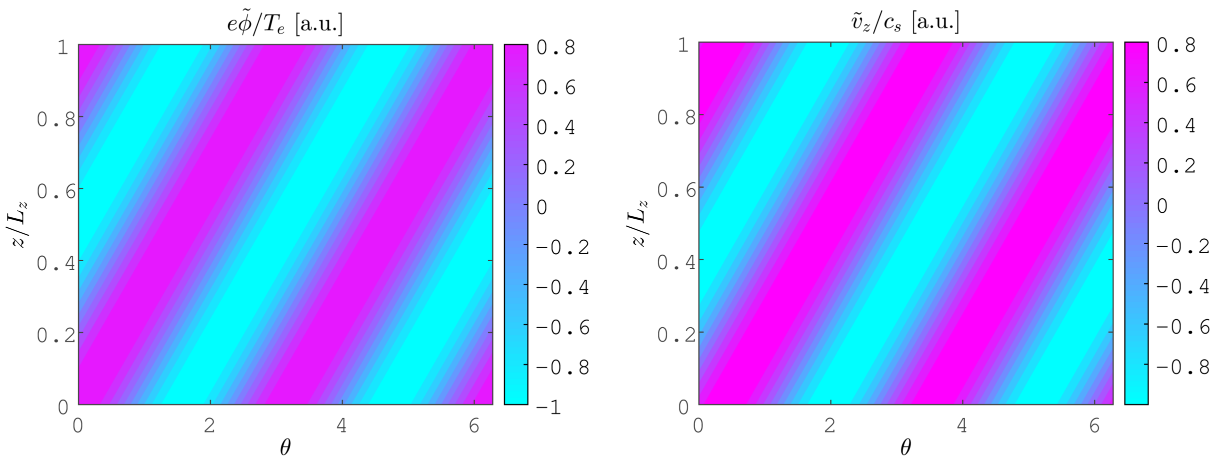

Fig. 2 Contour plot for unstable D'Angelo modes at a radial location. Surface of the cylinder is opened into 2d plane (θ, z). (a) potential fluctuation contour. (b) parallel velocity fluctuation contour. Both are plotted for $\langle v_z \rangle' < 0$. Due to necessary condition for instability, only modes with $k_y k_{\|} < 0$ can be unstable. As a result, fluctuation has a fixed pitch, as shown in the figure.

free energy in parallel flow shear. The unstable ion acoustic waves are coupled to drift wave branch ω_*. The coupling to drift waves introduces stabilizing effect on unstable ion acoustic waves [14].

The above discussion can be summarized into instability diagram, as shown in Fig. 1. The diagram is evaluated by solving Eq. 3 for marginal condition. The marginal condition is:

$$k_{\|} c_s \rho_s k_\theta \langle v_z \rangle' = k_{\|}^2 c_s^2 + \frac{\omega_{*e}^2}{4(1 + k_\perp^2 \rho_s^2)}. \tag{4}$$

The relation is plotted in Fig. 1 for: (a) $k_{\|} L_n = 1/50$ and (b) $k_y \rho_s = 0.8$. In both figures, different colors describe marginal condition for: ion acoustic waves without drift wave coupling (blue), the original D'Angelo modes (green), and the marginal condition for the model presented here (red). As we can see from the graph, the drift wave coupling is stabilizing for flow shear driven modes. Compared to the original work, the model in this paper includes ion finite inertia. As a result, drift wave frequency reduces from ω_{*e} to $\omega_{*e}/(1 + \rho_s^2 k_\perp^2)$. Thus stabilizing effect from drift waves is reduced. Above the red curve, D'Angelo modes are unstable and can support turbulent fluctuation. Below the curve, D'Angelo modes are stable and collisional drift waves support turbulent fluctuation.

An important feature of D'Angelo modes is that modes with a preferred pitch are selectively excited [21]. This is shown in Fig. 2. This property follows from the fact that the condition $k_y k_z \langle v_z \rangle' > 0$ must be satisfied in order to access to negative compressibility. For example, for $\langle v_z \rangle' < 0$, only modes with $k_y k_z < 0$ is unstable, while modes with $k_y k_z > 0$ is stable. In this case, potential fluctuation has a pattern as shown in Fig. 2. Here, potential is plotted in (θ, z) plane. The pattern of parallel velocity fluctuation is also plotted. In terms of phase, the parallel velocity fluctuation is shifted by $\delta = \tan^{-1}(\gamma/\omega_r) \cong \pi/4$. This is a relation such that the velocity fluctuation drives anomalous loss of parallel momentum transport.

4. Turbulence Energetics

In order to elucidate the coupled dynamics of drift waves and D'Angelo modes, here we discuss fluctuation energetics [22]. From the model equations, the evolution of the total fluctuation energy, $I \equiv \int d^3x\{(\rho_s\nabla_\perp e\tilde{\phi}/T_e)^2 + (\tilde{n}_e/n_0)^2 + (\tilde{v}_\parallel/c_s)^2\}/2$ is given as:

$$\partial_t I = \int d^3x(\mathcal{P} - \mathcal{D}), \tag{5a}$$

$$\mathcal{P} = c_s\rho_s\partial_y\frac{e\tilde{\phi}}{T_e}\frac{\tilde{n}_e}{n_0}\frac{\partial_x\langle n_e\rangle}{n_0} + c_s\rho_s\partial_y\frac{e\tilde{\phi}}{T_e}\frac{\tilde{v}_\parallel}{c_s}\frac{\partial_x\langle v_z\rangle}{c_s}, \tag{5b}$$

$$\mathcal{D} = D_\parallel\left\{\nabla_\parallel\left(\frac{\tilde{n}_e}{n_0} - \frac{e\tilde{\phi}}{T_e}\right)\right\}^2. \tag{5c}$$

Here $\mathcal{P}$ is the production term for turbulent fluctuation and $\mathcal{D}$ is the collisional dissipation. The production term takes the form of flux times gradient. For example, the first term is the product of turbulent particle flux and the gradient in the particle profile, while the second term is the product of turbulent momentum flux $\langle\tilde{v}_x\tilde{v}_\parallel\rangle$ and the gradient in parallel flow velocity. In this sense, the production term can be thought of as entropy production term for fluctuation. Importantly, the production term identifies the direction of energy flow (Fig. 3). For example, if the production term is positive, fluctuation gains energy from free energy source. On the other hand, if the production term is negative, fluctuation loses energy to drive secondary structures, such as parallel flows or peaked density profiles.

As an illustration of the fluctuation production, here we first look at fluctuations dominated by drift waves. In this case, by keeping leading order contribution in the particle and momentum transport channel, the production term is given as:

$$\langle\mathcal{P}\rangle = \frac{D_n}{L_n^2} - \sum_{\boldsymbol{k}}\frac{k_y\rho_s c_s k_\parallel}{k_\parallel^2 D_\parallel}\left|\frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_e}\right|^2\partial_x\langle v_z\rangle. \tag{6}$$

Here the average is evaluated by integrating in y and z direction. $D_n = \sum_{\boldsymbol{k}} c_s^2/(k_\parallel^2 D_\parallel)\rho_s^2 k_y^2|e\tilde{\phi}_{\boldsymbol{k}}/T_e|^2$ is the turbulent diffusivity. The first term is the production term associated with density relaxation. This term is positive definite; thus density gradient is driving turbulent fluctuation. This is plausible since we are investigating drift wave dominated regime. In contrast, the second term, which is due to parallel flow coupling, can be negative. In this case, drift wave turbulence can generate secondary flow structures in magnetic field direction. Importantly, to have a finite contribution in this channel, we must have $\overline{k_y k_\parallel} \neq 0$ where $\overline{(...)}$ denotes spectrum average. In other words, we need a symmetry breaking of drift wave turbulence in the magnetic field direction. This is very analogous to the nature of residual stress to drive intrinsic rotation in toroidal plasmas [6]. In the case of basic experiment, we note that experimental configuration can break the symmetry, since source is located at one end while sink is located at the other end. Thus the symmetry in the magnetic field direction of typical basic experiments is broken and it is quite likely to have nonzero residual stress. As a consequence, it is quite likely to produce a secondary flow structure along magnetic field by drift wave turbulence.

As the second illustration, we discuss production term for D'Angelo modes. In this case, the production term is evaluated as:

$$\langle\mathcal{P}\rangle = D_V\left(\frac{\langle v_z\rangle'}{c_s}\right)^2 - \sum_{\boldsymbol{k}}\frac{c_s k_\parallel\rho_s k_y\langle v_z\rangle'}{\gamma_{KH} + k_\parallel^2 D_\parallel}\left|\frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_e}\right|^2. \tag{7}$$

Here $D_V = \sum_{\boldsymbol{k}} c_s^2/(\gamma_{KH})\rho_s^2 k_y^2|e\tilde{\phi}_{\boldsymbol{k}}/T_e|^2$ is the eddy viscosity on parallel flows. The first term is positive definite. This term is associated with release of free energy in parallel flow velocity gradient. Turbulent fluctuation is produced from parallel flow shear. On the other hand, D'Angelo modes can create a secondary structure in density profile. This effect is captured in the second term. Note that the second term is explicitly dependent upon $\langle v_z\rangle'$, i.e. a tensor quantity, thus it may look peculiar at the first glance. However, the quantity is multiplied by another tensor quantity, i.e. $k_\parallel k_y$, thus the second term as a whole is a scalar. The second term is negative definite, when D'Angelo modes are unstable. This is since if D'Angelo modes are unstable, the condition $k_y k_\parallel\langle v_z\rangle' > 0$ must be satisfied so as to have 'negative compressibility' effect. As a consequence, D'Angelo modes can convert fluctuation energy to produce a secondary structure in density profile. In this case, up-gradient, net inward flux can arise and can peak density profile.

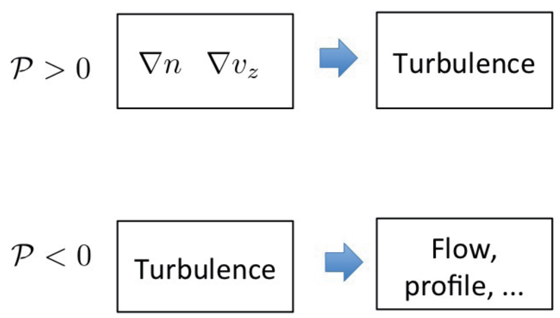

Fig. 3 A schematics for fluctuation production. When the production is positive, turbulence gains energy by releasing free energy stored in gradients. When the production is negative, turbulence loses energy and can drive a secondary structure.

5. Transport

In order to elaborate the impact of the coupled dynamics on transport processes, here we explicitly evaluate transport fluxes. By using a simplified quasilinear theory, we have:

$$\frac{\Gamma_n}{n_0 c_\mathrm{s}} = \sum_{\boldsymbol{k}} k_y \rho_\mathrm{s} \frac{-\omega + \omega_{*\mathrm{e}}}{k_\parallel^2 D_\parallel} \left| \frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_\mathrm{e}} \right|^2 + \sum_{\boldsymbol{k}} k_y \rho_\mathrm{s} \frac{c_\mathrm{s}^2 k_\parallel^2 - c_\mathrm{s} k_\parallel \rho_\mathrm{s} k_y \langle v_z \rangle'}{k_\parallel^2 D_\parallel \omega} \left| \frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_\mathrm{e}} \right|^2, \quad (8a)$$

$$\frac{\Pi_{rz}}{c_\mathrm{s}^2} = \sum_{\boldsymbol{k}} k_y \rho_\mathrm{s} \frac{(-\omega + \omega_{*\mathrm{e}}) c_\mathrm{s} k_\parallel}{k_\parallel^2 D_\parallel \omega} \left| \frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_\mathrm{e}} \right|^2 + \sum_{\boldsymbol{k}} k_y \rho_\mathrm{s} \frac{c_\mathrm{s} k_\parallel - \rho_\mathrm{s} k_y \langle v_z \rangle'}{\gamma_\mathrm{KH}} \left| \frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_\mathrm{e}} \right|^2 \Theta(\gamma_\mathrm{KH}). \quad (8b)$$

Here $\Theta(\gamma_\mathrm{KH})$ is the step function. The transport flux has both diagonal and off-diagonal terms. The first term in the particle flux is diagonal term, which is driven by density gradient. At the simplest level, this term is related to turbulent diffusivity. The second term is off-diagonal term that is related to gradient in parallel flow velocity. Importantly, when D'Angelo modes are unstable, the last term tend to drive inward flux. Later we show that the net inward flux is indeed possible for certain plasma parameters. The momentum flux also has diagonal and off-diagonal terms. The second term is diagonal term, which is driven by parallel flow velocity gradient. At the simplest level, this term is related to eddy viscosity on the flow. The first term is off-diagonal term. This is akin to residual stress, which is important to understand the generation mechanism of intrinsic rotation. Symmetry breaking is required to have non-vanishing residual stress.

To be more specific, here we discuss transport caused by D'Angelo modes. When D'Angelo modes are dominant, the turbulent flux reduces to:

$$\Pi_{rz} \cong -D_\mathrm{V} \langle v_z \rangle', \quad (9a)$$

$$\frac{\Gamma_n}{n_0 c_\mathrm{s}} \cong \sum_{\boldsymbol{k}} k_y \rho_\mathrm{s} \frac{\omega_{*\mathrm{e}}}{k_\parallel^2 D_\parallel} \frac{1 + 2k_\perp^2 \rho_\mathrm{s}^2}{2(1 + k_\perp^2 \rho_\mathrm{s}^2)} \left| \frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_\mathrm{e}} \right|^2 - \sum_{\boldsymbol{k}} \frac{2(1 + k_\perp^2 \rho_\mathrm{s}^2)}{k_\parallel^2 D_\parallel} (L_n k_\parallel \rho_\mathrm{s} k_y \langle v_z \rangle') \left| \frac{e\tilde{\phi}_{\boldsymbol{k}}}{T_\mathrm{e}} \right|^2. \quad (9b)$$

Here $D_\mathrm{V} = \sum_{\boldsymbol{k}} c_\mathrm{s}^2 \gamma_\mathrm{KH} / (\omega_\mathrm{KH}^2 + \gamma_\mathrm{KH}^2) \rho_\mathrm{s}^2 k_y^2 \left| e\tilde{\phi}_{\boldsymbol{k}} / T_\mathrm{e} \right|^2$ is the eddy viscosity on flows. When D'Angelo modes are dominant, momentum flux is down the gradient. The flux is characterized by the eddy viscosity. The particle flux consists of two terms. The first part is due to density gradient. This term yields diffusive, outward flux. The second term is off-diagonal term due to gradient in parallel flows. Importantly, this term is negative definite for peaked profile $L_n \propto -\mathrm{d}\langle n \rangle / \mathrm{d}x > 0$, since necessary condition for D'Angelo modes to be unstable requires $k_\parallel k_y \langle v_z \rangle' > 0$. Physically put, the condition is required for turbulence to mix free energy in parallel flow velocity gradient. For a given configuration, the parallel momentum flux $\propto k_y k_\parallel$ must have a corresponding sign to relax velocity profile. In this situation, the mixing in parallel momentum results in inward flux of particle. The inward particle can compete against the outward, diffusive flux. When the influx due to the parallel flow velocity becomes strong enough, net inward, up-gradient particle flux can result and can peak density profile. By using plasma parameters for D'Angelo instability: $\rho_\mathrm{s} k_y = 0.8$, $k_\parallel L_n = 0.06$, $e\tilde{\phi}/T_\mathrm{e} \sim 0.2$, $c_\mathrm{s} = 3 \times 10^3$ [m/sec], $L_n = 0.04$ [m], $v_z' = 2 \times 10^5$ [1/sec], and $D_\parallel = 1.4 \times 10^4$ [m^2/sec], we have $\Gamma_n \sim -0.17 \times 10^{21}$ [1/m^{2}sec]. Thus net inward flux and density peaking is possible for typical plasmas parameters, as observed in experiments. Finally, we note that $L_n < 0$ for hollowed profile, so the off-diagonal term produces outward flux. In this case, D'Angelo mode tends to further hollow the profile.

6. Summary and Discussion

In summary, we have discussed coupled dynamics of drift waves and D'Angelo modes. Plasma parameters that determines which modes are dominant are estimated from Fig. 1. Dynamics is characterized by calculating turbulent fluctuation production. The results are summarized in Table 1. When drift waves are dominant, fluctuation is driven by density gradient. Instability is set by phase shift, which is due to electron collision in this model. Transport is characterized by diffusive particle flux and residual stress on flows. In particular, to have non-zero residual stress, symmetry breaking in the parallel direction is required. When D'Angelo modes are dominant, fluctuation is driven by parallel velocity gradient. Instability is due to negative compressibility, which is only available for fluctuation with $k_y k_\parallel \langle v_z \rangle' > 0$. As a consequence, mode patterns are characterized by fixed pitch, as depicted in Fig. 2. Parallel momentum flux is down the gradient and is characterized by eddy viscosity. Particle flux consists of both density gradient driven diffusive flux and parallel flow velocity driven off-diagonal term. The off-diagonal particle flux is inward for unstable D'Angelo modes. For typical plasma parameters, net inward, up-gradient particle flux is possible, which can be thought of as an origin of density peaking behavior.

We discuss a few caveats on the model presented in this work and necessary future development. First of all, in this work we have not addressed the role of perpendicular flow. In particular, turbulence driven zonal flows can be important element in understanding the coupled dynamics. In this case, coupling among drift waves, zonal flows, and axial flows must be formulated. A possible approach for this is to calculate fluctuation energetics as well as potential fluctuation balance, which would yield momentum theorem akin to that by Charney and Drazin in geophys-

Table 1 A summary of analysis. Relevant driving free energy, relevant destabilizing mechanism, impact on transport are listed. In the presence of multiple driving source, transport interfere each other. For example, drift waves drive transport of parallel momentum, which can result in parallel flow inversion. Alternatively, parallel flow shear driven D'Angelo modes tend to drive inward particle flux, which can cause net inward particle flux to peak density profile.

	Drift waves	D'Angelo modes
Driving free energy	∇n	∇v_z
Destabilizing mechanism	Phase shift $\tilde{n} \propto (1 - i\delta)\tilde{\phi}$ (electron collision in this model)	Negative compressibility Necessary condition for instability: $k_y k_\parallel \langle v_z \rangle' > 0$
Transport: particle parallel momentum	Outward, diffusive particle flux Residual stress	Inward contribution in particle flux Eddy viscosity on flows

ical fluid dynamics [23–25]. Secondary, we note that the simplified quasilinear transport modeling presented in this work may not be applicable when plasmas are dynamically perturbed [26]. In this case, dynamical perturbation can immediately couple to turbulence dynamics to impact transport processes [27]. This effect may be relevant when we try to control particle transport via parallel flows by injecting neutral beam. In this case NBI would dynamically perturb plasmas and transport in these perturbed plasmas would require transport modeling beyond quasilinear theory. Finally, another important extension is to extend the analysis presented in this work to collisionless plasmas, where kinetic effect plays an important role. Indeed, some of the branches driven by parallel flows are characterized by resonance drive. Then we may ask a question of what happens when the resonance is strong enough. In this case, we would expect that phase space structures can form and phase space turbulence can develop [28]. Transport in this situation is described not by simplified quasilinear diffusive flux but by Lenard-Balescu flux with dynamical friction [29, 30]. Phase space structures may convert poloidal and toroidal flows to produce doubly connected flow structure [31].

With the caveat in mind, here we discuss a possible application of the ideas presented in this paper. In the context of fusion research, this type of parallel flow shear driven turbulence may be used for fueling control. When particle source is localized in peripheral region, the particles can be transported inward by using the parallel flow shear driven turbulence. In the context of astrophysical plasmas, we may be able to exploit this type of ides to explain collimation of jets. After injected, astrophysical jets may drive Kelvin-Helmholtz like instability as presented in this work. In that case, parallel flow shear driven instability may drive inward flux of plasmas and may introduce pinching effect, whereby collimation may result.

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[1] B.B. Kadomtsev, *Tokamak Plasma: A Complex Physical System* (IoP publishing, Bristol and Philadelphia, 1992).
[2] K. Itoh, S.-I. Itoh and A. Fukuyama, *Transport and Structural Formation in Plasmas* (IoP publishing, Bristol and Philadelphia, 1999).
[3] P.H. Diamond, S.-I. Itoh, K. Itoh and T.S. Hahm, Plasma Phys. Control. Fusion **47**, R35 (2005).
[4] B.B. Kadomtsev, *Plasma Turbulence* (Academic, New York, 1965).
[5] K. Ida and J.E. Rice Nucl. Fusion **54**, 045001 (2014).
[6] P.H. Diamond, Y. Kosuga, Ö.D. Gürcan, C.J. McDevitt, T.S. Hahm, N. Fedorczak, J.E. Rice, W.X. Wang, S. Ku, J.M. Kwon, G. Dif-Pradalier, J. Abiteboul, L. Wang, W.H. Ko, Y.J. Shi, K. Ida, W. Solomon, H. Jhang, S.S. Kim, S. Yi, S.H. Ko, Y. Sarazin, R. Singh and C.S. Chang, Nucl. Fusion **53**, 104019 (2013).
[7] W.E. Amatucci, J. Geophys. Res. **104**, 14481 (1999).
[8] B. Albertazzi, A. Ciardi, M. Nakatsutsumi, T. Vinci, J. Beard, R. Bonito, J. Billette, M. Borghesi, Z. Burkley, S.N. Chen, T.E. Cowan, T. Herrmannsdörfer, D.P. Higginson, F. Kroll, S.A. Pikuz, K. Naughton, L. Romagnani, C. Riconda, G. Revet, R. Riquier, H.-P. Schlenvoigt, I.Yu. Skobelev, A.Ya. Faenov, A. Soloviev, M. Huarte-Espinosa, A. Frank, O. Portugall and H. Pepin, J. Fuchs, Science **346**, 325 (2014).
[9] V.V. Garvrishchaka, S.B. Ganguli and G.I. Ganguli, Phys. Rev. Lett. **80**, 728 (1998).
[10] E. Agrimson, N. D'Angelo and R.L. Merlino, Phys. Rev. Lett. **86**, 5282 (2001).
[11] C. Teodorescu, E.W. Reynolds and M.E. Koepke, Phys. Rev. Lett. **88**, 185003 (2002).
[12] T. Kaneko, H. Tsunoyama and R. Hatakeyama, Phys. Rev. Lett. **90**, 125001 (2003).
[13] T. Kobayashi, "Parallel flow structure formation by turbulent momentum transport in linear magnetized plasmas," The 4th Asia Pacific Transport Working Group International Conference, BO2 (2014).
[14] N. D'Angelo, Phys. Fluids **8**, 1748 (1965).
[15] P.J. Cattor, M.N. Rosenbluth and C.S. Liu, Phys. Fluids **16**,

1719 (1973).
[16] L. Bai, A. Fukuyama and M. Uchida, Phys. Plasmas **5**, 989 (1998).
[17] L. Bai, A. Fukuyama and M. Uchida, Plasma Phys. Control. Fusion **40**, 785 (1998).
[18] M. Uchida, S. Sen, A.Fukuyama and D.R. McCarthy, Phys. Plasmas **10**, 4758 (2003).
[19] N. Mattor and P.H. Diamond, Phys. Fluids **31**, 1180 (1988).
[20] M. Wakatani and A. Hasegawa, Phys. Fluids **27**, 611 (1984).
[21] D.R. McCarthy, A.E. Booth, J.F. Drake and P.N. Guzdar, Phys. Plasmas **4**, 300 (1997).
[22] Y. Kosuga, P.D. Diamond and Ö.D. Gürcan, Phys. Plasmas **17**, 102313 (2010).
[23] J.G. Charney and P.G. Drazin, J. Geophys. Res. **66**, 83 (1961).
[24] P.H. Diamond, Ö.D. Gürcan, T.S. Hahm, K. Miki, Y. Kosuga and X. Garbet, Plasma Phys. Control. Fusion **50**, 124018 (2008).
[25] L. Wang, P.H. Diamond and T.S. Hahm, Plasma Phys. Control. Fusion **54**, 095015 (2012).
[26] S. Inagaki, T. Tokuzawa, N. Tamura, S.-I. Itoh, T. Kobayashi, K. Ida, T. Shimozuma, S. Kubo, K. Tanaka, T. Ido, A. Shimizu, H. Tsuchiya, N. Kasuya, Y. Nagayama, K. Kawahata, S. Sudo, H. Yamada, A. Fujisawa, K. Itoh, and the LHD Experiment Group, Nucl. Fusion **53**, 113006 (2013).
[27] S.-I. Itoh and K. Itoh, Sci. Rep. **2**, 860 (2012).
[28] M. Lesur, P.H. Diamond and Y. Kosuga, Plasma Phys. Control. Fusion **56**, 075005 (2014).
[29] T.H. Dupree, Phys. Rev. Lett. **25**, 789 (1970).
[30] Y. Kosuga, P.H. Diamond, L. Wang, Ö.D. Gürcan and T.S. Hahm, Nucl. Fusion **53**, 043008 (2013).
[31] Y. Kosuga, S.-I. Itoh, P.H. Diamond and K. Itoh, Plasma Phys. Control. Fusion **55**, 125001 (2013).