Simulation of Sawtooth Oscillation in Burning Plasma*)

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The sawtooth oscillation is one of the important instabilities driven by the plasma current in tokamak plasma. The sawtooth period is a key parameter that characterizes the effect of the sawtooth oscillation on the plasma behavior. For prediction of sawteeth in burning plasma, the sawtooth model in the 1.5-dimensional transport code TOTAL has been extended to include the effects of fast particles and the magnetic shear. By using the newly implemented model, we simulated sawteeth in the presence of the alpha particles as the fast ion in ITER. It is found that the sawtooth periods in DT plasma are longer than those in DD plasma. In DT plasma, the sawtooth period doesn't change monotonically but has a peak as a function of the average central ion temperature or the RF heating power. This is because the mechanisms of triggering sawteeth are different in the low RF heating power region and in the high RF heating power region.

Keywords: sawtooth oscillation, m = 1 mode, alpha particle, RF heating, ITER

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1. Introduction

In tokamak plasma, various instabilities are driven by the plasma current. The sawtooth oscillation is one of instabilities, and it is especially important for the fusion reactor because it occurs in the central region of plasma and directly limits the fusion power. It is also reported that the sawtooth oscillation prevents impurities from accumulating in the central region of plasma [1]. The sawteeth with long periods eject large energy from the plasma center and also induce other instabilities such as the neoclassical tearing modes (NTMs) [2]. It is estimated that the sawtooth periods will become longer due to the presence of the fast ions in ITER and DEMO [3], and it has been found that they depend on the magnetic shear at the $q = 1$ radius so that it is attempted to control them by the localized current drive on the $q = 1$ surface [4]. Therefore, it is important to estimate the sawtooth periods and the variations of the plasma profiles.

The sawtooth periods in ITER are analyzed by using a 1.5-dimensional (1.5-D) transport code TOTAL (toroidal transport analysis linkage) [5]. The Porcelli model [6] has been newly implemented to the code to study the effects of fast particles and the magnetic shear on the sawtooth crashes. In section 2, the simulation models are described. In section 3, the simulation conditions and the simulation results are described. First, we examine the difference of the sawtooth periods between DT and DD plasmas. Secondly, we examine the variation of the sawtooth periods in DT plasma when the RF heating power is changed. In section 4, we present a summary.

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2. Numerical Model

2.1 Transport model

The diffusivity χ is usually described as

$$\chi = \chi^{\mathrm{NC}} + \chi^{\mathrm{AN}}, \tag{1}$$

where χ^{NC} is the neoclassical part of the thermal diffusivity, and χ^{AN} is its anomalous part. For the anomalous part, we used the Bohm type model based on the model used in simulation of Joint European Torus (JET) plasmas [7], which is described as

$$\chi^{\mathrm{AN}} = \alpha_{\mathrm{B}} \frac{T_{\mathrm{e}}}{B_{\mathrm{t}}} q^2 \Big/ L_{\mathrm{p}}^*, \tag{2}$$

where the unit of χ^{AN} is m^2/s, T_{e} is the electron temperature in eV, B_{t} is the toroidal magnetic field in T, q is the safety factor, L_{p}^* is the pressure scale length normalized by the plasma minor radius, and $\alpha_{\mathrm{B}} = 3.3 \times 10^{-4}$. The same value of diffusivity is assumed for ions and for electrons.

2.2 Sawtooth model

We used two models for the sawtooth oscillation in order to describe the plasma profiles after the sawtooth crashes and triggering the sawtooth crashes. For the profiles after the sawtooth crashes, we used Kadomtsev model [8]. For the triggering model for sawtooth crashes, we newly implemented the Porcelli model.

The sawtooth crashes are triggered by the onset of the m = 1 mode (kink instability). The stability condition of the mode is determined by the effective potential energy functional δW that represents the potential energy changed by the mode. In the Porcelli model, δW is represented by the sum of three terms, the ideal MHD term $\delta W_{\rm MHD}$, the Kruskal-Oberman term $\delta W_{\rm KO}$ and the fast ion term $\delta W_{\rm fast}$. In the model, the sawtooth crashes are triggered when one of the following conditions is satisfied,

$$-\delta\hat{W}_{\rm core} = -\left(\delta\hat{W}_{\rm MHD} + \delta\hat{W}_{\rm KO}\right) > c_{\rm h}\omega_{\rm Dh}\tau_{\rm A}, \quad (3)$$

$$-\delta\hat{W} = -\left(\delta\hat{W}_{\rm core} + \delta\hat{W}_{\rm fast}\right) > 0.5\omega_{*{\rm i}}\tau_{\rm A}, \quad (4)$$

$$-c_\rho\hat{\rho} < -\delta\hat{W} < 0.5\omega_{*{\rm i}}\tau_{\rm A} \text{ and } \omega_{*{\rm i}} < c_*\gamma_\rho, \quad (5)$$

where $c_{\rm h}, c_\rho, c_*$ are constants, $\omega_{\rm Dh}$ is the fast ion precession frequency, $\omega_{*{\rm i}}$ is the diamagnetic frequency, $\tau_{\rm A}$ is the Alfvén time, and $\hat{\rho}$ is the ion Larmor radius normalized by the minor radius of the $q = 1$ surface r_1. $\delta\hat{W}_{\rm core}$ is the sum of the normalized ideal MHD term $\delta\hat{W}_{\rm MHD}$ and the normalized Kruskal-Oberman term $\delta\hat{W}_{\rm KO}$. $\delta\hat{W}_{\rm MHD}$ represents the normalized ideal MHD potential energy change without any kinetic effects, depending on the toroidal geometry and the plasma shaping, and is given by

$$\delta\hat{W}_{\rm MHD} = -9\pi(l_{\rm i1} - 1/2)\varepsilon_1^2(\beta_{\rm p1}^2 - \beta_{\rm pc}^2)/s_1$$
$$-18\pi(l_{\rm i1} - 1/2)^3\{(\kappa_1 - 1)/2\}^2/s_1, \quad (6)$$

where s_1 is the magnetic shear at the $q = 1$ radius, $l_{\rm i1}$ is the internal inductance at the $q = 1$ radius, $\varepsilon_1 = r_1/R$, and κ_1 is the elongation of the $q = 1$ surface. $\beta_{\rm p1}$ and $\beta_{\rm pc}$ are given by

$$\beta_{\rm p1} = (2\mu_0/B_{\rm p1}^2)\left[\langle p\rangle_1 - p(r_1)\right], \quad (7)$$

$$\beta_{\rm pc} = 0.3(1 - 5r_1/3a), \quad (8)$$

where μ_0 is the space permeability, $B_{\rm p1}$ is the poloidal magnetic field at the $q = 1$ radius, and $\langle p\rangle_1$ is the volume averaged total pressure within the $q = 1$ surface. The first term of the right hand of (6) represents the effects of the toroidal geometry, and the second term of that represents the effects of the plasma shaping. $\delta\hat{W}_{\rm KO}$ represents the stabilizing effects of the trapped thermal ions, and is given by

$$\delta\hat{W}_{\rm KO} = 0.6\left(\frac{5}{2}\int_0^1 {\rm d}x x^{3/2} p_{\rm i}(x)/p_{\rm i0}\right)\varepsilon_1^{1/2}\beta_{\rm i0}/s_1, \quad (9)$$

where $x = r/r_1$, $p_{\rm i}(x)$ is the ion pressure profile, $p_{\rm i0}$ is its peak value, and $\beta_{\rm i0}$ is the peak value of the ion toroidal beta. $\delta\hat{W}$ is the sum of $\delta\hat{W}_{\rm core}$ and the normalized fast ion term $\delta\hat{W}_{\rm fast}$. $\delta\hat{W}_{\rm fast}$ represents the stabilizing effect of the fast ions, and is given by

$$\delta\hat{W}_{\rm fast} = c_{\rm f}\varepsilon_1^{3/2}\left\{\left(-2\mu_0/B_{\rm p1}^2\right)\int_0^1 {\rm d}x x^{3/2}{\rm d}p_{\rm fast}/{\rm d}x\right\}\Big/ s_1, \quad (10)$$

where $c_{\rm f}$ depends on the ratio between the mode frequency and the fast ion precession frequency and reduces to unity in the limit $\omega << \omega_{\rm Dh}$, and $p_{\rm fast}$ is the fast ion pressure. γ_ρ represents the growth rate of the resistive internal kink mode in the semicollisional regime, and it is given by

$$\gamma_\rho \approx 1.1 s_1^{6/7}\hat{\rho}^{4/7}\Big/\left(\tau_{\rm A}S^{1/7}\right), \quad (11)$$

where S is the magnetic Reynolds number.

A sawtooth crash is triggered when one of the three conditions (3), (4) and (5) is satisfied. The condition (3) is satisfied when the driving force for the internal kink mode overcomes the fast ion stabilization. The condition (4) is satisfied when the driving force for the internal kink mode overcomes the diamagnetic rotation stabilization. The condition (5) is satisfied when the resistive internal kink mode is driven in semicollisional regime. In this paper, we refer to the left hand of the condition (5) as (5.a) and the right hand of the condition (5) as (5.b). For $c_{\rm h}, c_\rho, c_*$ and $c_{\rm f}$, $c_{\rm h} = 0.4$, $c_\rho = 1$, $c_* = 3$ and $c_{\rm f} = 1$ are used in this paper, and these values are the same as those in [5].

3. Simulation Results

3.1 Simulation conditions

We used the ITER parameters for the plasma parameters. The typical plasma parameters used in the simulation are shown in Table 1. The electron density profile is fixed to a nearly flat one with the central value of about 10^{20} m^{-3}. The radio-frequency (RF) heating is added with a constant power and with a profile of $\exp\{-(r/a)^2/0.6^2\}$. The evolution from a low temperature state with RF heating is simulated. The sawtooth period is evaluated after the plasma enters into a steady state with repeated sawtooth crashes.

3.2 Difference between DT and DD plasmas

Simulations on DT and DD plasmas were compared to study the effect of alpha particles on the sawtooth instability. The RF heating power was scanned from 12 to 70 MW in DT plasma to change the population of alpha particles. In DD plasma, the RF heating power of 30 to 170 MW was added to obtain the same temperature level as DT plasma. We define the average central ion temperature as

$$T_{\rm i}(0)_{\rm ave} = \left(T_{\rm i}(0)^{\rm before-crash} + T_{\rm i}(0)^{\rm after-crash}\right)\Big/2. \quad (12)$$

Figure 1 shows the dependence of the sawtooth periods on the average central ion temperature. The sawtooth periods in DT plasma are longer than those in DD plasma at the same temperature level. The sawtooth crashes are triggered by the condition (5) in the relatively low temperature region (I) and they are triggered by the condition (3) in the relatively high temperature region (II).

Figure 2 shows the time evolutions of the components of the each triggering condition at the point (i) (DT) and the

Table 1 The typical parameters of ITER.

R [m]	6.2
a [m]	2.0
$B_{\rm t}$ [T]	5.3
$I_{\rm p}$ [MA]	15
κ	1.7
δ	0.33

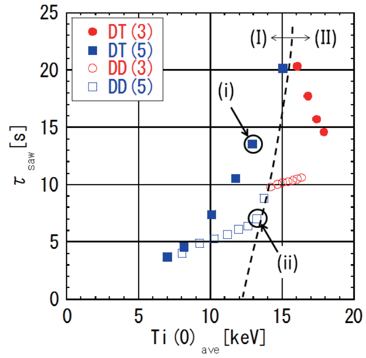

Fig. 1 Dependence of the sawtooth period on the average central ion temperature in DT and DD plasmas. The open symbols denote the results of DD plasma. The sawtooth crashes are triggered by condition (5) in region (I) and they are triggered by condition (3) in region (II).

point (ii) (DD) , at the same temperature level (region (I)), in Fig. 1. It takes a longer period in (i) for $-\delta\hat{W}$ to increase sufficiently to satisfy the condition (5) though the increase rates are similar in (i) and (ii). This is because $-\delta\hat{W}_{\rm fast}$ is negative due to the presence of the alpha particles in DT plasma while it is zero in DD plasma. That is, the pressure inside the $q = 1$ radius needs to increase larger after the sawtooth crash in (i) so that $-\delta\hat{W}_{\rm core}$ becomes larger in (i) by an amount of $-\delta\hat{W}_{\rm fast}$ than in (ii). Therefore, the difference in region (I) is caused by the presence of $-\delta\hat{W}_{\rm fast}$.

On the other hand, the sawtooth crashes are triggered by condition (3) in region (II). The $c_{\rm h}\omega_{\rm Dh}\tau_{\rm A}$ is positive in DT plasma due to the presence of the alpha particles while it is zero in DD plasma. That is, the right hand side of the condition (3) is larger in DT plasma, so that the sawtooth periods in DT plasma become longer than those in DD plasma by the duration for $-\delta\hat{W}_{\rm core}$ to become larger by an amount of $c_{\rm h}\omega_{\rm Dh}\tau_{\rm A}$. Therefore, the difference in region (II) is caused by the presence of $\omega_{\rm Dh}$.

3.3 Variations of the sawtooth periods in DT plasma

In Fig. 1, the sawtooth period has a peak as a function of the average central ion temperature or the RF heating power in DT plasma. Here its cause is studied. Figure 3 shows the variations of the sawtooth periods determined by each condition against the average central ion temperature in DT plasma. In this figure, three curves are obtained by assuming that the sawtooth crashes are triggered by only one of the three conditions, neglecting the other two conditions. The actual sawtooth periods are determined by the lower envelope of these three curves. The sawtooth periods are given by the condition (3) in the relatively high temperature region and they are given by the condition (5) in the relatively low temperature region.

To see the cause of negative temperature dependence of sawtooth periods by condition (3) in the high temperature region of Fig. 3, the time evolutions of the components of the condition (3) and $\beta_{\rm p1}$ at the point (iii) and the point (iv) are shown in Fig. 4. At (iv), the increase rate of $-\delta\hat{W}_{\rm core}$ is larger. This is because the electron and ion pressures increase more rapidly due to the higher RF heating power at (iv). Then, the first term of $-\delta\hat{W}_{\rm MHD}$ increases faster at (iv) than at (iii). The values of the second term of $-\delta\hat{W}_{\rm MHD}$ and $-\delta\hat{W}_{\rm KO}$ are also changed, but the effect of the first term of $-\delta\hat{W}_{\rm MHD}$ is larger than those of the second term of $-\delta\hat{W}_{\rm MHD}$ and $-\delta\hat{W}_{\rm KO}$ in this region. Therefore, $-\delta\hat{W}_{\rm core}$ at (iv) increase faster than that at (iii). The negative temperature dependence of sawtooth periods by condition (3) in the high temperature region of Fig. 3 is caused by change in the heating power.

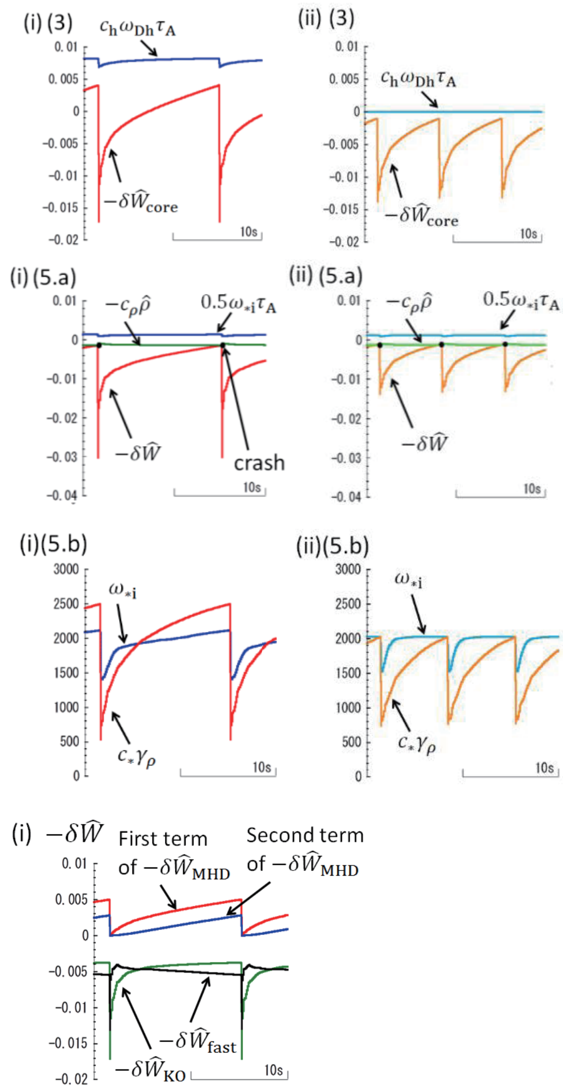

Fig. 2 The time evolutions of the components of the each condition and $-\delta\hat{W}$ and at the point (i) and the point (ii) in Fig. 1.

The cause of positive temperature dependence of sawtooth periods by condition (5) in Fig. 3 is also investigated. Figure 5 shows the time evolutions of the components of

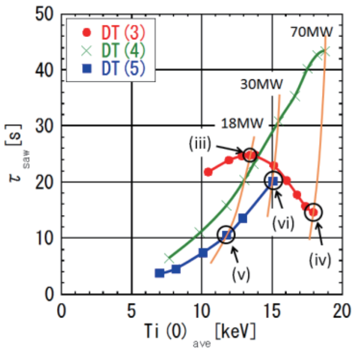

Fig. 3 Dependence of the sawtooth period determined by only one of the three conditions on the average central ion temperature in DT plasma, neglecting the other two conditions.

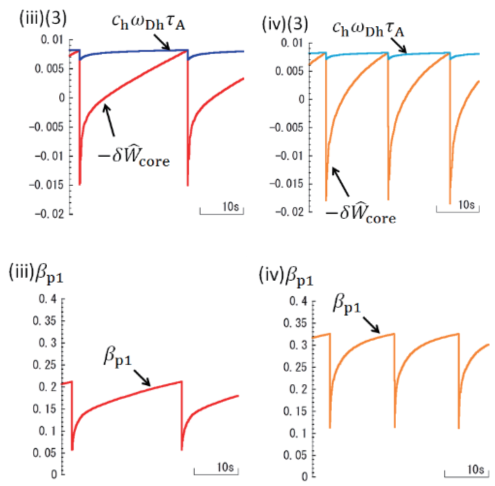

Fig. 4 The time evolutions of the condition (3) and β_{p1} at the point (iii) and the point (iv) in Fig. 3.

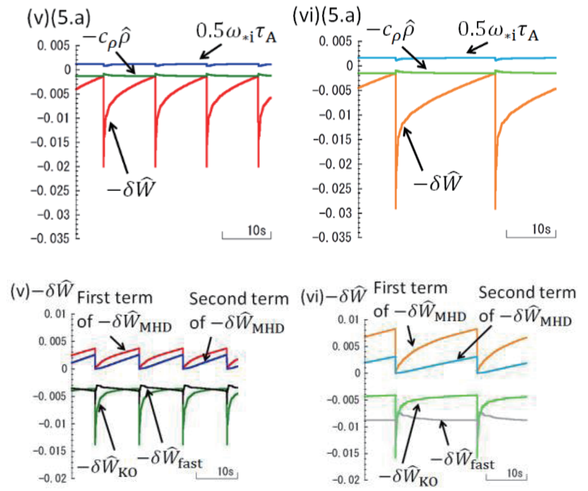

Fig. 5 The time evolutions of the components of the condition (5.a) and $-\delta\hat{W}$ at the point (v) and the point (vi) in Fig. 3.

the condition (5.a) and $-\delta\hat{W}$ at the point (v) and the point (vi) in Fig. 3. The gradient of $-\delta\hat{W}$ at (vi) is a little bit larger than that at (v), but the level of $-\delta\hat{W}$ after a sawtooth crash at (vi) is significantly lower than that at (v) as shown in the top side panels of Fig. 5. The difference in $-\delta\hat{W}$ after a sawtooth crash is caused by the difference in $-\delta\hat{W}_{\mathrm{fast}}$ as shown in the bottom side panels of Fig. 5. The $-\delta\hat{W}_{\mathrm{fast}}$ is lower at (vi) because of the larger alpha particle population due to larger RF heating power. The positive temperature dependence of sawtooth periods by condition (5) in Fig. 3 is caused by change in the population of alpha particles.

According to the above study, it is found that a peak of the sawtooth period appears because sawteeth are triggered by different mechanisms in the low RF heating power region and in the high heating power region.

4. Summary

For prediction of sawtooth in burning plasma, the sawtooth model in the 1.5-dimensional transport code TOTAL has been extended to include the effects of fast particles and the magnetic shear.

By using the newly implemented sawtooth model, we examine the difference of the sawtooth periods between DT and DD plasmas and the variation of the sawtooth periods in DT plasma when the RF heating power is changed in ITER.

The sawtooth periods in DT plasma are longer than those in DD plasma due to the presence of the alpha particles in DT plasma. In DT plasma, the sawtooth period has a peak as a function of the average central ion temperature when the RF heating power is increased. In the low RF heating power region, the resistive internal kink mode triggers sawteeth and then the sawtooth periods become long due to the enhanced stabilization effect of alpha particles when the RF heating power is increased. On the other hand, in the high RF heating power region, the internal kink mode triggers sawteeth and then the sawtooth periods become short due to the rapid increase of the plasma pressure when the RF heating power is increased.

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