Numerical Investigation on Accuracy and Resolution of Contactless Methods for Measuring j_C in High-Temperature Superconducting Film: Inductive Method and Permanent Magnet Method*)

Teruou TAKAYAMA, Atsushi KAMITANI, Ayumu SAITOH[1] and Hiroaki NAKAMURA[2]

Yamagata University, 4-3-16 Johnan, Yonezawa, Yamagata 992-8510, Japan

[1]*Graduate School of Engineering, University of Hyogo, Himeji, Hyogo 671-2280, Japan*

[2]*National Institute for Fusion Science, 322-6 Oroshi-cho, Toki 509-5292, Japan*

(Received 2 December 2011 / Accepted 2 February 2012)

The accuracy and the resolution of two types of the contactless methods for measuring the critical current density in a high-temperature superconducting (HTS) film have been investigated numerically. To this end, a numerical code has been developed for analyzing the shielding current density in the film with a crack. The results of computations show that the accuracy of two contactless methods is degraded remarkably due to the crack. Specifically, in the permanent magnet method, the maximum repulsive force acting on the film decreases when the magnet approaches near the crack. It is found that, even if the crack size is small, the maximum repulsive force decreases. This means that the crack can be detected. In the inductive method, although the threshold current decreases because of the crack, its value does not necessarily decrease for the case with a small crack size. In fact, the accuracy is not degraded when the inner radius of the coil contains the crack of the film. For this reason, we conclude that the smallest possible inner radius is preferable to detect the crack.

Keywords: contactless method, critical current density, crack detection, high-temperature superconductor, numerical simulation

DOI: 10.1585/pfr.7.2405017

1. Introduction

As is well known, a high-temperature superconductors (HTSs) are used for developing various devices and systems (e.g. nuclear fusion reactor, flywheel, and SQUID, etc.), and they are characterized by some parameters. Particularly, a critical current density j_C is one of the most important parameters, and it is important to measure the value of j_C accurately. The standard four-probe method [1] has been generally used for measuring j_C. In the method, electrodes are deposited with a silver paste to decrease the measurement error. After a large current source flows in an HTS sample, j_C can be evaluated accurately from nonlinear *I-V* characteristics. However, HTS characteristics may be degraded because of a heat generated between the electrodes and the sample. As a result, the process may lead to the destruction of a sample surface or to the degradation of superconducting characteristics.

As a contactless method for measuring j_C, Claassen *et al.* have proposed the inductive method [2]. By applying an ac current to a small coil placed just above an HTS film, they monitored a harmonic voltage induced in the coil. They found that, only when a coil current exceeds a threshold current I_T, the third-harmonic voltage develops suddenly. They conclude that j_C can be evaluated from the threshold current. In contrast to this, Mawatari *et al.* elucidated the inductive method on the basis of the critical state model [3]. From their results, they derived a theoretical formula between j_C and I_T. This method has been successfully employed as the measurement of the j_C-distributions and the detection of a crack [4].

On the other hand, Ohshima *et al.* proposed the permanent magnet method [5,6]. In the method, while moving a permanent magnet above an HTS film, the electromagnetic force acting on the film is measured. As a result, they found that the maximum repulsive force F_M is roughly proportional to j_C. This means that j_C is estimated from the measured value of F_M. This method has been successfully applied to the determination of j_C-distributions and the detection of a crack [7].

In the previous study, a numerical code was developed for analyzing the time evolution of a shielding current density in an HTS film without crack [8]. By using the code, two types of the contactless methods were reproduced for the case without any cracks. The results of computations showed that, in the inductive method, the critical current density j_C near the film edge cannot be accurately measured. In the permanent magnet method, even if the magnet is placed near the film edge, the maximum repulsive force is almost proportional to j_C [8]. However, any cracks were not at all taken into consideration in this study.

author's e-mail: takayama@yz.yamagata-u.ac.jp

*) This article is based on the presentation at the 21st International Toki Conference (ITC21).

The purpose of the present study is to develop a numerical code for analyzing the time evolution of the shielding current density in an HTS film with a crack and to simulate the permanent magnet method and the inductive method. In addition, we investigate the influence of the crack on the resolution and the accuracy of the two types of the contactless methods, and we investigate the possibility of the crack detection for two methods.

2. Governing Equation

In measuring a critical current density j_C by using contactless methods, a time-dependent magnetic field is applied to an HTS sample. In the following, we assume that a magnetic field $\boldsymbol{B}/\mu_0$ is applied to a square-shaped HTS thin film of the length a and the thickness b. Furthermore, the square cross-section of the film is denoted by Ω, and an outer boundary of Ω is expressed by C_0. Furthermore, we assume that a crack exists in Ω and its shape is given by an inner boundary C_1.

Throughout the present study, we adopt the Cartesian coordinate system $\langle O : \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z \rangle$, where z-axis is parallel to the thickness direction. Note that, in the inductive method, the origin O is chosen at the center of an HTS upper surface, whereas O is taken at the centroid of an HTS in the permanent magnet method.

As is well known, the YBCO superconductors have a strong crystallographic anisotropy: the current flow in the c-axis direction differs from that in the ab-plane, and the flow along c-axis is almost negligible. Here, the c-axis is the direction along z, and it is perpendicular to the ab-plane. On the basis of the fact, we assume the thin-layer approximation [9]: the thickness of the HTS film is sufficiency thin that the shielding current density can hardly flow in the thickness direction.

The shielding current density $\boldsymbol{j}$ is closely related to the electric field $\boldsymbol{E}$. The relation can be written as $\boldsymbol{E} = E(|\boldsymbol{j}|)[\boldsymbol{j}/|\boldsymbol{j}|]$, where a function $E(j)$ is given by the power law: $E(j) = E_C[j/j_C]^N$. Here, E_C is the critical electric field, and N is a constant.

Under the above assumptions, the shielding current density $\boldsymbol{j}$ can be written as $\boldsymbol{j} = (2/b)\nabla S \times \boldsymbol{e}_z$, and the time evolution of the scalar function $S(\boldsymbol{x}, t)$ is governed by the following integro-differential equation [9]:

$$\mu_0 \partial_t(\hat{W} S) + \partial_t \langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle + (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z = 0, \quad (1)$$

where a bracket $\langle \ \rangle$ denotes an average operator over the thickness of the film, and $\boldsymbol{E}$ is an electro magnetic field. In addition, $\hat{W} S$ is defined by $\hat{W} S \equiv \iint_\Omega Q(|\boldsymbol{x} - \boldsymbol{x}'|) S(\boldsymbol{x}', t) d^2\boldsymbol{x}' + (2/b) S(\boldsymbol{x}, t)$. Here, both $\boldsymbol{x}$ and $\boldsymbol{x}'$ are position vectors in the xy-plane. The explicit form of $Q(r)$ is described in [9].

The initial and boundary conditions to (1) are assumed as follows: $S = 0$ at $t = 0$, $S = 0$ on C_0, $\partial S/\partial s = 0$ on C_1, and $\oint_{C_1} \boldsymbol{E} \cdot \boldsymbol{t} ds = 0$. Here, s is an arclength along C_1. By solving the initial-boundary-value problem of (1), we can determine the time evolution of the shielding current density in an HTS film.

Discretized with the backward Euler method, the initial-boundary-value problem of (1) is reduced to the nonlinear boundary-value problem. In order to solve the problem, we adopt the Newton method and the finite element method. On the basis of the numerical method, a numerical code has been developed for analyzing the shielding current density in an HTS film with a crack. In order to simulate two types of the contactless methods, a magnetic flux density $\boldsymbol{B}/\mu_0$ is given by generating a permanent magnet or a coil.

Throughout the present study, the geometrical and physical parameters are fixed as follows: $a = 20$ mm, $b = 600$ nm, $E_C = 0.1$ mV/m, and $N = 20$. In addition, the critical current density j_C is assumed to be homogeneous.

3. Simulation of Contactless Methods

In the section, we investigate the influence of a crack on the accuracy of the permanent magnet method and the inductive method. A crack shape is a line segment with two end points $(x, y) = (0, \pm y_c^*)$ in the xy-plane. Here, the crack size is denoted by $L_c (\equiv 2y_c^*)$.

3.1 Permanent magnet method

In the permanent magnet method, a cylindrical permanent magnet of radius r_m and height h_m is placed above an HTS film. Moreover, a distance $L(t)$ between an magnet bottom and a film surface is given by $L(t) = L_{min} + (L_{max} - L_{min})(t/\tau_0 - 1)\tanh[200(t/\tau_0 - 1)]$, where L_{min} and L_{max} are the minimum and the maximum value of $L(t)$, respectively. In terms of the coordinate system for the present study, the symmetry axis of the permanent magnet can be expressed as $(x, y) = (x_m, y_m)$. For the purpose of characterizing the strength of the magnet, we use a magnetic flux density B_F at $(x, y, z) = (0, 0, b/2)$ for the case with $L = L_{min}$. In this subsection, the parameters are fixed as follows: $y_m = 0$ mm, $L_{max} = 20$ mm, $L_{min} = 0.5$ mm, $h_m = 3$ mm, $\tau_0 = 39$ s, and $B_F = 0.3$ T.

According to Ohshima's experimental results, a critical current density j_C can be estimated by using the formula: $j_C = \alpha(F_M/b)$ [5, 6], where F_M is the maximum repulsive force acting on an HTS sample, and α is the proportionality constant. In order to determine the proportionality constant numerically, we investigate the relation between j_C and F_M. Note that various values of F_M are evaluated by assuming the value of j_C. In Fig. 1, we show the dependence of the critical current density j_C on the maximum repulsive force F_M for various radius r_m. We see from this figure that, in spite of the radius r_m, F_M is roughly proportional to j_C. In order to determine the evaluated value $j_C^P(r_m)$ of j_C, a straight line is fitted to M data points (j_{C_j}, F_{M_j}) $(j = 1, 2, \cdots, M)$ by using the least-squares (see Fig. 1). As a result, we get the following three equations:

$$j_C^P \ (1.5\,\text{mm}) = 1.79 \times 10^{-7} (F_M/b), \quad (2)$$

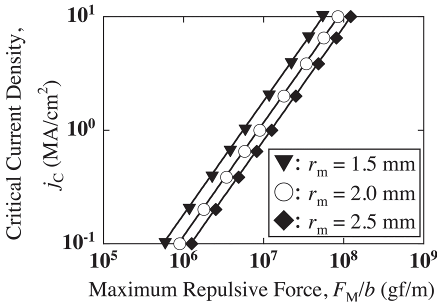

Fig. 1 Dependence of the critical current density j_C on the maximum repulsive force F_M.

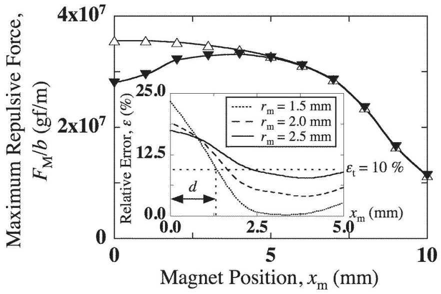

Fig. 2 Dependence of the maximum repulsive force F_M on the magnet position x_m for $r_m = 2$ mm, $j_C = 4$ MA/cm^2. Here, the symbols, △ and ▼, show $L_c = 0$ mm and $L_c = 12$ mm, respectively. The inset indicates the dependence of the relative error ε on x_m for $r_m = 2$ mm, $j_C = 4$ MA/cm^2 and $L_c = 12$ mm.

$$j_C^P\,(2.0\,\text{mm}) = 1.16 \times 10^{-7}(F_M/b), \tag{3}$$

$$j_C^P\,(2.5\,\text{mm}) = 8.07 \times 10^{-8}(F_M/b). \tag{4}$$

Note that (2)-(4) are applicable only to an HTS film without any cracks. Consequently, the critical current density j_C can be evaluated by substituting F_M to (2)-(4).

Let us first investigate the influence of a crack on the maximum repulsive force F_M. In Fig. 2, we show the dependence of the maximum repulsive force F_M on the magnet position x_m. We see from this figure that a crack tend to reduce F_M. This result qualitatively agrees with the experimental one. In addition, it is found that, F_M decreases with the crack size L_c for $1.6\,\text{mm} \leq L_c \leq 19.2\,\text{mm}$, On the other hand, F_M drastically decreases whether without or with a crack when the magnet approaches near the edge. Therefore, we suppose that a crack position can be detect by measuring F_M.

Next, let us investigate the accuracy of the permanent magnet method for various radius r_m. As the measure of the accuracy, we define a relative error: $\varepsilon \equiv |j_C^P - j_C|/j_C$. In the inset of Fig. 2, we show the dependence of the relative error ε on the magnet position x_m. This figure indicates the relative error ε decreases with increasing x_m. In particular, ε becomes the maximum value when the magnet approaches near the crack.

Finally, we investigate the resolution of the crack detection. To this end, we set $\varepsilon_t = 10\%$ as the a tolerance of the method (see Fig. 2). As a result, a distance d between the magnet and crack can be evaluated. The distance d a function as the radius r_m is depicted in Fig. 3. This figure indicates that the distance d monotonously increases with the radius r_m. We conclude that the resolution of the crack detection can be improved by using a smaller radius.

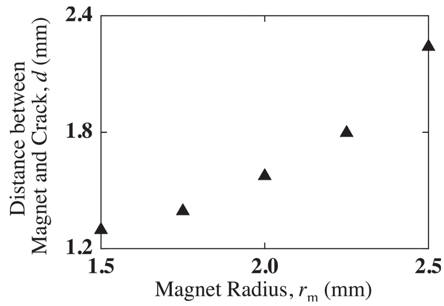

Fig. 3 Dependence of the distance d on the radius r_m for $j_C = 4$ MA/cm^2 and $L_c = 12$ mm.

3.2 Inductive method

A small N_c-turn coil is placed just above an HTS film, and an ac current $I(t) = I_0 \sin 2\pi f t$ flows in it. We assume that a vertical section of the coil is given by $r_{in} \leq r \leq r_{out}$ and $z_b \leq z \leq z_t$ with the cylindrical coordinates (r, θ, z). In order to determine the coil position, the symmetrical axis of the coil is shown by $(x, y) = (x_c, y_c)$.

According to Mawatari's theory [3], a critical current density j_C can be calculated from

$$j_C^I = 2F(r_{max})I_T/b, \tag{5}$$

where j_C^I denotes an estimated value of j_C. Also, $F(r_{max})$ is the maximum value of a primary coil-factor function $F(x)$ [3] determined from the configuration of the coil and the HTS . I_T is a lower limit of the coil current I_0 when a third-harmonic voltage V_3 begins to develop in the coil. An important point is that (5) is also applicable only to an HTS film without any cracks.

In simulating the inductive method, calculating a third-harmonic voltage V_3 from time evolution of the shielding current density, we must determine an I_0-V_3 curve. To evaluate the value of I_T, we use the conventional voltage criterion $V_3 = 0.1\,\text{mV} \Leftrightarrow I_0 = I_T$ [3]. Incidentally, the numerical method of V_3 is described in [10].

The parameters in this subsection are fixed as follows: $x_c = y_c = 0$ mm, $r_{out} = 2.5$ mm $z_b = 0.2$ mm, $z_t = 1.2$ mm, $N_c = 400$, $f = 1$ kHz, and $j_C = 1$ MA/cm^2. As a result, the values of $F(r_{max})$ can be obtained as follows:

$F(r_{max}) = 6.23 \times 10^4\,\text{m}^{-1}$; $r_{in} = 1.0$ mm,

$F(r_{max}) = 7.74 \times 10^4\,\text{m}^{-1}$; $r_{in} = 1.5$ mm,

$F(r_{max}) = 9.52 \times 10^4\,\text{m}^{-1}$; $r_{in} = 2.0$ mm.

In Fig. 4, we show the I_0-V_3 curves for various crack size L_c's. We see from this figure that, beginning to develops drastically from a certain value of I_0 for all crack size L_c's, the third-harmonic voltage V_3 increases with I_0. By

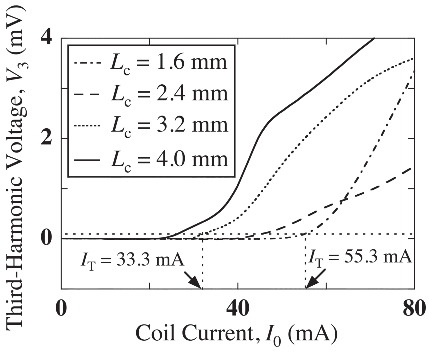

Fig. 4 I_0-V_3 curves for r_{in} = 1 mm.

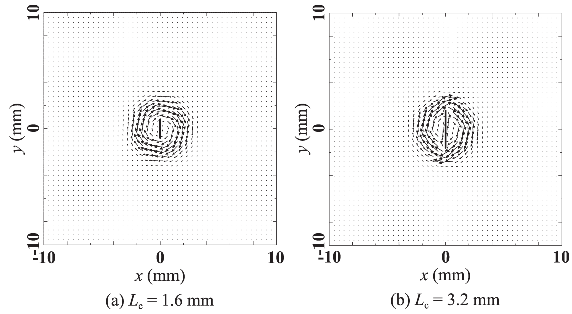

Fig. 5 The spatial distribution of the shielding current density for I_0 = 60 mA and r_{in} = 1 mm at time ft = 1.2. Here, the thick line indicates the crack.

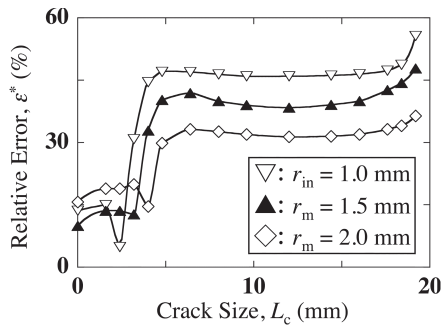

Fig. 6 Dependence of the relative error ε^* on the crack size L_c.

applying the voltage criterion to the I_0-V_3 curve for L_c = 3.2 mm, we get $I_T = 33.3$ mA (see inset of Fig. 4). This value is different from the analytic value I_T^A = 48.1 mA of I_T. Incidentally, I_T^A can be calculated from the formula $I_T^A = j_C b/[2F(r_{max})]$ derived from (5). On the other hand, the value of I_T for $L_c = 1.6$ mm is $I_T = 55.3$ mA by using the voltage criterion. This value is almost equivalent to that for $L_c = 0$ mm. This means that the crack size of $L_c = 1.6$ mm cannot be detected. The cause is a spatial distribution of the shielding current density.

In Figs. 5 (a) and (b), we show the spatial distributions of the shielding current density $\boldsymbol{j}$. We see from this figure that, for $L_c = 1.6$ mm, the spatial distribution of $\boldsymbol{j}$ is symmetric about the symmetry axis of the coil since $\boldsymbol{j}$ hardly affects the crack. This behavior is almost equal to that without crack. On the other hand, the flow of $\boldsymbol{j}$ for $L_c = 3.2$ mm is disorder due to the crack. From these results, we imply that the small crack size may be detected by changing the inner radius r_{in} of the coil.

Let us investigate the influence of the inner radius r_{in} on the crack detection. To this end, we define the relative error: $\varepsilon^* \equiv |j_C^I - j_C|/j_C$. In Fig. 6, we show the dependence of the relative error ε^* on the crack size L_c. This figure indicates that the relative error ε^* decreases for the small crack size L_c, whereas the value of ε^* becomes remarkably large for the case with the large L_c. From this result, it is found that, if the inner radius r_{in} almost contains the crack, the accuracy is not degraded. Therefore, the crack detection may be impossible by using the inductive method. Consequently, we conclude that, in order to detect any cracks of the film, the smallest possible inner radius is preferable.

4. Conclusion

1. The maximum repulsive force acting on the film decreases when the magnet approaches near the crack. In addition, the crack can be detected even if the crack size is small in the permanent magnet method.
2. A small crack size does not necessarily detect for the inductive method. This is mainly because, when the inner radius of the coil almost contains the crack of the film, the accuracy is not degraded. For this reason, we conclude that the smallest possible inner radius is preferable to detect the crack.

Acknowledgment

A part of this work was carried out under the Collaboration Research Program (NIFS11KNTS011 and NIFS10KNXN178) at National Institute for Fusion Science Japan.

[1] B.P. Martins, *Recent Developments in Superconductivity Research* (Nova Publishers, New York, 2006) p.68.
[2] J.H. Claassen, M.E. Reeves and R.J. Soulen, Jr., Rev. Sci. Instrum. **62**, 996 (1991).
[3] Y. Mawatari, H. Yamasaki and Y. Nakagawa, Appl. Phys. Lett. **81**, 2424 (2002).
[4] S.B. Kim, Physica C **463-465**, 702 (2007).
[5] S. Ohshima, K. Takeishi, A. Saito, M. Mukaida, Y. Takano, T. Nakamura, I. Suzuki and M. Yokoo, IEEE Trans. Appl. Supercond. **15**, 2911 (2005).
[6] A. Saito, K. Takeishi, Y. Takano, T. Nakamura, M. Yokoo, M. Mukaida, S. Hirano and S. Ohshima, Physica C **426**, 1122 (2005).
[7] S. Ohshima, K. Umezu, K. Hattori, H. Yamada, A. Saito, T. Takayama, A. Kamitani, H. Takano, T. Suzuki, M. Yokoo and S. Ikuno, IEEE Trans. Appl. Supercond. **21**, 3385 (2011).
[8] T. Takayama, A. Kamitani and H. Hiroaki, Plasma Fusion Res. **5**, S2113 (2010).
[9] A. Kamitani and S. Ohshima, IEICE Trans. Electron. **E82-C**, 766 (1999).
[10] T. Takayama and A. Kamitani, IEEE Trans. Appl. Supercond. **19**, 3573 (2009).