A New De-Noising Method of Laser-Produced Plasma Penumbral Images by Principal Component Analysis*)

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Penumbral imaging technique can be applied to highly penetrating radiations such that of as neutrons. In penumbral imaging, the source image can be recovered from its penumbral image by deconvolution. The method is an efficient imaging technique for fast ignition research. However, the γ rays produced by the fast-heating laser pollute the penumbral image as noise. Conventional deconvolution methods like the Wiener filter cannot obtain a clear reconstructed image from noisy penumbral image. In this paper, we propose a new reconstruction method by principal component analysis (PCA). The method can efficiently remove the noise by "training" images obtained from other experiments. We used the (2D)^{2}PCA method as a noise reduction method, which is one of the PCA methods. The efficacy of the proposed method is demonstrated by computer simulation.

Keywords: penumbral imaging, deconvolution, Wiener filter, fast ignition, principal component analysis

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1. Introduction

In inertial confinement fusion research, the neutron imaging is a key technique because the neutron images can provide direct information about the burn region; penumbral imaging [1–3] is a powerful technique for neutron imaging or other penetrating radiations. The aperture in the penumbral imaging is larger than the size of the source image. On the detector, the coded image of the source image is recorded as a penumbral image. The reconstructed image can be obtained by deconvolution of the penumbral image. Usually, the Wiener filter is used for the deconvolution. A limitation of the penumbral imaging is that the reconstruction process is sensitive to the noise of the penumbral image. The heuristic reconstruction method [4] can obtain clear reconstructed images. However, it requires a huge computation cost and many parameters for the calculation. Therefore, the signal-to-noise ratio (S/N) of the penumbral image should be increased for the reconstruction.

The uniformly penumbral array method [5, 6] can increase the S/N of the penumbral image by using several penumbral apertures. The several apertures are arranged by the *m*-sequence. On the detector, the coded image of the penumbral images is recorded as a coded penumbral image. The decoded image is obtained by the decoding operator, which is calculated from the *m*-sequence. The S/N of the decoded image can be significantly improved compared with the conventional penumbral images. However, in the use of the fast ignition [7], a lot of γ-rays are produced by interaction between the heating laser and the compressed fuel. As a result, the S/N on the detector becomes extremely low. The uniformly penumbral array method is not sufficient for the use of the fast ignition. If the Wiener filter is used for the reconstruction from the low S/N penumbral image, the reconstructed image will contain artifacts which prevent diagnosis of the fusion.

In this paper, we propose a new noise reduction method by using the principal component analysis (PCA) method for the penumbral imaging. In the method, the noise of the penumbral image is efficiently removed by using basis functions which are obtained from other penumbral images. The efficacy of the proposed method is demonstrated by the computer simulations.

2. Penumbral Imaging

A basic concept of penumbral imaging is shown in Fig. 1. The information of the source image is recorded in the gray region, which is called "a penumbra". It is easy to show that the encoded image $P(x, y)$ is given by Eq. (1):

$$P(x, y) = \iint A(x - x', y - y') \cdot O(x', y') dx' dy', \tag{1}$$

where $A(x, y)$ is a point spread function (PSF) of the aper-

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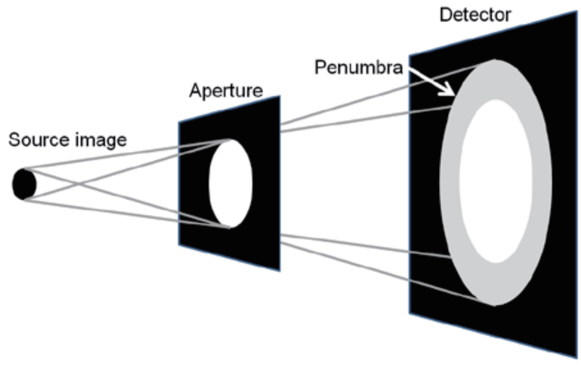

Fig. 1 A fundamental schematic of penumbral imaging.

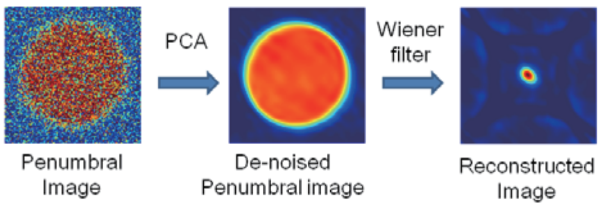

Fig. 2 The flow of the proposed method.

ture, $O(x, y)$ is a function describing the source, respectively. This integration is a convolution between $A(x, y)$ and $O(x, y)$.

The source image can be recovered from the penumbral image by deconvolution. The Wiener filter is usually used for deconvolution to reduce the noise amplification. However, the reconstructed image is polluted by the noise when the S/N of the penumbral image is low.

3. Proposed Method

3.1 Basic concept

The basic concept of the proposed method is shown in Fig. 2. In the proposed method, PCA is used in the noise reduction process of the penumbral imaging. The reconstructed image is obtained from the de-noised penumbral image.

A difference between the conventional method and the proposed method is shown in Fig. 3. In the proposed method, we introduce the concept of the "machine learning". We previously obtain basis functions from other experimentally obtained penumbral images, and this procedure is called "training". Unlike Fourier transform or wavelet-based methods, in the proposed PCA-based method, the basis functions are learned by the PCA algorithm. The obtained basis functions can separate the signal component and the noise component from the noise included penumbral image. By using the obtained basis functions, we can obtain a de-noised penumbral image. We finally obtain the reconstructed image from the de-noised penumbral image.

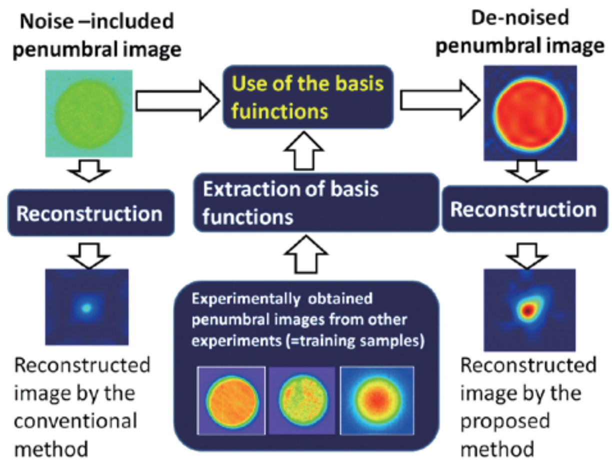

Fig. 3 A difference between the conventional method and the proposed method.

3.2 One-dimensional principal component analysis

PCA is a data representation method. The calculation of PCA is close to the singular value decomposition (SVD) method. However, PCA essentially obtains the vectors to maximize variance of the data. The vectors can be previously obtained from other data, which is called "training data", and its generalization ability is high. Therefore, PCA has recently been used in a lot of research fields, such as computer vision and pattern recognition. PCA was proposed for analysis of 1-Dimensional vector data. Given 2-Dimensional data, it should be unfolded to a 1D vector in order to allow it to be processed by PCA.

Let principal axes $w_1, w_2, \cdots w_n$ be in n-dimensional space, where n corresponds to the number of pixels of the image. A projection $\boldsymbol{y} = [y_1, y_2, \cdots, y_n]^{\mathrm{T}}$ to each axis from a point $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$ can be expressed as;

$$\begin{cases} y_1 = w_{11}x_1 + w_{12}x_2 + \cdots + w_{1n}x_n = \boldsymbol{w}_1^{\mathrm{T}}\boldsymbol{x} \\ y_2 = w_{21}x_1 + w_{22}x_2 + \cdots + w_{2n}x_n = \boldsymbol{w}_2^{\mathrm{T}}\boldsymbol{x} \\ \qquad \vdots \\ y_n = w_{n1}x_1 + w_{n2}x_2 + \cdots + w_{nn}x_n = \boldsymbol{w}_n^{\mathrm{T}}\boldsymbol{x} \end{cases}, \tag{2}$$

where

$$w_i^{\mathrm{T}} w_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}. \tag{3}$$

Therefore, $\boldsymbol{y}$ can be expressed as:

$$\boldsymbol{y} = \boldsymbol{W}^{\mathrm{T}}\boldsymbol{x}, \tag{4}$$

where $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_n]$ is a projection matrix, which is also the matrix of eigenvectors of a covariance matrix of the sample vector x. In this case, the sample vector is a set of some learning data and $\boldsymbol{W}$ is called "the basis functions". By using $\boldsymbol{W}$, to the penumbral image $\boldsymbol{x}_{\mathrm{p}}$ for removal of the noise, its projected data $\boldsymbol{y}_{\mathrm{p}}$ is;

$$\boldsymbol{y}_{\mathrm{p}} = \boldsymbol{W}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{p}}, \tag{5}$$

where $\boldsymbol{y}_{\mathrm{p}}$ is called a feature vector of $\boldsymbol{x}_{\mathrm{p}}$. In general, we remove the noise by using the top m eigenvectors arranged in the descending order of eigenvalues. The de-noised penumbral image can be obtained from $\boldsymbol{x}_{\mathrm{p}} = \boldsymbol{W}\boldsymbol{y}_{\mathrm{p}}$ by

$$\hat{\boldsymbol{x}}_{\mathrm{p}} = \sum_{i=1}^{m} \boldsymbol{w}_i^{\mathrm{T}} \boldsymbol{x}_{\mathrm{p}} \boldsymbol{w}_i. \tag{6}$$

3.3 Two-dimensional principal component analysis (2D-PCA)

In one-dimensional PCA, usually the unfolded vectors are very long, the dimension of the covariance matrix is so huge and its matrix size is $n^2 \times n^2$. Therefore, it is very difficult to calculate the bases in the unfolding vector subspace. Yang, *et al.*, proposed a 2D-PCA method [8] to overcome the problem. This method is to calculate the bases in the column-mode subspace of the 2D images instead of finding the bases in the long unfolding subspace. Therefore, the 2D data can be directly used in the training without the unfolding pre-processing. The bases on the column-subspace can be obtained by the image covariance matrix $\boldsymbol{G}$ [8] as follows:

$$\boldsymbol{G} = \frac{1}{s} \sum_{i=\mathbf{1}}^{s} (\boldsymbol{x}_i - \bar{\boldsymbol{x}})^{\mathrm{T}} (\boldsymbol{x}_i - \bar{\boldsymbol{x}}), \tag{7}$$

where $\bar{\boldsymbol{x}}$ is an average of the training samples and s is the number of training samples. By carrying out PCA with respect to $\boldsymbol{G}$, its matrix of eigenvectors $\boldsymbol{U} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_n]$ is obtained. The feature matrix $\boldsymbol{Y}$ of the penumbral image $\boldsymbol{x}_{\mathrm{p}}$ is expressed as:

$$\boldsymbol{Y} = \boldsymbol{x}_{\mathrm{p}} \boldsymbol{U}. \tag{8}$$

When the de-noised penumbral image $\hat{\boldsymbol{x}}_{\mathrm{p}}$ from $\boldsymbol{x}_{\mathrm{p}}$ is obtained, we use we use a feature matrix $\boldsymbol{Y}_m = [\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_m]$ and a similarly truncated matrix $\boldsymbol{U}_m$ with the top m eigenvectors. It can be expressed as a feature vector of top m-th eigenvectors of the feature vector $\boldsymbol{Y}$. It can be expressed as;

$$\hat{\boldsymbol{x}}_{\mathrm{p}} = \boldsymbol{Y}_m \boldsymbol{U}_m^{\mathrm{T}}. \tag{9}$$

2DPCA is essentially working in only the direction of the row of image. By simultaneously considering the row and column directions, 2-Directional 2DPCA, i.e. $(2\mathrm{D})^2$PCA [9] is used for efficient noise removal.

4. Computer Simulation

We carried out computer simulations to validate the applicability of the proposed method. The source image used in the simulations is shown in Fig. 4 (a). The source image consists of Gaussian distribution and super-Gaussian distribution. The penumbral image of the source image is shown in Fig. 4 (b). The size of the aperture used for the convolution is 48 pixels. The penumbral image including noise is Fig. 4 (c). Its signal-to-noise ratio (S/N) is 3.5 [dB].

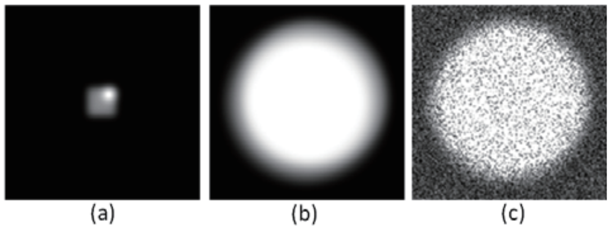

Fig. 4 Source image (a), penumbral image from the source image (b), and penumbral image including noise (c) in the computer simulation.

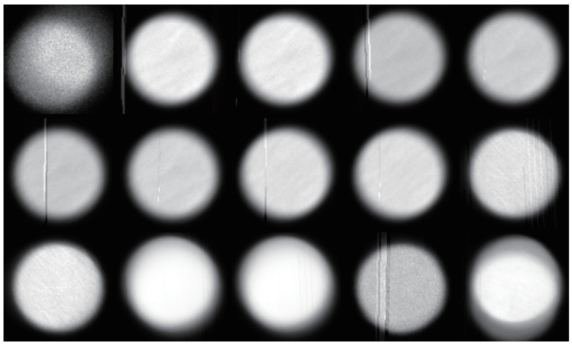

Fig. 5 Penumbral images for the training.

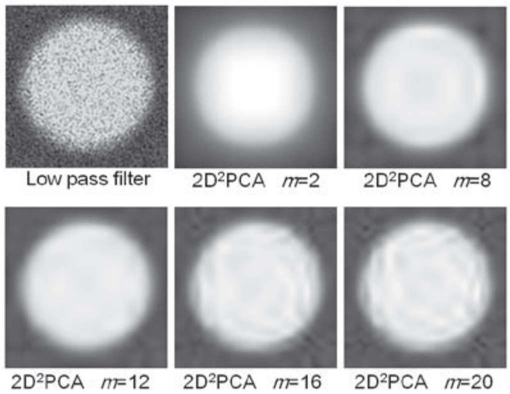

Fig. 6 The penumbral images by the low pass filter and the proposed method.

Some penumbral images used for the training are shown in Fig. 5. We used 60 experimentally obtained penumbral images to obtain basis functions. To remove directional dependences, these images are rotated per 45 degrees. So eight penumbral images are obtained from one penumbral image. Finally the 480 penumbral images are used for the training.

The de-noised penumbral images are shown in Fig. 6. The penumbral image using the low pass filter still contains the noise. On the other hand, the penumbral images using the proposed method do not contain the noise. When the number of basis functions (m) is small, the penumbral image is blurred. When m is large, the penumbral image is made clear. It can be seen that the basis functions ob-

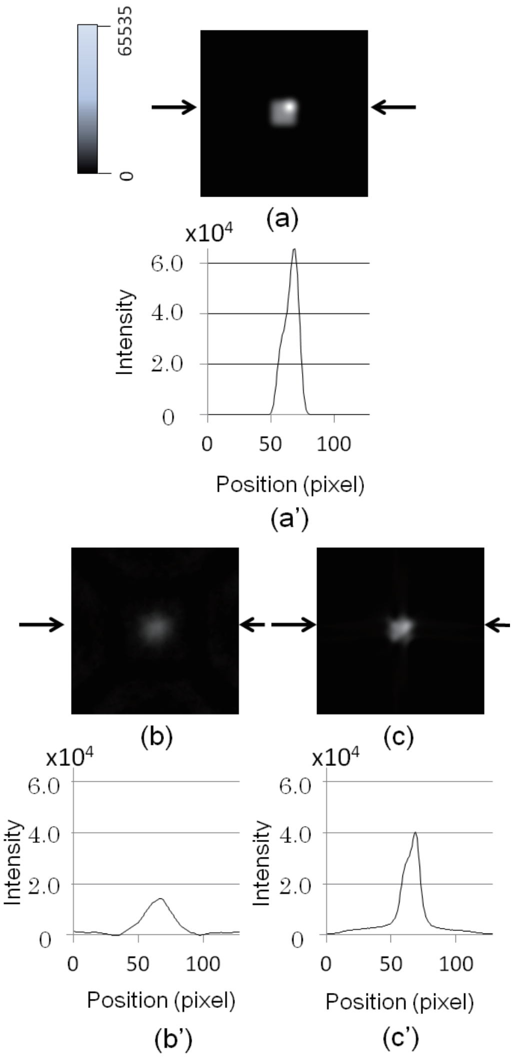

Fig. 7 The reconstructed images and their profiles. (a)(a′): source image, (b)(b′): reconstructed image by the Wiener filter, (c)(c′): reconstructed image by the proposed method.

tained from the experimental penumbral image can divide the signal component and the noise component.

The reconstructed images of the penumbral images and their profiles are shown in Fig. 7. The reconstructed image by the proposed method (Fig. 7 (c)) is clearer than the one by the low pass filter. The reconstructed image by the proposed method is clearer than the one by the conventional method. The core region in the image can be seen observed. Also, the profile by the proposed method (Fig. 7 (c′)) is sharper than the one by the conventional method (Fig. 7 (b′)).

Table 1 Image errors of the reconstructed images.

Methods	Image Error
Low-pass filter	4892
Proposed method (PCA)	2067

The image errors between the source image and each reconstructed image are shown in Table 1. The image error *er* is defined as:

$$er = \frac{\left\| O - \hat{O} \right\|^2}{NPX}, \tag{10}$$

where O is the source image (Fig. 4 (a)) and $\hat{O}$ is each reconstructed image by the conventional method and the proposed method, respectively. NPX is the number of pixels. The image error of the proposed method is smaller than the one of the conventional method.

In conclusion, the proposed method can produce a clear reconstructed image and it is found that the basis functions obtained from the experimental penumbral image can efficiently remove the noise. The technique will also be applied to measurements of laser-produced plasma experiments. This study is supported by the Rising Star Program for Subtropical Island Sciences of the University of the Ryukyus, Japan.

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