Multi-Input Multi-Output (MIMO) Control System with a State Equation for Fusion Reactors

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In future fusion reactors, plasma control can be anticipated to be quite complicated because actuators and diagnostics would be limited because of extreme environmental conditions, such as high neutron fluxes. In addition, control parameters and actuators are not in simple one-to-one correspondences (e.g., NBI power affects not only plasma current but also fusion power). This results in the need of using multi-input multi-output control systems. To confront this problem, we have developed a control system design that involves an state equation. In this research, simulations were performed in which three plasma parameters (fusion power, plasma current, and plasma density) were controlled using three actuators (NBI power, amount of gas puff, and inductively driven current). Parameters for these actuators were determined from the state equation, and the plasma parameters were simultaneously controlled with sufficiently high accuracy.

1. Introduction

In designing DEMO or commercial fusion reactors, the following aspects should be taken into consideration. First, simultaneous control will be required for many parameters related to the core plasma and in-vessel components. For example, to supply steady state electric power, fusion power must be controlled at the rated value, and to avoid various instabilities, control of the core plasma might be indispensable. In addition, the divertor plasma should be given attention to maintain soundness of the divertor plate. Second, relationships between control parameters and actuators are quite complicated, i.e., one actuator generally affects several plasma parameters. For example, NBI power affects not only plasma current but also fusion power. Finally, the actuators and diagnostics that can be installed in a reactor will be limited because of critical environments, such as high heat flux or high neutron flux or both. These problems should be taken into consideration [1–3] in constructing control systems for DEMO and commercial reactors. To address these problems, we should consider what parameters should be controlled, what actuators and diagnostics can be installed, and what control logic should be applied. These issues are interlinked, as shown in Fig. 1.

Categorizations of control parameters, actuators, and diagnostics have been discussed elsewhere [1, 4, 5]. In this study, we consider the control logic for the future reactor. Because various plasma parameters are affected by several actuators and because relationships between control parameters and actuators are complicated, a multi-input
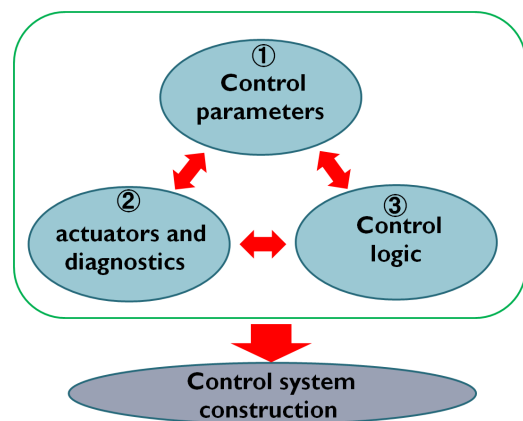

Fig. 1 Interacting factors that complicate the design and operation of control systems for fusion reactors.

multi-output (MIMO) control system should be used in future reactors. In JT-60 plasma experiments, two parameters are simultaneously controlled by two actuators: the minimum value of the safety factor q_{min} controlled by LHCD and the ion temperature gradient (ITG) is controlled by perpendicular NB injection. In this case, the two parameters, q_{min} and ITG, can be controlled independently because they are only weakly coupled [6]. In contrast, if we want to control fusion power and the qmin value with NBI and gas puffing, then a complicated control logic must be introduced because the two actuators simultaneously affect both fusion power and q_{min}. We have endeavored to perform a multiple control simulation by using a 1.5D transport code [7]. In the simulation, we succeeded in control-

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ling two parameters using PID theory in which the PID gains were determined from the output characterization. As is well known, PID theory is an efficient, useful, and familiar method for single-input single-output (SISO) systems. However, for MIMO systems, it is difficult to determine PID gains. For example, in SISO systems, three gain parameters are sufficient for the P, I, and D terms, while in a 2×2 MIMO system [6, 7], twelve ($= 4 \times 3$), PID gains must be determined because four gain parameters in the 2×2 control matrix must be known for each P, I, and D term. It is hard to determine twelve PID gains from only an output characterization.

In this study, we introduce a control logic based on a state equation. In modern control theory, application of a state equation to a MIMO control system is a familiar approach [8]. In Sec. 2, we briefly explain feedback control using a state equation. In Sec. 3, we provide an example of how to determine PID gains with the state equation. In Sec. 4, simulation results are presented, and Sec. 5 contains a discussion and summary.

2. Application of a State Equation

The general form of a state equation is given by

$$\dot{x} = F(x, u), \tag{1}$$

$$\dot{y} = G(x, u), \tag{2}$$

where x is a state vector, u is an actuator vector, and y is an output vector. A state equation represents the physical model of the real system that is to be controlled. Here, we try to control the real system with the state equation, and any difference between the real system and the state equation will be dealt as a disturbance, as shown in Fig. 2. In the figure, r is a reference value, y is the output, e is the error between r and y, and d is a disturbance. Figure 2 shows that the model error between the state equation and the real system can be interpreted as a disturbance. To use the state equation, the controller can also be designed to minimize the effects of model error.

Parameters for the actuator u would be determined from (1) and (2). In general, the actuator u is nonlinearly coupled to the state vector x and the output y. Since feedback control might be expected around an equilibrium state
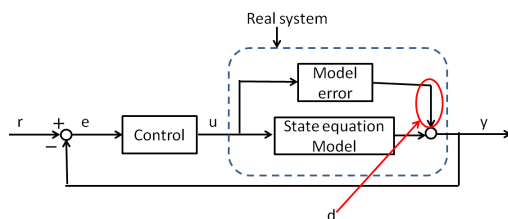

Fig. 2 Feedback loop with model error. Model error between the real system and the state equation model can be dealt as a disturbance.

with a small perturbation, we linearize the state equation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta x = A\Delta x + B\Delta u, \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta y = C\Delta x + D\Delta u, \tag{4}$$

where A, B, C, D are coefficient matrices given by

$$A = \frac{\partial F}{\partial x}, \; B = \frac{\partial F}{\partial u}, \; C = \frac{\partial G}{\partial x}, \; D = \frac{\partial G}{\partial u}. \tag{5}$$

Using this linearized state equation, parameters for the actuator vector can be easily solved. Next, we consider a feedback model for the output parameters by introducing the following simple equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta y = -K\Delta y, \tag{6}$$

where $\Delta y = y - y_{\text{ref}}$ is defined as the deviation from the reference value. Here, only proportional control is taken into consideration, and K is, in general, a diagonal matrix related to the characteristic time of each component in the state equation. From (3)-(6), one can acquire a suitable actuator vector u.

3. MIMO Control of Core Plasma Parameters

In fusion reactors, several parameters should be simultaneously controlled with a limited number of actuators. Here, by using a point model for the core plasma, we derive actuator parameters for a MIMO system. Let us introduce three control parameters for the core plasma: fusion power P_{fus}, averaged plasma electron density $\langle n_e \rangle$ and total plasma current I_p. We also introduce three actuators: NBI, gas puffing, and induction current. As shown in the following equations, these control parameters are strongly coupled to the actuators. The state equation for the core plasma is described as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} I_p \\ N \\ W \end{pmatrix}$$
$$= \begin{pmatrix} -\dfrac{I_p}{\tau_j} + \dfrac{1}{\tau_j}\left(C_{\text{bs}}\epsilon^{0.5}\beta_p I_p + \dfrac{\gamma}{n_{20}R}P_{\text{NBI}} \right) + \dot{I}_{\text{ind}} \\ -\dfrac{N}{\tau_p} - \dfrac{n^2}{2}\langle\sigma v\rangle V + N_{\text{puff}} \\ -\dfrac{W}{\tau_e} + \dfrac{E_\alpha}{4}n^2\langle\sigma v\rangle V - C_B n_{20}^2 T_{10}^{1/2} V + P_{\text{NBI}} \end{pmatrix}$$
$$= F(x, u), \tag{7}$$

and

$$y = \begin{pmatrix} I_p \\ P_{\text{fus}} \\ \langle n_e \rangle \end{pmatrix} = \begin{pmatrix} I_p \\ \dfrac{N^2}{4V}\langle\sigma v\rangle \\ \dfrac{N}{V} \end{pmatrix} = G(x), \tag{8}$$

$$x = \begin{pmatrix} I_{\mathrm{p}} \\ N \\ W \end{pmatrix}, \tag{9}$$

$$u = \begin{pmatrix} \dot{i}_{\mathrm{ind}} \\ P_{\mathrm{NBI}} \\ N_{\mathrm{puff}} \end{pmatrix}, \tag{10}$$

where N is the total number of particles and W is the total stored energy. Values for the parameters are as follows [9]:

$$\gamma = 0.25, \quad C_{\mathrm{bs}} = 0.782, \quad C_{\mathrm{b}} = 0.032, \tag{11}$$

$$\tau_{\mathrm{p}} = 1\,\mathrm{s}, \quad \tau_{\mathrm{j}} = 100\,\mathrm{s}, \tag{12}$$

$$\beta_{\mathrm{p}} = 0.7, \quad B_{\mathrm{T}} = 5.3\,\mathrm{T}, \tag{13}$$

$$R = 6.2\,\mathrm{m}, \quad a = 2.0\,\mathrm{m}, \quad \kappa = 1.7, \tag{14}$$

$$V = 830\,\mathrm{m}^3, \quad A_{\mathrm{i}} = 2.5, \tag{15}$$

$$\tau_{\mathrm{e}} = HH \\ \times 0.0562 A_{\mathrm{i}}^{0.19} R_{\mathrm{p}}^{1.39} a^{0.58} \kappa^{0.78} B_{\mathrm{T}}^{0.15} I_{\mathrm{p}}^{0.93} n_{19}^{0.41} P_{\mathrm{tot}}^{-0.69}. \tag{16}$$

Since this state equation is nonlinear, linearization around the equilibrium point is performed. In this case, the target value is chosen as the equilibrium value of vector x, and from Eq. (7), the equilibrium value of vector u can be determined. The linearized state equation can be summarized as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta x = A\Delta x + B\Delta u, \tag{17}$$

$$\Delta y = C\Delta x. \tag{18}$$

The symbol Δ indicates the deviation from the equilibrium point for each parameter. We set the reference point to the equilibrium point $y_{\mathrm{ref}} = y_{\mathrm{eq}}$ and require the controller to satisfy the equation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta y = -K(y - y_{\mathrm{ref}}) = -K\Delta y, \tag{19}$$

$$K = \begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{20}$$

The unit of K is an inverce second (s^{-1}), and diagonal elements are determined from the inverse of the characteristic time for each state variable. In this situation, each component of the vector Δy is expected to be dampened by its characteristic time. Thus, to get a suitable controller, we change the equations as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}y = \frac{\mathrm{d}}{\mathrm{d}t}\Delta y = C\frac{\mathrm{d}}{\mathrm{d}t}\Delta x = C\frac{\mathrm{d}}{\mathrm{d}t}x, \tag{21}$$

thus,

$$\frac{\mathrm{d}}{\mathrm{d}t}y = CA\Delta x + CB\Delta u, \tag{22}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}y = CAC^{-1}\Delta y + CB\Delta u. \tag{23}$$

From Eq. (19), we can obtain the actuator value as follows:

$$\Delta u = -(CB)^{-1}\left(K + CAC^{-1}\right)(y - y_{\mathrm{ref}}). \tag{24}$$

This is just a proportional controller. A proportional (P) controller, without the integral (I) and differential (D) controllers, cannot inhibit a disturbance or the effects of model errors. Thus, the I and D controllers should be added. On doing so, the controller is given as follows:

$$\Delta u = (CB)^{-1}\left(K + CAC^{-1}\right)(y - y_{\mathrm{ref}}) \\ - (CB)^{-1} K_2 \int_0^t (y - y_{\mathrm{ref}})\mathrm{d}\tau \\ - (CB)^{-1} K_3 \frac{\mathrm{d}}{\mathrm{d}t}(y - y_{\mathrm{ref}}), \tag{25}$$

where

$$K_2 = \begin{pmatrix} 0.001 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \tag{26}$$

$$K_3 = \begin{pmatrix} 1.5 \times 10^{-4} & 0 & 0 \\ 0 & 0.015 & 0 \\ 0 & 0 & 0.015 \end{pmatrix}. \tag{27}$$

The components in K_2 and K_3 are determined to make the I and D terms comparable with the P term. Finally, a detailed adjustment is performed by trial and error. In general, if we perform PID control for three control parameters with three actuators, we need a 3×3 matrix for each P, I and D gain, i.e., $(3\times3)\times3 = 27$ components must be determined. However, by introducing the state equation model, PID gains involving only diagonal matrices could be available, requiring the determination of only $3 \times 3 = 9$ components. In addition, the characteristic time of each state equation variable is practical for determining PID gains.

4. MIMO Simulation for Core Plasma

For this MIMO system with a point model for core plasma, a feedback simulation was performed with the software Matlab/Simulink, which is an excellent tool for control simulation and controller design. Here (7) is solved by Matlab/simulink and actuators are evaluated from (25). Typical results for the time evolutions of the control parameters and actuator values are shown in Figs. 3 and 4, respectively, where the reference values were set as follows:

$$y = \begin{pmatrix} I_{\mathrm{p}} \\ P_{\mathrm{fus}} \\ \langle n_{\mathrm{e}} \rangle \end{pmatrix} = \begin{pmatrix} 15\,\mathrm{MA} \\ 400\,\mathrm{MW} \\ 1.0 \times 10^{20}/\mathrm{m}^3 \end{pmatrix}. \tag{28}$$

The initial control parameters are as follows:

$$x = \begin{pmatrix} I_{\mathrm{p}} \\ N \\ W \end{pmatrix} = \begin{pmatrix} 15\,\mathrm{MA} \\ 8.3 \times 10^{23} \\ 300\,\mathrm{MW} \end{pmatrix}. \tag{29}$$
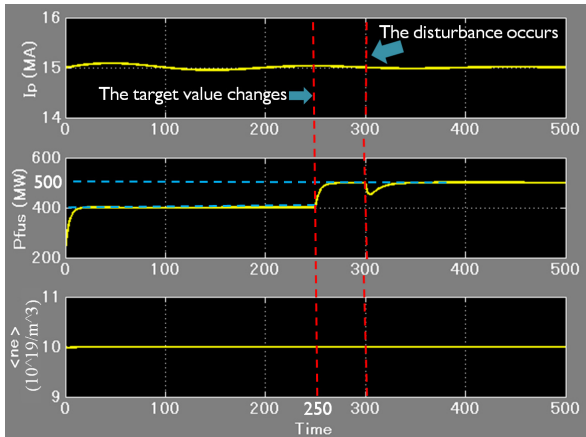
Fig. 3 Simulation results for time evolution of plasma current I_p, fusion power P_{fus} and plasma electron density $\langle n_e \rangle$. I_p and $\langle n_e \rangle$ are maintained at their target values, while beginning at 250 s, P_{fus} follows its target value from 400 to 500 MW and recovers from the disturbance at 300 s.

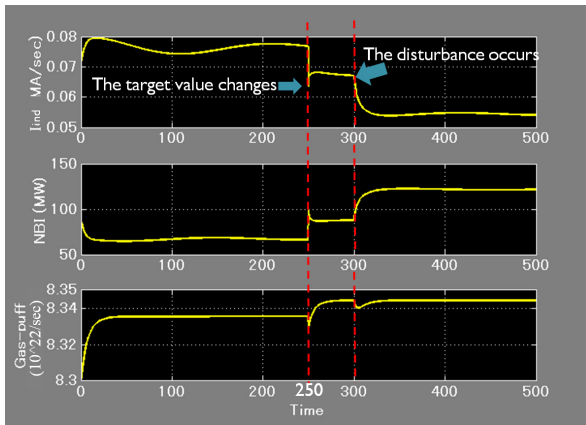Fig. 4 Simulation results for the time evolution of the induced current $\dot{I}_{cs}$, NBI power and amount of gas puff. At 250 s and 300 s, NBI power changes to drive the fusion power to its target. Simultaneously, the other two actuators change to keep I_p and $\langle n_e \rangle$ constant.

In Fig. 3, during the period $t = 0$ - 250 s, control parameters were maintained at target values with no offset. The controller was designed from the linearized form of the state equation given in (6).

At $t = 250$ s, the target value for fusion power was changed from 400 to 500 MW, while keeping values of the other two control parameters constant. In this case, the fusion power smoothly changed to the target value within 30 s with no offset. Simultaneously, the plasma current and plasma density remained constant. At this time, Figure 4 shows that the NBI power increased from $P_{NBI} = 66$ to 87 MW, so as to change the fusion power. Simultaneously, the induced current and the gas puff amount changed to keep the plasma current and plasma density at their reference values.

Next, at $t = 300$ s, a deterioration in plasma confinement was simulated by changing the confinement enhancement factor HH from 1 to 0.95, thereby simulating a disturbance in plasma performance. Although the fusion power decreased slightly, it recovered within 40 s, and the deviation in fusion power was approximately 10%. Simultaneously, NBI was increased to recover fusion power, while the induced current and gas puff amount were decreased to keep plasma current and plasma density constant.

5. Discussion and Summary

In this research, we performed a MIMO simulation for a fusion reactor in which three control parameters (i.e., fusion power, plasma current, and plasma density) were selected, and three actuators (i.e., NBI, gas puff, and induced current) were employed. By introducing an state equation on a point model for a core plasma, actuator values were smoothly determined. In addition, since the number of PID gains was significantly reduced, values for PID gains could be determined by a simple procedure.

In this simulation, the controller was designed within the framework of a physical model for a tokamak reactor, but the method is also suitable for helical or other types of fusion reactors. A similar 0D helical control simulation was reported in [10]. While in [11,12], a plasma parameter profile control experiment was reported in which parameter profile information was factored into well-known functions, and the controller was designed using a state equation. However, in those studies, many diagnostics were used, and it might prove difficult to use the same methods in a future reactor. For this reason, in the future reactor, a plasma simulator will be necessary for profile control in which a plasma simulator would be expected to serve as an alternative to diagnostics.

In this research, the effects of model error (i.e. the error between the model and the real system) were ignored. However, in a real reactor, the effects of model error will be critical. A control system that decreases the effects of model error will be required in the real reactor. For this problem, so-called robust control should be introduced into fusion reactor control. In Ref. [13], an H2 control simulation was performed with a 0D state equation and Matlab/Simulink; H2 control theory is one such robust control theory. In H2 control theory, the effects of model error are evaluated as the H2 norm, and the controller is designed to minimize the H2 norm. More recently, a more advanced H-infinity control theory has been developed. In the H-infinity theory, the effects of model error are evaluated as the H-infinity norm. To operate the future reactor, a robust control theory will be necessary. Therefore, an H-infinity control simulation with the 0-D time evaluate equation will be part of our future work.

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