## Nonlocal Ponderomotive Force in a Super Gaussian Laser Beam and the Conditions for Long Time Scale Interaction

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We have applied the theory of the nonlocal ponderomotive force which we derived recently using the noncanonical Lie perturbation approach to investigate a long time scale particle motion in a super Gaussian laser beam. In such a flat-top beam profile, the local field gradient is diminished near the axis, so that the conventional ponderomotive formula is hardly applied. Numerical analyses of the interaction time and its dependence on the initial position and momentum of particles show that the nonlocal effect of the ponderomotive force, which is associated with higher order spatial derivatives, regulates the dynamics predominantly and sensitively.

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In recent years, the delicate control of laser field patterns in plasmas is anticipated. For instance, the super Gaussian beam, in which the ponderomotive force is significantly weakened near the axis owing to the flat-top transverse beam profile, is considered preferable in maintaining long interaction between laser and particles and also in achieving efficient particle acceleration via the laser piston and/or Coulomb explosion mechanism [1, 2]. In such a case, the ponderomotive force estimated from the conventional formula proportional to the local field gradient is diminished [3-5]. Then, a residual higher order force associated with the nonlocal field profile becomes important. As a theory of the ponderomotive force applicable to such a regime, we have recently derived a new formula for the relativistic ponderomotive force in a transversely localized laser field that includes the higher order nonlocal effect [6,7]. The new formula can be utilized to analyze the particle motion in such a flat-top laser beam.

In Refs. [6] and [7], the oscillation center equation of motion describing the higher order nonlocal ponderomotive force is derived on the basis of the noncanonical phase space coordinate  $Z^{\mu} = (\eta; X, Y, Z, P_x, P_y, p_{\eta})$  where  $\eta = \omega t - k_z z$  is the phase,  $\omega$  the angular frequency,  $k_z$ the wave number, **X** and **P** the oscillation center position and mechanical momentum of the oscillation center respectively,  $p_{\eta} = p_z - \gamma mc$  is a constant of motion in the uniform laser field,  $p_z$  the mechanical momentum of the particle and  $\gamma$  the relativistic factor. The equation is derived on the basis of the noncanonical Hamiltonian mechanics incorporated with Lie transformation [8]. Here, the laser field is assumed to be transversely nonuniform and linearly polarized and is given by the normalized vector potential  $a = |q|A/mc^2$  as  $a = a_x(x) \sin \eta \hat{e}_x$  where qand m are the charge and rest mass of the particle, respectively, and c the speed of light. In the oscillation center equation, the nonlocal effects are taken into account up to the third order of the expansion parameter  $\epsilon \sim l/L$  where  $L^{-1}(X) \equiv \partial_x \ln a_x(x)|_{x=X} = a_x^{-1}\partial_X a_x$  is the scale length of the gradient of the laser field amplitude,  $l \equiv a_x(X)/k_z\zeta_0$ the particle excursion length and  $\zeta_0$  a constant by which the initial value of  $p_{\eta}$  is defined as  $p_{\eta 0} \equiv -mc\zeta_0$ . The equation is derived on the basis of the expansion of the laser field amplitude around x = X:

$$a_x(x) = a_x(X) \left[ 1 + \epsilon \frac{\tilde{x}}{L} + \epsilon^2 \frac{\tilde{x}^2}{2!R} + \epsilon^3 \frac{\tilde{x}^3}{3!T} + \cdots \right],$$
(1)

where  $\tilde{x} \equiv X - x$ . Here,  $R^{-1}(X) \equiv a_x^{-1}\partial_X^2 a_x$  and  $T^{-1}(X) \equiv a_x^{-1}\partial_X^3 a_x$  are the curvature of the field amplitude and its derivative, respectively, and  $l^2/R \sim \epsilon^2$  and  $l^3/T \sim \epsilon^3$  are assumed. By using the Lie transformation  $Z^{\mu} \mapsto Z'^{\mu}$  and proper gauge functions, we can remove small oscillations up to  $\mathcal{O}(\epsilon^3)$  from the *X* direction so that the secular equations of motion describing the ponderomotive force up to  $\mathcal{O}(\epsilon^3)$  are obtained as

$$\frac{dX'}{d\eta} = \frac{P'_x}{mc\zeta_0 k_z} \left( 1 + \epsilon^2 \frac{3}{2} \frac{l^2}{L^2} \right),$$
(2)  
$$\frac{dP'_x}{d\eta} = -\frac{mca_x}{2} \left[ \epsilon \frac{l}{L} + \frac{\epsilon^3}{8} \left( \frac{7}{2} \frac{l}{L} \frac{l^2}{R} + \frac{l^3}{T} + \frac{1}{2} \frac{l^3}{L^3} \right) \right],$$
(3)

which are the central results in Ref. [6] (Eqs. (10) and (11)). Equation (3) involves terms represented by second and third spatial derivatives; therefore, the force depends

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Fig. 1 Transverse laser field profiles for j = 2, i.e., the Gaussian case  $\exp(-x^2/w^2)$  (dotted line), and j = 4, i.e., the super Gaussian case  $\exp(-x^4/w^4)$  (solid line).



Fig. 2 Numerical simulation for a particle interacting with the laser beam having the envelope  $a_0 \exp(-x^4/w^4)$  where  $a_0 = 4$  and the wavelength  $\lambda = 1 \,\mu\text{m}$ . The oscillation center trajectories obtained by the ponderomotive formulae up to the first (red line) and third (blue line) orders are shown, together with the particle trajectory calculated by the exact equation of motion (black line) for (a)  $w = 6 \,\mu\text{m}$  and (b) 3  $\mu\text{m}$ .

not only on the local field gradient, but also on the curvature and its variation.

Here, we investigate the higher order nonlocal effect originating from the super Gaussian beam structure on the oscillation center dynamics based on Eqs. (2) and (3). We assume the laser field given by  $a_x(x) = a_0 \exp(-x^j/w^j)$ where  $a_0$  is the amplitude at the beam center and w the scale length of the field envelope. For instance, j = 2 corresponds to a Gaussian beam and j = 4 a super Gaussian having a gentle amplitude variation near the axis, as shown in Fig. 1. In the case of j = 4, the first to third derivatives vanish at the center, resulting in significant weakening of the ponderomotive force. Therefore, here we mainly consider j = 4 to maintain the long interaction.

The numerical solutions obtained using Eqs. (2) and (3) up to the third order  $\epsilon^3$  (case (I), blue line) and the first order  $\epsilon$  (case (II), red line) are shown in Fig. 2 for (a)  $w = 6 \,\mu\text{m}$  and (b)  $w = 3 \,\mu\text{m}$ , together with the solution obtained using the exact equation of motion, i.e.,  $d\mathbf{p}/d\eta = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c) \,\omega/\gamma$  (black line). Here, we assume  $a_0 = 4$ , laser wavelength  $\lambda = 1 \,\mu\text{m}$  and the initial condition  $(X', P'_x) = (0, 0.001 mc)$ . In both Figs. 2 (a) and (b), the trajectories for case (I) (up to  $\mathscr{O}(\epsilon^3)$ ) show an almost exact agreement with those of the direct numerical calculation, whereas the trajectories for case (II) (up to  $\mathscr{O}(\epsilon)$ ) exhibit a significant difference in the ejection time (the time that the oscillation center reaches to the laser beam radius, X' = w). Using the relation  $d\eta/dt = \omega\zeta_0/\gamma$ , the ejection times for case (I) and (II) are obtained as t = 630 and 1750 fs respectively for  $w = 6 \mu m$ , and t = 200and 870 fs respectively for  $w = 3 \mu m$ . We have also confirmed that the Gaussian profile (j = 2) provides a shorter interaction time, e.g., 140 fs for  $w = 6 \mu m$  in case (I), and the relative difference of the ejection time between case (I) and (II) is less than 1%, suggesting that the first order force plays a dominant role. In other words, nonlocal effects increase significantly with increasing values of the polynomial *j*.

The physics leading to such a difference is explained as follows: The conventional first order formula looks only at a narrow region through the local gradient, which is very weak in the present super Gaussian case. On the other hand, the third order formula has a capability to include the global extent of the profile up to around the beam radius  $X' \sim w$ . The third order formula can then capture the rapid change in the field amplitude near the beam radius even when the oscillation center is located near the center, and the formula can represent the nonlocal effect as a residual ponderomotive force, which enhances the ejection. For this reason, the first order formula used in case (II) significantly overestimates the interaction time, as in Figs. 2(a) and (b). The dynamics are found to be described analytically by an exponential function for case (I) whereas by the Jacobi elliptic function sn for case (II) which will be discussed separately.

Based on the above results, we further investigate the transverse initial condition that allows for keeping the interaction without suffering ejection over a given phase advance  $\Delta \eta$ . Here, we impose the condition applied in Fig. 2 (a), i.e.,  $w = 6 \mu m$ , and  $a_0 = 4$  and  $\Delta \eta = 300$ , which corresponds to a 1 psec time duration. In Fig. 3 (a), the result is plotted in normalized phase space for initial condition,  $(X'_0/\lambda, P'_{x0}/mc)$ . The blue squares and red circles show the results obtained numerically by using the third order (case (I)) and first order (case (II)) ponderomotive formulae, respectively. The hatched area corresponds to the region allows for the long interaction. Namely, only the particles with initial conditions in the hatched area can keep the interaction during 1 psec. The hatched region is about 16 times larger in case (II) than in case (I), suggesting that the higher order terms are effective in determining the particle dynamics near the axis. We also investigated the maximum initial momentum  $P'_{x0}$  for keeping the interaction during  $\Delta \eta = 300$  with various beam radii w for both cases (I) and (II). The numerical result for each case is shown in Fig. 3 (b) using blue squares and red circles. Here, the initial position is set as X' = 0. It is found that  $P'_{x0}$  decreases exponentially as  $P'_{x0} \sim 1/\exp(\lambda^2/w^2)$  for case (I) (blue line), whereas  $P'_{x0}$  exhibits more gentle dependence  $P'_{x0} \sim (w/\lambda)^{5/3}$  for case (II) (red line). These results suggest that the new particle motion associated with



Fig. 3 (a) The allowed area for the initial position and momentum in the perpendicular direction to maintain the interaction keeping  $X' < w = 6 \,\mu m$  during  $\Delta \eta = 300$  evaluated by the third order (case (I), blue squares) and first order (case (II), red circles) formulae. (b) The initial momentum  $P'_{x0}$  that leads to X' = w at  $\eta = 300$  for each w in case (I) (blue squares) and (II) (red circles). The other parameters are the same as those used in Fig. 2, i.e.,  $a_x = a_0 \exp(-x^4/w^4), a_0 = 4 \text{ and } \lambda = 1 \,\mu\text{m}.$ 

the third order terms predominantly and sensitively regulates the dynamics.

In summary, we applied the ponderomotive formula that includes the nonlocal effect up to the third order of  $\epsilon$  to study particle motion in a super Gaussian laser beam, which successfully prolongs the interaction time compared with the Gaussian counterpart. In this profile, since the local field gradient diminishes near the beam center, the higher order terms represented by the curvature of the field envelope and its variation dominate the dynamics. A comparison with the direct integration of the particle orbit indicates the validity of the formula derived in Ref. [6] with a Volume 8, 1201160 (2013)

sufficient convergence of the expansion series up to  $\mathcal{O}(\epsilon^3)$ . We also investigated the initial particle position and momentum that allow for keeping the interaction for a given phase advance. The conventional formula is found to overestimate the allowed area in phase space, suggesting the importance of the higher order terms. The result obtained here could be verified by measuring the amount of X-rays, which may reflects the number of interacting particles [9]. The new formula is expected to be applicable to channeling problems where the long time scale interaction between self-focused laser beam and particles influenced by the ponderomotive force play a key role [10].

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