2-D Particle-In-Cell Simulations of the Coalescence of Sixteen Current Filaments in Plasmas*)

Kazuki IWATA, Takayuki HARUKI[1)] and Masahiro SATO[1)]

Department of Intellectual Information Engineering, Faculty of Engineering, University of Toyama, Toyama 930-8555, Japan

[1)]*Graduate School of Science and Engineering, University of Toyama, Toyama 930-8555, Japan*

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A dense plasma focus device can produce dense and high energy plasma in a short time. Recently, it has been proposed that the device could be applied to fusion for clean energy production (focus fusion). In order to understand the behavior of the plasma current filaments in the device, two-dimensional, relativistic, fully electromagnetic, particle-in-cell simulations were performed. Sixteen plasma current filaments were initially located on the edge of a circle in our model. They begin to interact with each other while pinching, and then coalesce in the vicinity of the center of system. In the pinch phase (during the coalescence), there appears dense plasma, whose maximum number density is 10 times larger than the initial value. The ions are accelerated, but the rate of the number of them is somewhat small. After that, the current becomes unstable and jumps out from the center. These results are useful for understanding the coalescence process of current filaments.

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1. Introduction

Focus fusion is one of the ways to ignite a fusion with a Dense Plasma Focus (DPF) device, which can produce very high dense plasmas. The ultimate goal is to achieve the ignition of p-B fusion: $\mathrm{p}+{}^{11}\mathrm{B} \rightarrow 3\alpha + 8.68\,\mathrm{MeV}$. There are many advantages of p-B fusion. Here, we will list three of them. First of all, hydrogen and boron exist as fuel in abundance on Earth. Secondly, no neutrons are produced in this pathway, so we can be free from any radioactive waste problems. The third one is, it is possible to directly convert α particles into electric energy. However, as is well known, a high plasma temperature is required to reach ignition (ion temperature $T_{\mathrm{i}} \geq 100\,\mathrm{keV}$).

A DPF device is a simple device improved from Z-pinch device and used frequently as source of x-ray, high energy electron, ion and neutron beams. This device was firstly developed in the 1960s by N. V. Filippov (USSR) and by J. W. Mather (USA). Two coaxially located electrodes are set in a vacuum chamber with a capacitor bank. One electrode is set as an inner one at the center. The others (outer electrodes) form a circle surrounding the central inner electrode. By discharge, gas filled out in a chamber is ionized (plasma) and a current sheath is produced. The sheath runs down along the inner electrode while ionizing and sweeping up neutral gas (*run-down* phase). The sheath reaches the end of the inner electrode then changing into filaments, which coalesce (or merge) into one filament (*run-in* phase). Finally, the plasma is so strongly compressed that very high dense regions appear (*pinch* phase).

Recently significant experimental results on p-B fusions have been reported. E. Lerner *et al.* have showed D-D focus fusion reactions from deuterium ions with energies of more than 150 keV in a DPF device (its experiment is called *FF-1*) [1]. Plasmoids (high dense plasma regions) with number densities $\sim 3\times10^{25}\,\mathrm{m}^{-3}$ were observed. Plasmas were confirmed in spatial scales 300 - 500 μm (the radius of plasmoids) for durations 7 - 30 ns. C. Labaune *et al.* also have showed p-B laser fusion reactions in a two-laser beam system, where laser-accelerated proton beams hit a laser-produced boron plasma [2]. The produced plasma number density is $10^{26}\,\mathrm{m}^{-3}$, whose value is higher than previous result [1].

On the other hand, many numerical studies have been done with a Particle-In-Cell (PIC) simulation. Nielsen *et al.* firstly investigated that a single plasma current is pinched and becomes a helical structure (kink instability) with three-dimensional (3-D) PIC simulations [3]. Similarly, Haruki *et al.* showed that a current is unstable against both sausage and kink instabilities and high-energy particles are observed [4]. After that, Mizuguchi *et al.* investigated that fast magnetosonic shock waves produced in a pinch current can accelerate particles [5]. Many studies on the coalescence dynamics of multiple currents in the cross-section of a DPF device have been done. They include the case of two current filaments [6] and four and eight current filaments [7] in a proton-boron-electron plasma. In any case, particle heating was observed in these processes.

author's e-mail: m1471105@ems.u-toyama.ac.jp

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Furthermore Schmidt *et al.* performed implicit PIC simulations from run-down to pinch phases in the longitudinal cross-section (the *r*-*z* plane) [8].

In this paper, we will focus on the phases from run-in to pinch, in particular the coalescence process of sixteen current filament, in a DPF device. Plasmoids associated with sausage and kink instability were observed [1], but the detailed physics is unknown. Here we will show the formation of very dense plasmas and the stability of pinched current filament by using 2-D PIC codes.

2. Simulation Model

The simulation code used here is a 2-D, relativistic, fully electromagnetic PIC code, modified from the TRISTAN code [9]. A simulation domain is spatially 2-D, but electric and magnetic fields and particle velocity are 3-D vectors. In this code, the electron and ion dynamics are described as not fluids but particles. Both equations of motion are given by

$$\frac{\mathrm{d}\boldsymbol{v}_{s,i}}{\mathrm{d}t} = \frac{q_s}{m_s}\left(\boldsymbol{E} + \boldsymbol{v}_{s,i} \times \boldsymbol{B}\right),$$

$$\frac{\mathrm{d}\boldsymbol{r}_{s,i}}{\mathrm{d}t} = \boldsymbol{v}_{s,i},$$

where $\boldsymbol{v}$ and $\boldsymbol{r}$ are particle velocity and position, and $\boldsymbol{E}$ and $\boldsymbol{B}$ are electric and magnetic fields, q, m and t are a charge, mass and time, respectively. The subscripts s and i denote the plasma species (s = i for ions, e for electrons) and particle index, respectively. These equations are solved for each particle along with Maxwell's equations:

$$\frac{\partial \boldsymbol{E}}{\partial t} = c^2 \nabla \times \boldsymbol{B} - \frac{1}{\epsilon_0}\sum_s \boldsymbol{j}_s,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}.$$

Here c is speed of light and $\boldsymbol{j}_s = \sum_i q_s \boldsymbol{v}_{s,i}$ is the particle current density. $\nabla \cdot \boldsymbol{B} = 0$ and $\nabla \cdot \boldsymbol{E} = \rho_e/\epsilon_0$ are automatically satisfied at all times due to the nature of the numerical scheme (ρ_e is electric charge density). Initially, the plasma is electrically neutral.

Our simulation model is shown in Fig. 1. We assumed that the plane is the cross-section perpendicular to an axial direction in a DPF device. Sixteen electron-ion (hydrogen) plasma columns (gray circles in Fig. 1) are forming a circle in a vacuum. Note that our model is in 2-D plane so the plasma columns are represented as circles.

The lengths in two dimensions was $L_x = L_y = 1000\Delta$. $\Delta = 1$ is the numerical grid size corresponding to an electron Debye length, $\lambda_{\mathrm{De}} = v_{\mathrm{te}}/\omega_{\mathrm{pe}}$ (v_{te} and ω_{pe} are an electron thermal velocity and an electron plasma frequency, respectively). The center of system was set at $(x_c, y_c) = (500\Delta, 500\Delta)$. Plasmas with a radius $r = 50\Delta$ are located on the edge of a circle with a radius $R = 300\Delta$. The location of each plasma is obtained from $(x_c + R\cos(k\theta),\ y_c + R\sin(k\theta))$. Here k is a filament index:

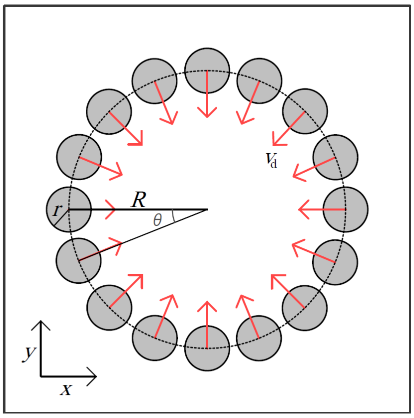

Fig. 1 Our simulation model. The gray circles represent cylindrical plasmas, which are forming a circle. r and R are radii of each plasma and the circle formed by the cylindrical plasmas. θ is a angle between between neighboring plasmas. v_d is drift velocity inward to the center of system (red arrows).

$k = 0, 1, \cdots, N_L - 1$ ($N_L = 16$ is the number of circles), and $\theta = \pi/8$ is an angle between neighboring current filaments.

The time step was taken to be $\omega_{\mathrm{pe}}\Delta t = 0.05$ ($\Delta t = 1$ is a time interval). The durations of our simulations were $4000\Delta t$, corresponding to $\omega_{\mathrm{pi}}t = 20.0$ (ω_{pi} (= 0.5) is an ion plasma frequency). Ion-to-electron mass ratio was chosen by $m_i/m_e = 100$ to reduce computer resources. The particle number densities were $n_e = n_i = n_0 = 100$ per cell. Therefore, the total particle number was $\approx 1.2 \times 10^7$ electron-ion pairs. The electron and ion skin depths were $d_e = c/\omega_{\mathrm{pe}} = 10\Delta$ and $d_i = c/\omega_{\mathrm{pi}} = 100\Delta$, respectively. Initial Plasma temperatures were set as the same $T_i = T_e$. The electron thermal velocity was also $v_{\mathrm{te}}/c = 0.1$.

First, in order to produce initial current filaments J_z, an external electric field E_z was applied onto all particles in the whole system. The profile of external field was defined by $E_z = 0.5E_0\,[\tanh((t-50)/20) - \tanh((t-1050)/20)]$, where $E_0 = v_{\mathrm{te}}\omega_{\mathrm{pe}}m_e/|q_e|$. Next, to be close our model to real DPF device, we imposed the initial drift velocity $v_d/c = 0.2$ onto all particles assumed to be accelerated from run-down to pinch phases (See red arrows in Fig. 1). Absorbing boundaries were applied to both the electromagnetic fields and the particles [10]. These fields and particle velocities start to be dumped from outside of the radius 420Δ at the center. This is in order to suppress the reflection of outgoing waves emitted from the initially produced current.

In this paper, normalization is mainly based on ion quantities to discuss the behaviour of ions. Time, spatial length, number density are normalized by the inverse of ion plasma frequency $\omega_{\mathrm{pi}}^{-1}$, ion skin depth d_i, initial particle number density n_0, respectively. Velocity is also normalized by c.

3. Simulation Results

We will show our simulation results from *run-in* to

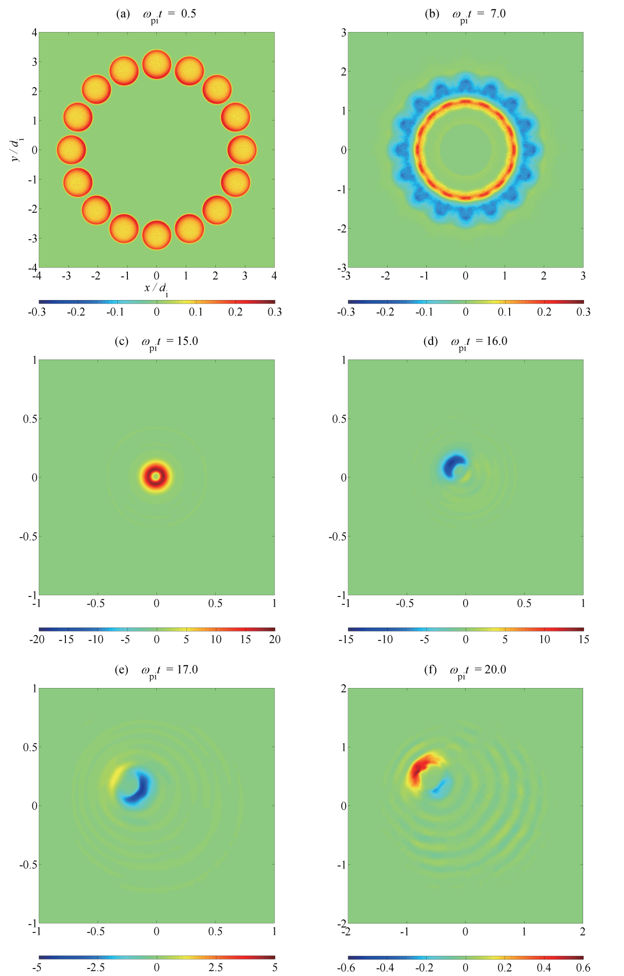

Fig. 2 The time development of spatial distributions of current densities j_z at (a) $\omega_{pi}t = 0.5$, (b) 7.0, (c) 15.0, (d) 16.0, (e) 17.0 and (f) 20.0.

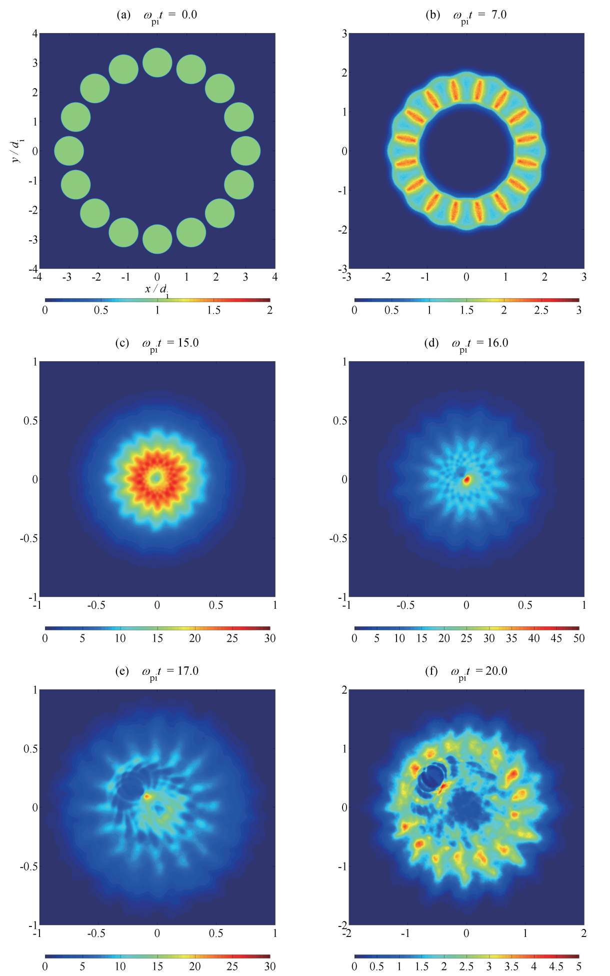

Fig. 3 The time development of spatial distributions of ion number densities n_i at (a) $\omega_{pi}t = 0.0$, (b) 7.0, (c) 15.0, (d) 16.0, (e) 17.0 and (f) 20.0.

pinch phases in a DPF device. Initially sixteen plasmas driven by an external electric field become the currents. Some outgoing waves are emitted from them, but dumped in absorbing boundary regions. We confirmed that these waves cannot be reflected from boundaries in the initial stage. In addition, due to inward drift velocity, current filaments move into the center and begin to coalesce each others there.

Figure 2 shows the time development of spatial distributions of current densities j_z. Here j_z is normalized by $n_0|q_e|c$. We observed appearance of sixteen current filaments by an external electric field as seen in Fig. 2 (a). In Fig. 2 (b), current filaments interact with each other while pinching and there a hollow structure appears. At the same time, negative currents (blue) are formed surrounded by the hollow-structured positive current (red). As seen in Fig. 2 (c), we could observe the focused current at the vicinity of the center of system. However, the current did not pinch completely. After that, the current becomes unstable and jumps out from the center in Figs. 2 (d-f). We also could observe that the current direction oscillates repeatedly (See Figs. 2 (d-f)) and some waves concentrically propagate.

In this case, it was found that the current did jump from the center to upper left. To confirm the direction of current break-up, we varied the seed of pseudo random number (initial parameter controlling the beginning of random numbers) used in this code [11]. Changing the seed corresponds to change particle thermal noise. As a results, the current jumped to not only the upper left but also various directions. Subsequently no stable currents remain to keep their structure in our model. We found that high magnetic pressure on the lower-right region pushes the current out to upper left.

Figure 3 shows the time development of spatial distributions of ion number densities n_i. All panels correspond to that of Fig. 2, but only Fig. 3 (a) is the initial stage. As seen in Fig. 3 (a), initial sixteen plasma current filaments are forming a circle in a vacuum. These ions (and electrons) become currents by an external electric field and move inward due to the initial drift. In Fig. 3 (b), the interaction of current filaments makes dense plasma region between them. These ions begin to coalesce near the center (See Fig. 3 (c)). Figure 3 (d) shows the maximum plasma pinch, whose number density is 45 times the initial one. As shown in Figs. 2 (d-f), the current break-up means that

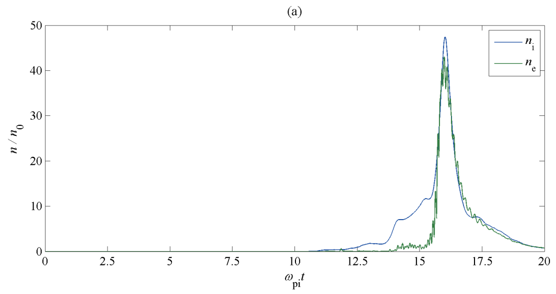

Fig. 4 The time development of ion (blue) and electron (green) number densities at the center (x_c, y_c).

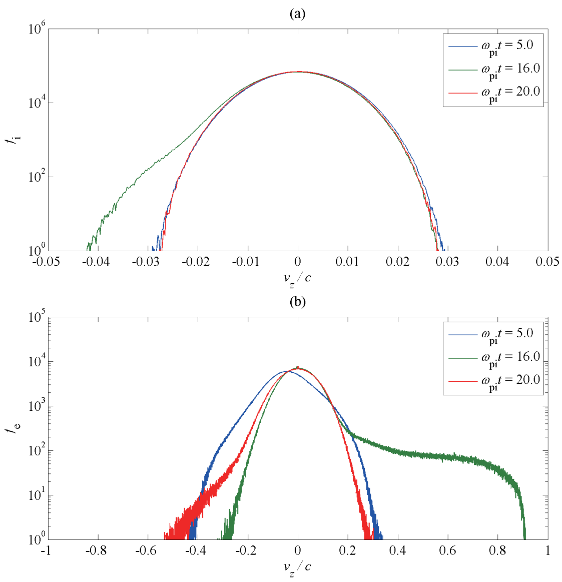

Fig. 5 The particle velocity distributions for (a) ions and (b) electrons in the z direction at $\omega_{pi}t = 5.0$ (blue), 16.0 (green) and 20.0 (red).

ions cannot be confirmed there. Figs. 3 (e-f) shows the ions tend to radially diffuse in the later phase.

Figure 4 shows the time development of ion and electron number densities at the center. In the early stage, both number densities are zero because of the vacuum state. After $\omega_{pi}t = 10.0$, ion density (blue) begins to increase and reaches the maximum value $n_i/n_0 = 45$ around $\omega_{pi}t = 16.0$. Similarly, electrons (green) are pinched at the same time. We estimated that the electron number density is $n_e/n_0 = 40$ at the maximum. And then both plasmas tend to diffuse until the end of simulations.

Figure 5 shows both ion and electron particle velocity distributions in the z direction at different times. Note that an external electric field was imposed from $\omega_{pi}t = 0.25$ to 5.25. The blue lines show the phase when the external electric field is switched off and then plasma currents are produced. At $\omega_{pi}t = 16.0$, the ions could be accelerated in the negative z direction with the maximum $v_{iz}/c = -0.04$. While electrons could be accelerated at the maximum $v_{ez}/c = +0.9$. Therefore ion particle acceleration was observed, but the rate of the number of them was somewhat small. These results also represent the opposite direction of the initially produced currents. It is suspected that the direction of current is flipping (the current is oscillating) due to the external electric field.

4. Conclusions

In order to understand the physics from run-in to pinch phases, we investigated the coalescence process of sixteen current filaments by the use of 2-D PIC code. In our model, to simply produce currents, an external electric field E_z was used in a similar way as in Ref. [3]. Plasmas were forcibly given by the drift velocity inward to the center for pinch.

Firstly, sixteen current filaments can coalescence in the vicinity of the center of system. Exactly saying, the merged current shows a hollow structure, while ions can be pinched. Secondly, in the pinch phase, the maximum ion number density is 45 times higher than the initial value. This result agree well with the order of $n_i/n_0 \sim 10$ suggested in the experimental study. Thirdly, both electrons and ions can be accelerated in the pinch phase (at the coalescence of current filaments). However, the rate of the number of accelerated ions is somewhat small. After that, the current becomes unstable and jumps out from the center. Hence these results could be useful for understanding the physics of dense plasma focus.

On the other hand, it has already been reported that the ions could be accelerated to $v_{iz} \sim 0.09c$ at the maximum in the pinch phase of single current filament [4]. Compared with the previous study, the maximum velocity $v_{iz} \sim 0.04c$ in the coalescence phase of sixteen current filaments could be somewhat small in this simulation. Hence, in addition to clear the detail mechanism of particle acceleration in our simulation, the comparison between single pinch and the coalescence process of currents will be needed.

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