# Calculation of Ion Orbits in the Divertor/Dipole Regions of the GAMMA10 A-Divertor*) 

Isao KATANUMA, Shun MASAKI, Shuhei SATO, Kazuto SEKIYA, Makoto ICHIMURA and Kiyoshi IMAI<br>Plasma Research Center, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan

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#### Abstract

The numerical code for calculation of ion orbits was developed in order to calculate the ion orbits in the divertor and dipole regions of the GAMMA10 A-divertor. The flux coordinates ( $\psi, \theta, \chi$ ) have been adopted, where the magnetic field $\boldsymbol{B}$ is represented by $\boldsymbol{B}=\nabla \psi \times \nabla \theta=\nabla \chi$. It is found that ion orbits behave chaotic around the magnetic null.


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## 1. Introduction

The GAMMA10 tandem mirror is planning to install the divertor and dipole mirror cells in it, which is called the GAMMA10 A-divertor tentatively, where the magnetic field lines are plotted in Fig. 1. The purpose of installing these cells is to perform the simulation experiments on the divertor physics of a big torus. The ions diffused radially in the central cell of the GAMMA10 A-divertor are expected to escape through x-point in the divertor mirror cell and then enter the dipole region. In order to make clear this situation, the ion orbits have to be traced numerically in the GAMMA10 A-divertor.

The research on the divertor in open magnetic system had begun as a part of the research on looking for the axisymmetric magnetic system with the character stable to flute modes. Thus the equilibrium and stability were investigated along this object [1, 2]. However, recently the original purpose of installing a magnetic divertor has been changed to be one of performing the simulation experiments of the divertor in the closed system. Thus the investigation of transport on the GAMMA10 A-divertor is in progress [3-5]. The study of ion orbits in the divertor and dipole regions is a part of the investigation of the GAMMA10 A-divertor with the aim of plasma wall interaction.

## 2. Basic Equations

The time evolution of ion position is described in the $(\psi, \theta, \chi)$ coordinates as

[^0]\[

$$
\begin{align*}
\frac{\mathrm{d} \psi}{\mathrm{~d} t} & =v_{r} r B_{z}-v_{z} r B_{r}, \\
\frac{\mathrm{~d} \theta}{\mathrm{~d} t} & =\frac{v_{\theta}}{r},  \tag{1}\\
\frac{\mathrm{~d} \chi}{\mathrm{~d} t} & =v_{r} B_{r}+v_{z} B_{z} .
\end{align*}
$$
\]

On the other hand the equation of motion is described in the cylindrical $\left(v_{r}, v_{\theta}, v_{z}\right)$ coordinates as

$$
\begin{align*}
\frac{\mathrm{d} v_{r}}{\mathrm{~d} t} & =\frac{q}{m c}\left(v_{\theta} B_{z}-v_{z} B_{\theta}\right)+\frac{v_{\theta}^{2}}{r} \\
\frac{\mathrm{~d} v_{\theta}}{\mathrm{d} t} & =\frac{q}{m c}\left(v_{z} B_{r}-v_{r} B_{z}\right)-\frac{v_{r} v_{\theta}}{r}  \tag{2}\\
\frac{\mathrm{~d} v_{z}}{\mathrm{~d} t} & =\frac{q}{m c}\left(v_{r} B_{\theta}-v_{\theta} B_{r}\right)
\end{align*}
$$

The spatial mesh is necessary in order to save the computation time when calculating ion orbits. The information of magnetic field lines is important in the open system, so that the spatial mesh corresponding to $(\psi, \theta, \chi)$ coordinates has been introduced.

Figure 2 plots the mesh adopted in the calculation. The $\chi$-axes are just along the magnetic field lines, which have the relation $\boldsymbol{B}=\nabla \chi$. The $\psi$-axes cross each $\chi$ axis perpendicularly. Here $\nabla \psi$ satisfies the relation $\boldsymbol{B}=$ $\nabla \psi \times \hat{e}_{\theta} / r$, where $\hat{e}_{\theta}$ is the azimuthal unit vector. The $\theta$ axes do not appear because the divertor magnetic field is axisymmetric. The merit of this mesh is that the potential $\phi$ is able to be represented by $\phi=\phi(\psi, \chi)$ easily instead of $\phi=\phi(r, z)$ in the axisymmetric magnetic system and thus the electric force $\nabla \phi$ can be included in the equation of motion with the relation of $\partial \phi / \partial r=r B_{z} \partial \phi / \partial \psi+B_{r} \partial \phi / \partial \chi$ and $\partial \phi / \partial z=-r B_{r} \partial \phi / \partial \psi+B_{z} \partial \phi / \partial \chi$. The information of magnetic field in the GAMMA10 A-divertor is stored at each mesh point. The magnetic field at a particle position


Fig. 1 The GAMMA10 A-divertor magnetic field lines with coils.


Fig. 2 Computational mesh.


Fig. 3 Schematic diagram of divertor and dipole regions in the GAMMA10 A-divertor. The arrows show the flow of ions escaping into the dipole region. Here the vacuum chamber is designed to pass through just on the floor.
is determined by the fifth order Lagrange's interpolation of neighboring mesh points. The equation of motion is calculated by the 4th order Runge-Kutta-Gill method numerically.

The code test was carried out by the comparison of "the interpolation of neighboring meshes" with "the calculation of magnetic field at the particle position with the help of the magnetic field calculation code". For example, an ion orbit with kinetic energy 5 keV was traced at $2.0 \times 10^{4}$ step. It was found that the difference between both results was negligibly small. The conservations of kinetic energy, angular momentum and magnetic moment of the ion were found to be good enough.


Fig. 4 Bird's-eye plot of divertor and dipole regions in the GAMMA10 A-divertor, where the divertor plate is installed.

## 3. Divertor and Dipole Regions

Figure 3 plots the schematic diagram of flow of ions near the separatrix, where the separatrix is a magnetic flux tube, the magnetic field lines on which pass through $x$ point.

Figure 4 illustrates the divertor-plate installed in the dipole region. All the ions entered in the dipole region are expected to hit finally the divertor plate installed in the dipole region. It is necessary to follow ion orbits in order to verify the above situation. 10,000 ions are placed on the separatrix at $(\theta=0, z=280 \mathrm{~cm}$ ), where is the inner mirror throat of the divertor mirror cell in the GAMMA10 A-divertor. Here x-point is at ( $r \simeq 48 \mathrm{~cm}, z \simeq 520 \mathrm{~cm}$ ) in the present GAMMA10 A-divertor. Initial ion temperature is set 1 keV , where each ion velocity was given with help of random numbers generated numerically. Ions move along the separatrix with Larmor motion and then reach near xpoint. The magnetic moment of each ion, which conserves in the strong magnetic field, breaks near x-point and so almost all the ions enter the dipole region in Fig. 5.

## 4. Numerical Results

Figure 5 is a snap shot of ion spatial positions at $t=5.2 \mathrm{msec}$. The $(r, z)$ positions of ions are plotted in Fig. 5 (a) and $(x, y)$ positions [or $(r, \theta)$ positions] mapped at $z=520 \mathrm{~cm}$ are plotted in Fig. 5 (b). Initial position of ions is denoted by an open circle in Figs. 5 (a) and 5 (b). The ions spread in the $\theta$-direction just like diffusion but not like


Fig. 5 Ion temporal positions ( $\bullet$ ) at $t=5.2 \times 10^{-3} \mathrm{msec}$. (a): $(r, z)$ positions of ions integrated in the $\theta$-direction, and (b): $(x, y)$ positions mapped at $z=520 \mathrm{~cm}$.


Fig. 6 One ion trajectory until $t=2.6 \mathrm{msec}$. (a): ion trajectory projected on the $(x, y)$ plane, and (b): radial trajectory as a function of time.
drift. It is found that the ions behave chaoticly near x-point and the azimuthal drifts generate mainly around x-point due to large magnetic field line curvature there. The characteristic time of ion diffusion in the $\theta$-direction is about 0.1 msec , that is ions drift at the amount of $\pi$ in the $\theta$ direction in 0.1 msec . If the divertor plate were installed in the dipole region, ions with $T_{\mathrm{i}} \simeq 1 \mathrm{keV}$ would be lost in the time scale of 0.1 msec .

Figure 6 plots the history of an ion orbit at $t=$ 2.6 msec . Here Fig. 6 (a) is the history which is looked in the axial direction, and Fig. 6 (b) is the history of radial position as a function of time $t$. Ion stays around xpoint almost all the time and the ion sometimes penetrates into the dipole region irregularly. Here the straight hor-


Fig. 7 Time change of total number of ions hit the divertor-plate.
izontal line in Fig. 6 (b) denotes the radial position of xpoint. The amount of azimuthal drift in the dipole region is smaller than that near $x$-point. The ion motion is found to be chaotic around x-point in Fig. 6.

Figure 7 plots the time variation of the total amount of ions hit the divertor-plate, where various cases of the initial ion temperature ( $T_{\mathrm{i}}$ ) are calculated. The divertorplate, which has the wide of $\Delta \theta=30^{\circ}$ in the $\theta$-direction, is set just shown in Fig. 4. It is found that ions hit the divertor plate in about 2 msec . Here the ions arriving at the outer mirror throat are reflected into the divertor artificially in the calculation.

The 10,000 ions, which have a Maxwell velocity distribution with $T_{\mathrm{i}}$ and are set at the initial position shown in Fig. 5, are followed at $t=2.6 \mathrm{msec}$, where ions are assumed to die if they hit the divertor-plate or touch the vacuum chamber. Because for high temperature $T_{\mathrm{i}} \geq 4 \mathrm{keV}$ ions some of them touch the vacuum chamber and die there before they hit the divertor-plate, total amount of ions hit the divertor-plate begins to saturate at $t \simeq 0.3 \mathrm{msec}$ in Fig. 7.

It is important to investigate the heat flux at the divertor plate in the dipole region. In order to estimate the flux of ions hit the divertor plate it is important to make clear the ion energy distribution died at the divertor plate. The ion motion in the axisymmetric system can be understood with help of the following equations of motion

$$
\begin{align*}
\frac{\mathrm{d}^{2} r}{\mathrm{~d} t^{2}} & =-\frac{\partial \Phi}{\partial r}, \quad \frac{\mathrm{~d}^{2} z}{\mathrm{~d} t^{2}}=-\frac{\partial \Phi}{\partial z}  \tag{3}\\
r \frac{\mathrm{~d} \theta}{\mathrm{~d} t} & =\frac{-q}{m c r}\left(\psi-\psi_{0}\right)
\end{align*}
$$

where $\Phi$ is the pseudo-potential defined by

$$
\begin{equation*}
\Phi \equiv \frac{q^{2}}{2 m^{2} c^{2}}\left(\frac{\psi-\psi_{0}}{r}\right)^{2} \tag{4}
\end{equation*}
$$

Here $\psi_{0}$ is a constant which is related to the coordinate of ion guiding center position. Ion motion in an axisymmetric magnetic field is represented by the motion in the pseudo-potential in Eq. (3). The equi-contour surfaces of $\Phi$ corresponding to 9 keV and 1 keV are plotted in Fig. 3, where ions with kinetic energy of 9 keV are confined in the dipole region without hitting vacuum chamber.


Fig. 8 Ion radial energy density profiles died at the divertorplate. Here the vertical dashed line is the radial position of a magnetic field line passing through x-point, and $r=149 \mathrm{~cm}$ is a magnetic field line which passes at $x=-180 \mathrm{~cm}$ where is the floor position shown in Fig. 3. Please note that the vacuum chamber is designed to pass through just on the floor as shown in Fig. 3.

Figure 8 plots the radial energy distribution of ions died at the divertor-plate. Here energy is normalized by one particle. The energy distribution in the $\theta$-direction was found to be uniform. Ions with $T_{\mathrm{i}} \leq 3 \mathrm{keV}$ hit the divertor plate without touching the vacuum chamber. But some of ions with $T_{\mathrm{i}} \geq 4 \mathrm{keV}$ touch the vacuum chamber before hitting the divertor plate.

## 5. Summary and Discussion

The GAMMA10 tandem mirror is planning to install an axisymmetric divertor and dipole mirror cells in it. The main purpose of installing the divertor and dipole cells is to perform the simulation experiment of the divertor of a big torus. The divertor plate is set in the dipole region, where plasma diffused in the central of the GAMMA10 A-divertor is expected to move along the magnetic field lines on the separatrix to x -point. The ions arrived at x point enter into the dipole region because of the break of magnetic moment.

In this paper, it is confirmed that the above scenario stands up all right. The ions with $T_{\mathrm{i}} \leq 3 \mathrm{keV}$ were found to hit the divertor plate without touching the vacuum chamber in a temporal design of the GAMMA10 A-divertor.

Now we discuss on the ion axial loss through the outer mirror throat and the effects of the coulomb collisions and
ion-neutral collisions.
Ion confinement time $\tau_{\mathrm{c}}$ in a mirror is given as $\tau_{\mathrm{c}}=$ $\sqrt{\pi} R L / v_{\mathrm{i}}$ in the collisional case [6], where $R=B_{\mathrm{M}} / B_{\mathrm{c}}\left(B_{\mathrm{M}}\right.$ and $B_{\mathrm{c}}$ are the magnetic field at outer mirror throat and at the center in the periphery, respectively), $L$ the axial length of the divertor mirror region, $v_{i}$ the ion thermal velocity. The ion confinement time is expected to be very good in the divertor because $B_{\mathrm{c}} \simeq 0$ near x-point. In fact Fig. 5 (a) indicates that none of ions arrive at the outer mirror throat $(z=760 \mathrm{~cm})$ at $t=5.2 \times 10^{-3} \mathrm{msec}$.

The ion density profile along a magnetic field line is almost constant in the case of the ion velocity distribution with isotropic energy. The electrostatic potential, therefore, is expected to be constant along a magnetic field line.

The break of ion magnetic moment conservation near x-point plays a role of collisions for ions, so that the above assumption of the collisional case and of the ion distribution function with isotropic energy is applied the estimation of confinement time.

As long as the angular momentum of an ion conserves in axisymmetric divertor region, the transfer of momentum by a coulomb collision between two ions does not change the ions' radial orbits in the dipole and divertor regions. Therefore, the effect of coulomb collisions is not so important in the present calculation.

The collisions with neutral atoms change the angular momentum and especially the charge exchange changes ions energy substantially. The ions with 1 keV have the thermal velocity $v_{i} \simeq 3.1 \times 10^{5} \mathrm{~m} / \mathrm{sec}$, so that the mean flight length $L_{\mathrm{h}}$ of 1 keV ions in 2 msec is about $L_{\mathrm{h}} \simeq$ $6.2 \times 10^{2} \mathrm{~m}$. The cross-section $\sigma_{\mathrm{cx}}$ of charge exchange between several keV Hydrogen ions and cold Hydrogen atoms is $\sigma_{\mathrm{cx}} \simeq 10^{-19} \mathrm{~m}^{2}$. In order to neglect the effect of charge exchange the density $n_{0}$ of hydrogen atoms is required to be $n_{0} \sigma_{\mathrm{cx}} L_{\mathrm{h}} \ll 1$, which gives $n_{0} \ll 1.6 \times 10^{16} / \mathrm{m}^{3}$. The neutral density $n_{0}$ is expected to be much less than $10^{16} / \mathrm{m}^{3}$ in GAMMA10 A-divertor by introducing the liquid $H_{\mathrm{e}}$ cryo-panel, so the effect of neutral atoms is not so important in the present calculation.
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[^0]:    author's e-mail: katanuma@prc.tsukuba.ac.jp
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