Fast and Robust Reconstruction of Penumbral Images by Combining Multiple Wiener Filters*)

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Penumbral imaging is a powerful technique for imaging with penetrating radiation such as neutrons and hard x rays. The reconstructed image can be obtained by the deconvolution of the detected image. Usually a Wiener filter is used for the reconstruction. The limitation of the Wiener filter is that it will introduce a significant distortion when the detected image contains noise. On the other hand, heuristic methods can obtain clear reconstructed image from noisy penumbral images. However, it significantly takes computation time. We proposed a new reconstruction method in the penumbral imaging to obtain clear reconstructed image by combining the multiple Wiener filters and the method can reduce computation time. The effectiveness of the proposed method has been demonstrated by computer simulation and real laser plasma experiment.

Keywords: penumbral imaging, deconvolution, Wiener filter, heuristic method, computation time

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1. Introduction

Penumbral imaging [1] is powerful technique for neutron imaging or other penetrating radiations [2, 3]. The technique uses facts that spatial information can be recovered from the shadow or penumbra that an unknown source casts through a simple large circular aperture. The reconstructed image can be obtained by the deconvolution of the detected image. Usually a Wiener filter is used for the reconstruction. A limitation of the Wiener filter is that it will introduce a significant distortion if the penumbral image is distorted by the noise. Coded penumbral imaging approach can significantly improve its signal-to-noise ratio [4]. However, the method needs extremely wide detector therefore the cost would be expensive. Heuristic reconstruction methods [5] can significantly reduce the distortion from the noisy penumbral image. However, it takes huge computation cost because it is based on iterative methods. Especially, the heuristic method cannot apply to the penumbral images whose the number of the pixels is large because the computation time is proportional to the number of the pixels of the image. Due to the computation cost, the methods cannot be easily applied for the deconvolution. Therefore we have to develop a new method for reconstruction from noisy penumbral images to reduce the distortion and computation time. In this article, we propose a new reconstruction method to reduce distortion by combining multiple Wiener filter.

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2. Penumbral Imaging and Proposed Method

The basic concept of the penumbral imaging technique is shown in Fig. 1. The encoded image consists of a uniformly bright region surrounded by a penumbra (hatched region). Information on the source is encoded in this penumbra. It is easy to show that the encoded image $P(\boldsymbol{r})$ is given by Eq. (1),

$$P(\boldsymbol{r}) = A\left(\frac{L_1}{L_1 + L_2}\boldsymbol{r}\right) * O\left(-\frac{L_1}{L_2}\boldsymbol{r}\right), \tag{1}$$

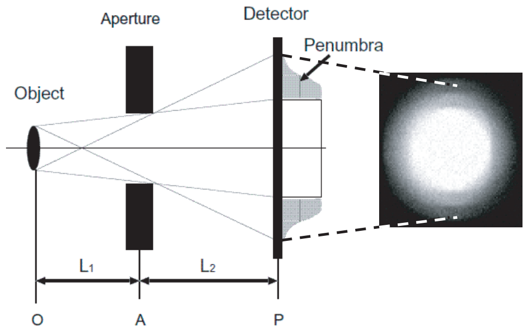


Fig. 1 The basic concept of penumbral imaging.

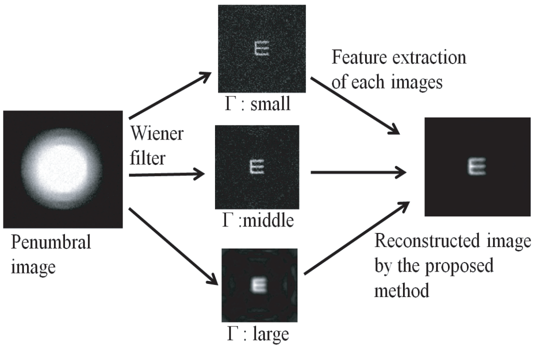


Fig. 2 The basic concept of the proposed method. First reconstructed images are obtained by changing the parameter of the Wiener filer. The final reconstructed image is obtained from the first reconstructed images.

where $A(\boldsymbol{r})$ is the aperture function or point spread function (PSF); $O(\boldsymbol{r})$ is the function describing the source; $\boldsymbol{r}$ is positional information in spatial domain. L_1 and L_2 are the distances from source to aperture and from aperture to detector, respectively. L_2/L_1 is the magnification of the camera; and $*$ denotes the convolution. If the exact point spread function $A(\boldsymbol{r})$ is *a priori* known, the source function $O(\boldsymbol{r})$ may be deconvoluted.

Usually, Wiener filter is used for the reconstruction. The Wiener filter $W(u)$ is defined as,

$$W(\boldsymbol{u}) = \frac{1}{A_F(\boldsymbol{u})} \times \left(1 + \frac{\Gamma}{A_F(\boldsymbol{u})}\right)^{-1}, \tag{2}$$

where $A_F(\boldsymbol{u})$ is the Fourier transform of the PSF, and Γ is a constant proportional to the noise-to-signal power density ratio. $\boldsymbol{u}$ is the spatial frequency of $\boldsymbol{r}$. The reconstructed image can be obtained by using an adequate Γ. Usually, Γ is selected by manual since the SN ratio often does not clear in experiments. When the SN ratio of the penumbral image is small, Γ is set to be large for the reconstruction. However, the artifact arises in the reconstructed image. The heuristic method can obtain the clear reconstructed image even if the signal-to-noise of the penumbral image is low.

The basic concept of the proposed method is shown in Fig. 2. We obtain the several reconstructed images by changing Γ of the Wiener filter. Each reconstructed images are obtained by changing Γ. When the Γ is small, its reconstructed image is not clear but there are few artifacts. On the other hand, when Γ is large, its reconstructed image is clear but there are lot artifacts. In the proposed method, we calculate geometric mean of reconstructed images obtained from different Γ. By calculating the geometric mean, we can obtain common signals between each reconstructed images which are obtained from small Γ and large Γ. Therefore, the distortion is reduced in the finally obtained reconstructed image. In this proposed method, the reconstructed images are multiplied in each pixel, and the multiplied image is regularized as the final reconstructed image.

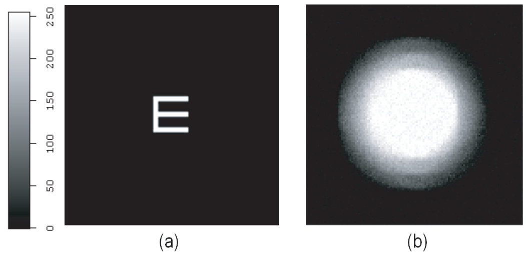


Fig. 3 Phantom (a) and its penumbral image (b) in the computer simulation.

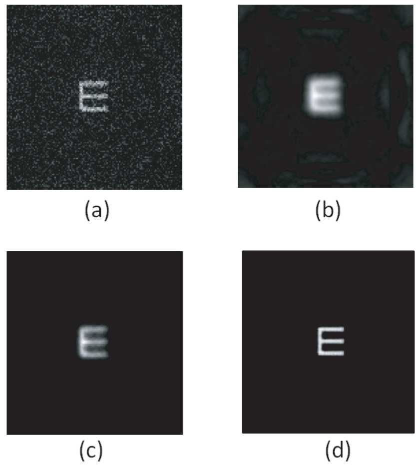


Fig. 4 Reconstructed images in the computer simulation. (a) Wiener filter Γ = 3000 (b) Wiener filter Γ = 30000 (c) Proposed method (d) Heuristic method.

3. Simulation Results

We carried out computer simulations to validate the applicability of the proposed method. The phantom used in the simulations is shown in Fig. 3 (a), which consists of 128×128 pixels. L_1 and L_2 are set as 1, respectively.

The radius of the aperture is 40 pixels. The penumbral image, which is obtained by Eq. (1), is shown in Fig. 3 (b). Its number of pixel is 128×128. A Gaussian noise is also added in the penumbral image and its SN ratio is 25 [dB].

Fig. 4 shows the reconstructed results by conventional Wiener filter, the proposed method, and the heuristic method, respectively. Their number of pixel is 128×128. The reconstructed image by small Γ (Fig. 4 (a)) is degraded by the noise. On the other hand, the reconstructed image by large Γ (Fig. 4 (b)) is degraded by the artifact. However, the reconstructed image by the proposed method (Fig. 4 (c)) is improved in compared with Fig. 4 (a) and Fig. 4 (b). Fig. 4 (d) is the reconstructed image by the heuristic method.

The image errors between source image and each reconstructed images are shown in Table 1. The image error er is defined as:

$$er = \frac{\left\| O - \hat{O} \right\|^2}{N}, \tag{3}$$

Table 1 Image errors by the proposed method and the heuristic method in the simulation.

Reconstruction Methods	Image error ($\times 10^{-3}$)
Proposed method	4.962
Wiener filter (Γ=3000)	6.551
Wiener filter (Γ=30000)	12.181
Heuristic method	1.985

Table 2 Computation times of the simulation by the proposed method and the heuristic method.

Reconstruction Methods	Computation time (sec)
Proposed method	3
Heuristic method	1644

where O is the source image (Fig. 3) and $\hat{O}$ is each reconstructed images by the conventional method and the proposed method, respectively. N is the number of the pixel. The image error of the proposed method is smaller than the ones of the Wiener filter. Although the error is larger than the one of the heuristic method, it is obvious that the reconstructed image is clear by Fig. 4 (c).

In order to make a quantitative evaluation of computation time, we measured computation times for the reconstruction by the heuristic method and the proposed method. The computation times are shown in Table 2. The simulations are done by a Pentium 3.2 GHz based PC equipped with 2 GB of RAM and running Fedora Linux. The results show that the computation time is reduced to 1/548 (= 3/1644).

From the computer simulation results, it found that the proposed method can clear reconstructed image from the noisy penumbral image in short computation time.

4. Experimental Results

We applied the proposed technique to x-ray imaging of laser-imploded targets with a penumbral camera. The experiments were carried out at the frequency doubled (0.53 μm) 12 beams Nd glass laser facility, GEKKOXII, at Osaka University, Japan [5].

The experimental setup for real laser-plasma experiments is shown in Fig. 5. The pinhole image is taken in same direction of penumbral camera. Thus we can compare the pinhole image and the reconstructed image from the penumbral image. The reconstructed image is better when the reconstructed image is near to the pinhole image. The distance between the target and aperture is 117 mm. The distance between the aperture and the detector is 1008 mm. Therefore the magnification of the camera is 8.6.

The experimentally obtained penumbral image is shown in Fig. 6 (a). For comparison of S/N of the penumbral image, a typical penumbral image which can obtain the reconstructed image by the conventional Wiener filter is shown in Fig. 6 (b). It can be seen that experimentally obtained penumbral image (Fig. 6 (a)) is degraded by the noise and its SNR is low compared with Fig. 6 (b). The pinhole image obtained by the experiment is shown in Fig. 7 (b). The reconstructed images by conventional Wiener filter are shown in Fig. 7 (c) and (d).

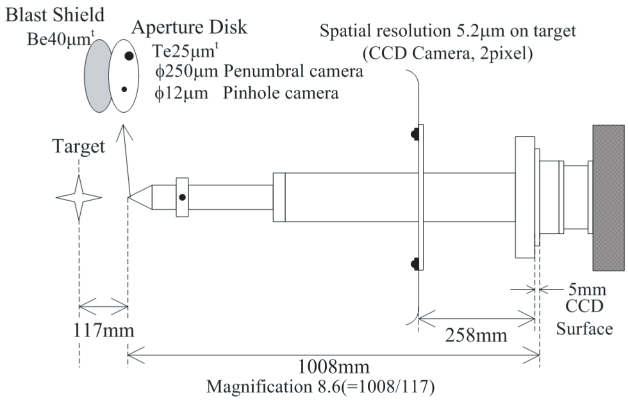
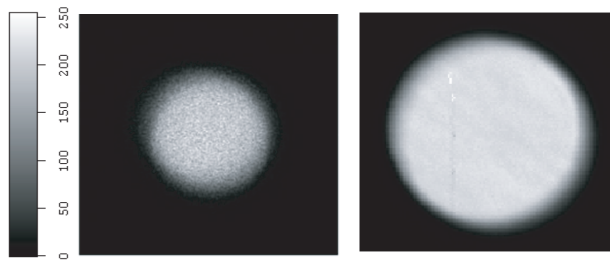


Fig. 5 Illustration of the demonstration experimental setup. Magnification of the camera is 8.6.



(a) (b)

Fig. 6 Penumbral image obtained from the demonstration experiment (a) and typical penumbral image which can reconstruct (b). The penumbral image obtained from the experiment (a) is noisy compared with the typical penumbral image (b).

The reconstructed image by the proposed method is shown in Fig. 7 (e), which can be obtained from two reconstructed image (Fig. 7 (c) and (d)). It can be seen that the noise will introduce significant distortions in the reconstructed images by the conventional methods (Fig. 7 (c) and (d)). On the other hand, From Fig. 7 (d), it can be seen that the target image can be reconstructed and there are no artifact by the parameter Γ of the Wiener filter. Fig. 7 (f) is the reconstructed image by the heuristic method. From the reconstructed images by the conventional Wiener filter, both of the images contain the artifact and noise because the S/N of the penumbral image is low. On the other hand, the reconstructed image by the proposed method (Fig. 7 (e)) does not almost contain the noise or the artifact. Furthermore, the reconstructed image by the proposed method is similar to pinhole image Fig. 7 (b) compared with conventional reconstructed images (Fig. 7 (c) and (d)). The quality of the reconstruction by the proposed

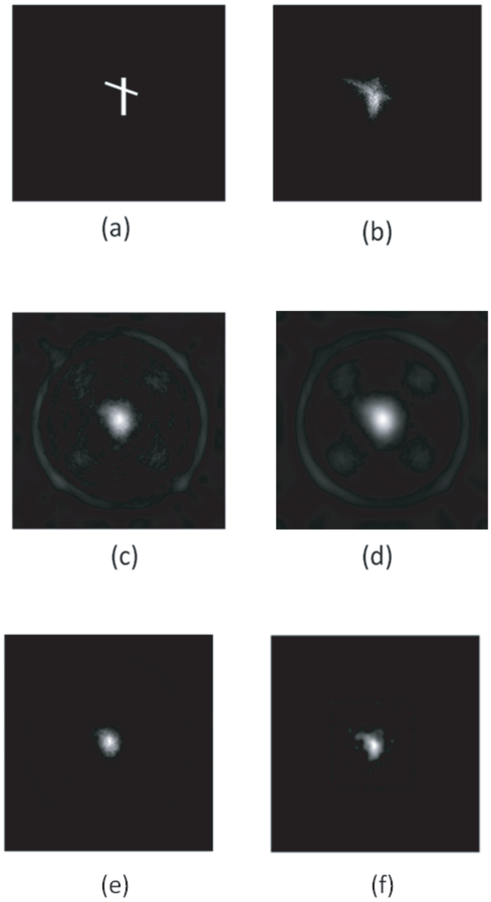


Fig. 7 The target image: (a) desired image, (b) pinhole images, (c) the reconstructed image by the conventional Wiener filter (Γ = 30000). (d) the reconstructed image by the proposed method (Γ = 100000), (e) the reconstructed image by the proposed method, (f) the reconstructed image by the heuristic method.

Table 3 Computation times of the experimental penumbral image by the proposed method and heuristic method.

Reconstruction Methods	**Computation time (sec)**
Proposed method	10
Heuristic method	83403

method is almost equal to heuristic reconstruction. The computation times of the proposed method and the heuristic method are shown in Table 3. The computation time is significantly reduced. In conclusions, the proposed method can obtain clear reconstructed images in a short time from noisy penumbral images. The technique will be also applied to measurements of laser-produced plasma density distribution and other experiments. This study is supported by the Rising Star Program for Subtropical Island Sciences of the University of the Ryukyus, Okinawa, Japan.

[1] K.A. Nugent and L. Davis, Opt. Commun. **49**, 393 (1984).
[2] A.P. Fews, M.J. Lamb and M. Savage, Laser Part. Beams **12**, 1 (1994).
[3] Y.-W. Chen, N. Miyanaga, M. Yamanaka, H. Azechi, S. Ishikawa, T. Yamanaka, S. Nakai and S. Tamura, Opt. Commun. **73**, 337 (1989).
[4] T. Ueda, S. Fujioka, S. Nozaki, R. Azuma, Y.-W. Chen and H. Nishimura, Rev. Sci. Instrum. **81**, 073505-1 (2010).
[5] S. Nozaki, Y.-W. Chen, Z. Nakao, R. Kodama and H. Shiraga, Rev. Sci. Instrum. **74**, 2240 (2003).