FDTD Simulation Analysis for Improving Two-Dimensional Electron Density Distribution Measurement using Phase Imaging in GAMMA 10*)

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A phase-imaging interferometer is used to measure the electron density distribution in the plug region in GAMMA 10. The electron density is derived by the Abel transform technique. We attempt to determine the plasma density distribution for an asymmetric Abel transform by using finite-difference time-domain simulations. Moreover, we try to construct the asymmetric Abel transform for an asymmetric plasma distribution.

Keywords: electron density, phase imaging method, asymmetric Abel transform, FDTD code, phase difference

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1. Introduction

GAMMA 10 is the largest tandem mirror device and uses plasma confinement by not only magnetic mirrors but also by electrostatic potentials. It consists of central, anchor, and plug/barrier mirror cells. A phase-imaging interferometer system [1] is set horizontally to measure the upper half of the plasma in the plug cell. The phase-imaging method is a technique for measuring the phase difference between microwaves passing through the plasma and reference microwaves, and it depends on the line-integrated electron density. We can obtain two-dimensional (2D) line-integrated electron density distributions by using a 2D detector. A 2D plasma density distribution is useful for understanding the mechanisms of plasma confinement and fluctuation phenomena.

The Abel transform technique is used to determine the line-integrated electron density distribution to derive the radial electron density distributions. In previous measurements, we used the Abel transform assuming an axisymmetric plasma density distribution. However, the plasma profile is not always axisymmetric. Thus, using the Abel transform method on a nonaxisymmetric plasma may yield a large error in the calculated electron density. Therefore, we try to calculate the electron density using an asymmetric Abel transform [2, 3], in which an asymmetric plasma density distribution is assumed. To improve the accuracy of the electron density distribution obtained from the measured radial electron line-integrated density distribution, we use the finite-difference time-domain (FDTD) method [4]. In this study, we focus on establishing an improved calculation method for the electron density distribution.

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2. Phase-Imaging Method

Figure 1 shows a schematic diagram of the phase-imaging device. The incident microwave (69.85 GHz) is divided by a directional coupler. One wave is transmitted through the plasma, and its phase is changed. The other wave is a reference wave that retains the initial phase. The device detects the phase difference $\Delta\phi(x)$ between these two waves. The incident wave is extended into a sheet beam by a plane mirror and a parabolic mirror, and it is irradiated to the upper half of the plasma cross-section. Next, the wave is concentrated into the detector array by mirrors and lenses on the receiver side. The reference wave is transmitted through waveguides and incorporated into the phase detector circuit with the transmitted wave. A heterodyne system is used for direct reading in this phase-imaging device.

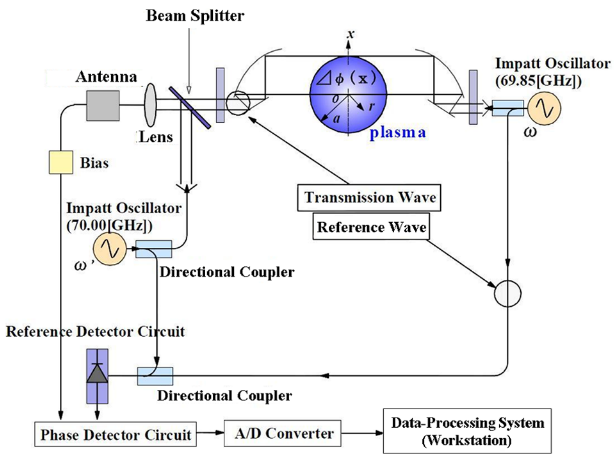

Fig. 1 Schematic diagram of phase-imaging device.

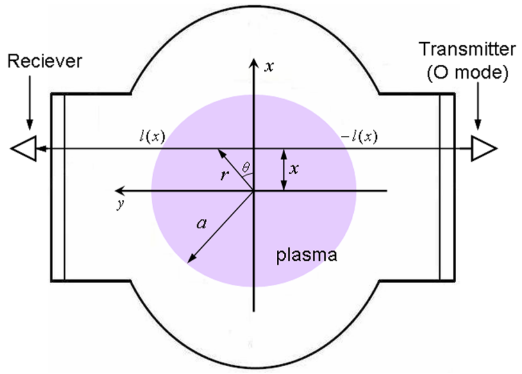

Fig. 2 Schematic view of GAMMA 10 cross section in the plug cell.

3. Electron Density Measurements

Figure 2 shows a schematic view of the GAMMA 10 cross-section in the plug cell. Here we consider the incident wave toward the positive direction of the y axis. The phase difference $\Delta\phi(x)$ owing to propagation in the plasma is written as

$$\Delta\phi(x)=\int_{-l(x)}^{l(x)}(k_0-k_\mathrm{p}(r))\mathrm{d}y=2\cdot\frac{2\pi}{\lambda}\int_0^{l(x)}(1-\tilde{n}(r))\mathrm{d}y, \tag{1}$$

where k_0, k_p, and $\tilde{n}(r)$ are the wave number in vacuum, wave number in the plasma, and refractive index, respectively. The electron density of the plug region ($n_\mathrm{e}\approx 1\times10^{12}\,\mathrm{cm}^{-3}$) is much smaller than the cut-off density ($n_\mathrm{c}\approx 6\times10^{13}\,\mathrm{cm}^{-3}$),

$$\Delta\phi(x)\cong\frac{2\pi}{\lambda n_\mathrm{c}}\int_0^{l(x)}n_\mathrm{e}(r)\mathrm{d}y. \tag{2}$$

Therefore, we can obtain the radial distribution of the electron density with an Abel transform.

4. FDTD Simulations

4.1 Simulation method

We attempted to determine the plasma distribution by using an FDTD simulation. The details are as follows. We assume a plasma density distribution in the analysis area and compare the phase differences obtained by the simulation and experimentally. If the phase differences agree, the plasma profile is calculated by the Abel transform. If not, another plasma density distribution is set, and the simulation is run again. The basic equations to be solved are as follows:

$$\partial\boldsymbol{B}/\partial t=-\nabla\times\boldsymbol{E}/\mu_\mathrm{r}, \tag{3}$$

$$\partial\boldsymbol{E}/\partial t=(c^2\nabla\times\boldsymbol{B}-\sigma E/\varepsilon_0-\boldsymbol{J}/\varepsilon_0)/\varepsilon_\mathrm{r}, \tag{4}$$

$$\partial\boldsymbol{J}/\partial t=\varepsilon_0\omega_\mathrm{pe}^2\boldsymbol{E}-e\boldsymbol{J}\times\boldsymbol{B}_0/m_\mathrm{e}-\nu\boldsymbol{J}. \tag{5}$$

Here $\boldsymbol{E}$, $\boldsymbol{B}$, c, σ, $\boldsymbol{J}$, ν, $\boldsymbol{B}_0$, e, m_e, and ω_pe represent the electric field, magnetic field, speed of light, electric conductivity, current density, collision frequency, external magnetic field, elementary charge, electron mass, and plasma frequency, respectively. The absorbing boundary condition was suggested by Uno [4]. The dielectric constant ε and permeability μ are given by

$$\varepsilon=\varepsilon_0\varepsilon_\mathrm{r},\mu=\mu_0\mu_\mathrm{r}, \tag{6}$$

where ε_0, μ_0, and ε_r, μ_r are the dielectric constant in vacuum, vacuum permeability, and dimensionless quantities, respectively.

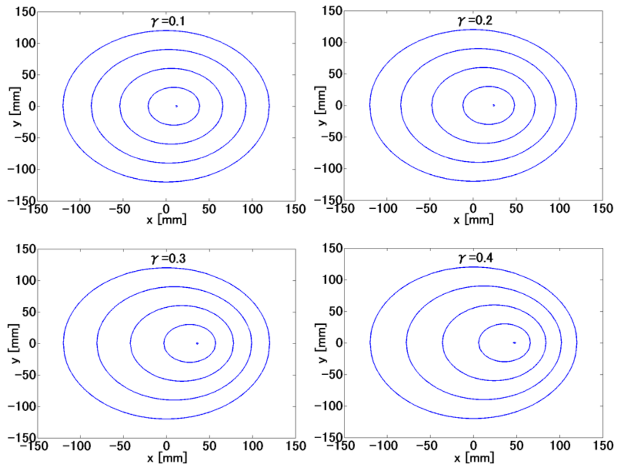

Fig. 3 Asymmetric plasma contour distribution in terms of γ, which is used in the FDTD method.

4.2 Simulation model

The analysis area is in 2D space (x, y). The frequency of the incident wave is 69.85 GHz. In Fig. 1, the incident wave is reflected by a plane mirror and a parabolic mirror (radius of curvature 796.7433 mm) and reaches the detector through the plasma, mirrors, and lenses. Parameters such as the distances and angles related to the arrangement of the experimental device are set to match the actual device. The asymmetric plasma contour distribution profile is described by

$$(x-\gamma(r_\mathrm{max}-r))^2+y^2=r^2. \tag{7}$$

Here γ and r_max are the deviation parameter of the density profile and the maximum radius of the plasma, respectively. The maximum radius of the plasma in the plug region is $r_\mathrm{max}=120\,\mathrm{mm}$. Figure 3 shows contour plots of the 2D plasma density profile for different values of γ.

Note that the value of γ in eq. (7) is not necessarily uniquely determined. The four patterns in Fig. 3 are examined in this study. The plasma density is assumed to be $n(r)=n_0\exp(-(r/80)^2)$. The equation is presented as $n(x)=n_0\exp(-((x-\gamma r_\mathrm{max})/(80(1-\gamma)))^2)$ by using $x=\gamma r_\mathrm{max}+r-\gamma r$. Here $n_0=1.5\times10^{12}$. Figure 4 shows this density distribution in the x direction.

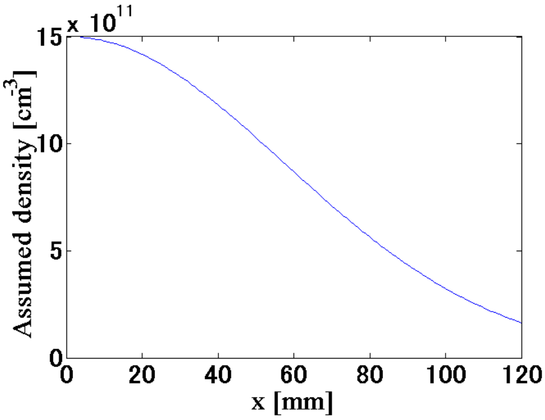

Fig. 4 Assumed density for $\gamma = 0$.

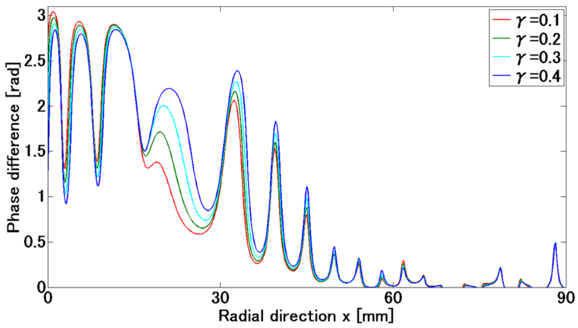

Fig. 5 Phase difference for each γ.

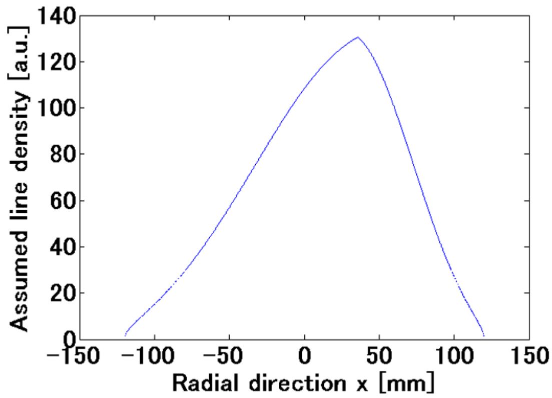

Fig. 6 Discrete observed values for$\gamma = 0.3$.

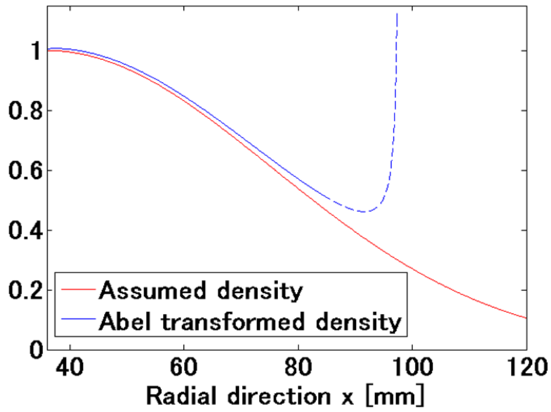

Fig. 7 Assumed and Abel-transformed densities for $\gamma = 0.3$.

4.3 Result

Figure 5 shows the result of phase analysis obtained by the FDTD calculation of the electromagnetic wave propagation at the detected position. The calculation unit width of the cell size and normalized unit parameter are 0.2 mm and 3.0×10^{11}, respectively.

The positions $x = 0$, 30, 60, and 90 mm in Fig. 5 correspond to the detected positions in the experiments and to the x axis in Fig. 3. Because the phase difference depends on the electron density, vibration of the phase difference should not appear. If we use a smaller calculation unit width, the vibration may be smaller. In Fig. 5, when γ is small, the phase difference is large at the observing position at approximately $x = 0$ mm; however, it is small at $x > 15$ mm because the smaller γ yields a larger electron density at the observing position at approximately $x = 0$ cm. Moreover, the larger γ yields a larger electron density at approximately $x > 15$ cm than at approximately $x = 0$ cm.

5. Asymmetric Abel Transform

5.1 Validity check

We derived the equation for the Abel transform for an asymmetric plasma distribution using eq. (7) and checked its validity. For this, we assumed a density distribution and compared it to the density obtained by an Abel transform calculation of the discrete observed value obtained from the assumed density. The discrete observed value of eq. (7) is described by

$$I(x) = 2\int_0^{\sqrt{r_{\max}^2 - x^2}} n(r)\mathrm{d}y. \qquad (8)$$

Here $n(r)$ is the plasma density. The integral of eq. (8) is replaced by r. For $x > \gamma r_{\max}$,

$$I(x) = 2\int_{r_0}^{r_{\max}} n(r)\frac{(1-\gamma^2)r - \gamma(x - \gamma r_{\max})}{\sqrt{r^2 - (x - \gamma(r_{\max} - r))^2}}\mathrm{d}r, \qquad (9)$$

where $r_0 = (x - \gamma r_{\max})/(1-\gamma)$. Figure 6 shows eq. (9) for $\gamma = 0.3$, where the density is assumed to be $n(r) = \exp(-(r/80)^2)$. The left side of the peak density ($x <$ 36 mm) is derived assuming symmetry.

Figure 7 shows the assumed and Abel-transformed densities for $\gamma = 0.3$. The Abel-transformed density corresponds to the assumed density at approximately $x <$ 85 mm.

The reason for the rapid increase in the difference at approximately $x > 90$ mm is that the assumed line density in Fig. 6 is missing some values at approximately $x >$ 90 mm. This is just a problem in the calculation of eq. (9). The validity could be confirmed up to $x = 85$ mm.

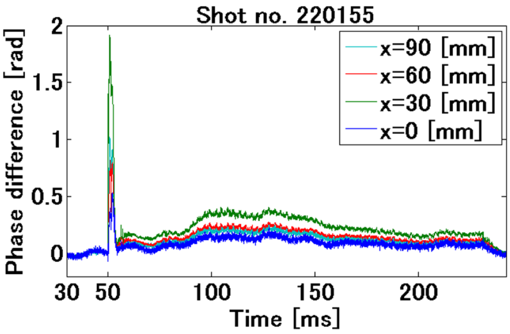

Fig. 8 Phase differences for each radial position measured experimentally.

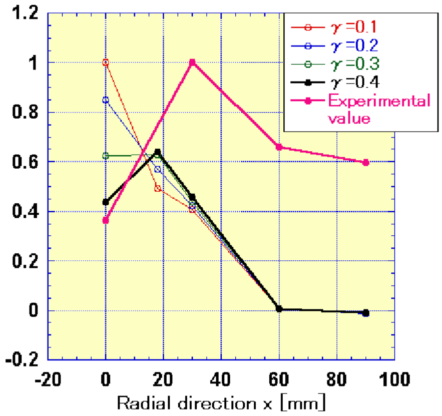

Fig. 9 Radial profiles of the phase differences.

5.2 Discussion

We compare the phase differences obtained by the FDTD calculation and experimentally to determine γ. Figure 8 shows the phase differences obtained experimentally.

We begin by comparing experimentally obtained phase differences at t = 100 ms and by the FDTD method at the radial positions of x = 0, 18, 30, 60, and 90 cm for each γ. The maximum value of the phase difference is normalized to one. Figure 9 compares the ratios of the radial phase difference profiles obtained by the FDTD method and experimentally; the calculation result for γ = 0.4 is similar to that obtained experimentally.

We understand that the optimal plasma distribution assumed by the asymmetric Abel transform is approximately γ = 0.4 in this shot. Although it may be considered as cases of larger γ corresponds to experimental results, it is

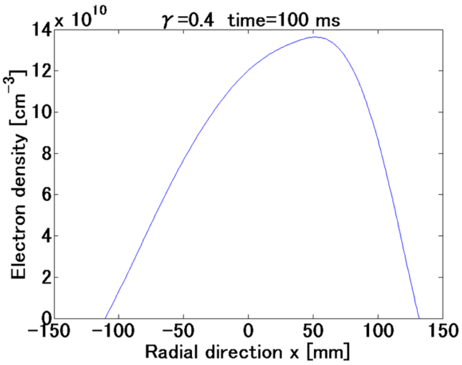

Fig. 10 Asymmetric electron density distribution.

hard to accept this if the other radial density profile measurements are considered. The main reason for the difference is a calculation error. In future research, more exact calculation is required. Figure 10 shows the electron density distribution derived for γ = 0.4.

In this shot, an electron density radial profile having a peak density position at x = 52 mm was obtained.

6. Summary

The purpose of this study was to improve the analytical accuracy by an inductive technique. A simulated model of the phase-imaging device was constructed using the FDTD method. We compared the experimentally observed and simulated phase differences and defined an asymmetrical plasma distribution using an asymmetric Abel transform. Moreover, we derived the equation for an Abel transform that describes an asymmetric plasma distribution and checked its validity. Using the asymmetric Abel transform, a plasma density profile having a peak density position at x = 52 mm was obtained. The possibility of a method of measuring an asymmetrical density distribution using FDTD was suggested.

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