Virtual Reality Visualization of Frozen-in Vector Fields*)

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Magnetic field lines in ideal magnetohydrodynamics (MHD) are frozen into the fluid. Three-dimensional (3-D) visualization of the frozen-in magnetic field lines are useful for MHD dynamo study since it extracts stretching, twisting, and folding components of the flow. A 3-D, interactive, and immersive visualization method for the frozen-in field lines based on the virtual reality (VR) technology is proposed. A VR system with the head and hand tracking functions provides an ideal environment for the frozen-in line visualization since the seeding of the initial 3-D curve and stereoscopic observation of its motion are naturally realized in the VR space.

Keywords: virtual reality, CAVE, visualization, frozen-in field

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1. Introduction

Visualization of three-dimensional (3-D) vector fields is always a hot topic in scientific visualization [1, 2]. To grasp three-dimensional structure and its temporal variation of flows or other vector fields produced by large scale computer simulation is critically important in simulation research. Among various methods developed for the vector field visualization, tracer particle method is a classical one. In this method, motion of massless particles conveyed by the target flow, or vector field in general, is observed.

A generalization of the tracer particle method from 0-D (point) to 1-D (line) is the timeline method in which three-dimensional lines are advected by the flow. In experimental fluid dynamics using water, the timeline visualization method is realized by, for example, the hydrogen bubble method [3]. For numerical data, Mininni *et al.* [4] implemented this visualization method in VAPOR visualization framework.

A challenge of the timeline method as a practical tool for large scale simulation data is its seeding strategy. The freedom of the initial condition (curves in a 3-D space) for a timeline is much larger than that of a tracer particle. In this paper, we propose to use a virtual reality (VR) technology for the implementation of the timeline method. A VR system with hand tracking capability provides an ideal tool for the seeding step of the timeline method. One can easily specify the initial curve just by literally moving his/her hand.

Another challenge of the timeline method is that the advected line may result in a highly complicated 3-D structure that is difficult to be grasped through common visual displays. The VR's stereoscopic view with the automatic view point adjustment by means of a head tracking system provides a natural solution. Application of the VR for the particle method was reported in [5, 6].

An interesting point of the timeline method is that it can be regarded as a visualization of the so-called frozen-in vector fields in fluid dynamics or in magnetohydrodynamics (MHD).

Time development of magnetic field $\boldsymbol{B}$ under flow $\boldsymbol{u}$ is described by [7];

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}), \tag{1}$$

where the fluid's electrical resistivity is supposed to be negligibly small, that is a good approximation for the liquid metals in planetary cores or hydrogen plasma of the stellar interior.

The vector field $\boldsymbol{B}$ defined by Eq. (1) has a remarkable character called "frozen-in", i.e., a line of force of the field $\boldsymbol{B}$ is attached or frozen into the fluid flow $\boldsymbol{u}$. In other words, a line of force is identical to the timeline.

Suppose a magnetic field $\boldsymbol{B}$ in a fluid $\boldsymbol{u}$, see Fig. 1 (a), and take a line of force of $\boldsymbol{B}$ that threads through a position A at time $t = t_0$. And take another point B that is connected by the line at this time. The fluid parcel placed at position A at $t = t_0$ is conveyed by the flow $\boldsymbol{u}(\boldsymbol{x})$. Similarly, the fluid parcel at B is conveyed by the flow there and these two particles are conveyed to points A' and B', respectively, at $t = t_1$. See Fig. 1 (b). If we draw another line of force of $\boldsymbol{B}$ at $t = t_1$ from A', as shown in Fig. 1 (c), then the resulting line always threads through the point B'; see Fig. 1 (d).

Since any pair of fluid parcels that are connected by a line of force of $\boldsymbol{B}$ at $t = t_0$ is connected by a single field line at any later time $t = t_1$ (see Fig. 2), we can identify the field lines at any time as the same line, and it can be interpreted as if it is conveyed by the flow $\boldsymbol{u}$. In ideal MHD, this is called Alfvén's theorem [7].

Another example of the frozen-in vector field is the vorticity $\boldsymbol{\omega}$ ($\equiv \nabla \times \boldsymbol{u}$) of an inviscid fluid. The time devel-

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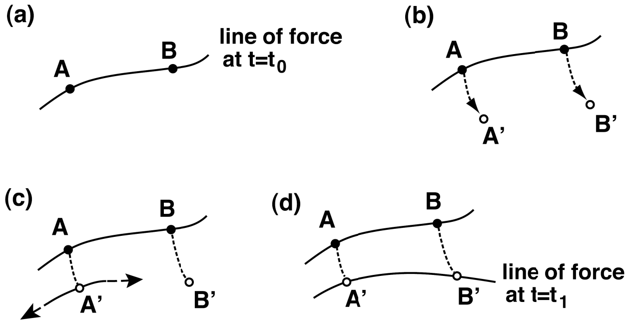

Fig. 1 Concept of the frozen-in. (a): Two points A and B are connected by a line of force of a vector field such as magnetic field. (b): Fluid parcels at A and B are conveyed by the flow to positions A' and B'. (c): New line of force started from A'. (d): The new line of force is always goes through B'. The two points A' and B' are always connected by the line of force.

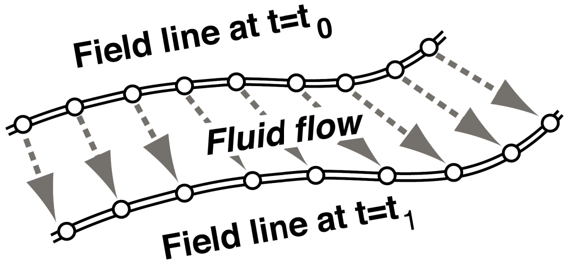

Fig. 2 A field line that is frozen in the flow. The frozen-in line is identical to the sequence of tracer particles.

opment of $\boldsymbol{\omega}$ is the same as Eq. (1), with substituted $\boldsymbol{\omega}$ instead of $\boldsymbol{B}$ [7]. The line of force of vorticity field (or vortex line) is also frozen in the flow and it is called Helmholtz's theorem in fluid mechanics. Generally, any solenoidal vector field $\boldsymbol{B}$ that follows, or is defined by, Eq. (1), is frozen in the flow $\boldsymbol{u}$.

2. Visualization of Frozen-in Vector

The concept of the frozen-in of a magnetic field line plays a significant role in the understanding of the so-called MHD dynamo process. The MHD dynamo is an energy conversion process from the flow energy into the magnetic energy. Magnetic fields of planets, starts, the Earth, and the Sun are all generated by the MHD dynamo process taking place in the interior of the bodies. Although detailed understanding of these astrophysical MHD dynamo process is still unknown and hot topic of research, its basic process can be intuitively explained by means of the frozen-in of the magnetic field lines.

Suppose a ring of magnetic field line of force in an MHD fluid; see Fig. 3 (a). If a pair of anti-parallel flow exists in the fluid as shown in (b), the ring of field line is expanded by the flow since the line is frozen-in the fluid. And if the fluid happens to have twisting and folding flow components as shown in (c), the expanded ring is twisted and then folded, again due to the frozen-in nature. The result of this process is, as shown in (d), a doubled ring of the field line.

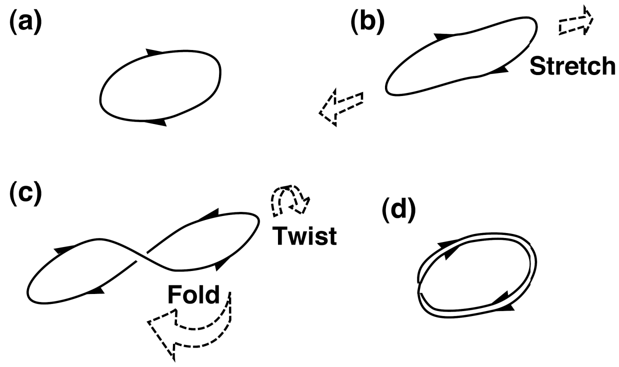

Fig. 3 Stretching, twisting, and folding process of a frozen-in field line. Initial magnetic field line that had a ring shape is doubled by the process, meaning that the magnetic intensity is doubled by the flow. This is the MHD dynamo process visualized by the frozen-in field line.

Since the number of field lines threading through a unit area is proportional to the amplitude of the magnetic field there, the doubled ring of the field line means doubled magnetic intensity. Since a magnetic field line has a tension force like a rubber band, the stretching flow against it transfers energy. The magnetic field was amplified by the fluid through the stretching, twisting, and folding flow. This is a schematic, but essentially accurate, explanation of the MHD dynamo. This process is believed to be taking place in planets and stars.

To study the origin of the magnetic field of planets and stars, large scale MHD simulations have been extensively performed on high performance computers [8, 9].

Although the visualization of frozen-in magnetic field lines is identical to the timeline method that is traditionally used in the flow visualization, the frozen-in visualization method has not been actively used in the MHD dynamo study. This would be explained by the difficulty of the user-interface, rather than the visualization algorithm itself.

3. Implementation of Frozen-in Line Visualization in VR

One of the technical challenges of the frozen-in or timeline method is, as in the case of the tracer particle method, the user interface for the seeding. The initial condition, or initial 3-D curve, of a timeline should be specified in 3-D space. The 2-D GUI based user interface is obviously inadequate for "drawing" a 3-D curve. The VR technology with a hand tracking system provides an ideal basis for our purpose. The VR's stereoscopic view with the head tracking system also provides an ideal environment for observing a possibly complicated 3-D structure and motion of the frozen-in line.

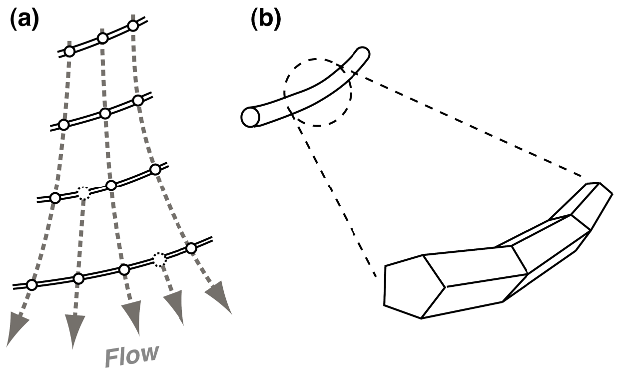

Fig. 4 Implementation of VR timeline method in CAVE's VR space. (a): When the discretized line is stretched and a distance between two tracer points goes beyond a critical value, a new tracer is inserted. (b): A timeline is rendered by a polygon tube.

We have implemented the 3-D, interactive, and immersive timeline method as one of the visualization modules of VFIVE framework [5, 10–12], which is a general purpose, freely available VR visualization software on the CAVEs [13]. Main development and tests are performed on a one-screen VR system with a rear stereo projector with head and hand tracking system (Intersense IS-900). The screen size is W3260 mm × H2500 mm. The CAVELib ver.3.2 is used as the basic API for the VR.

The algorithm of the VR timeline visualization is a straightforward application of the concept of the sequence of tracer particles shown in Fig. 2. The seed line at $t = t_0$ is discretized into small line segments.

As time goes on, the distance between two consecutive pair of line segments may increase if the flow has a stretching component there. In our program, we insert a new line segment, or a new tracer, at the middle of the pair, i.e., a linear interpolation of the pair coordinates, when the distance between the pair goes beyond a critical value; See Fig. 4 (a). The whole line is rendered as a thin tube composed of polygons as shown in Fig. 4 (b) with OpenGL.

Aubert *et al.* [14] have developed a visualization method for magnetic field, Dynamics Magnetic Field line Imaging technique (DMFI), and applied it to the geodynamo simulation data in a spherical shell region. The tracing algorithm of a magnetic field in DMFI method is different from ours. An anchor point with the maximum magnetic energy along the line is traced in DMFI. The field line advection in [4] uses the same strategy.

The seeding or the initial condition of the line is specified by hand (or wand) motion in the VR space. A short virtual beam emitted from the wand tip (Fig. 5 (a)) is used to draw a trajectory by 3-D "drag" with a pressed wand button; see panels (b) to (d) of Fig. 5. After the 3-D curve drawing, the tracing (or integration of equation of motion) of the line takes off.

To test the VR timeline method thus implemented, we have applied it to an analytically defined flow, or an ABC

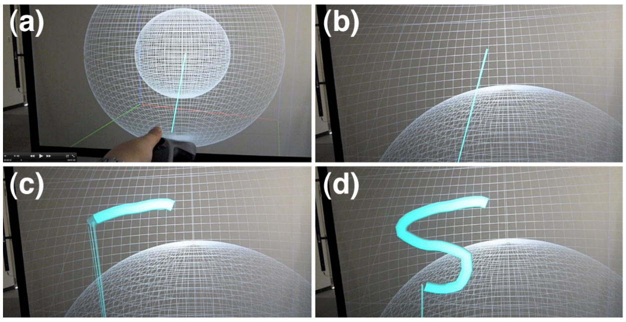

Fig. 5 Seeding or drawing step of the initial curve for the VR timeline method. (a): A virtual beam is emitted from the user's hand (or wand). (b) to (d): An S-shaped 3-D curve is drawn by the hand motion.

flow; $(u_x, u_y, u_z) = (\cos y + \sin z, \cos z + \sin x, \cos x + \sin y)$. The ABC flow is known as a chaotic flow and its stream line structure is investigated in detail [15]. The chaos in this case means that a frozen-in line can be exponentially stretched by the flow when it goes through bifurcation points of the velocity. The panels (a) to (e) of Fig. 6 show a sequence of snapshots when a user is drawing a 3-D initial curve by the wand "drag". The initial curve in this case was rather long with intention of finding the bifurcation points. The subsequent snapshots (f) to (l) show time development of the frozen-in line under the ABC flow. We could observe the chaotic behavior of the flow. By making use of the so-called navigation function in the VR space installed in VFIVE, the user can rotate, Fig. 6 (k), or fly to zoom into the timeline object, Fig. 6 (l).

Another application of the VR timeline method is shown in Fig. 7. The flow in this case is an output of an MHD dynamo simulation to understand the Earth's magnetic field generation process, or geodynamo simulation for short [8, 9]. In this case, the timeline can be interpreted as a magnetic field line in the Earth's core that is frozen in the convection flow of the liquid metal of the core.

The short virtual beam emitted from the wand tip (Fig. 7 (a)) is again used for the seeding step. In this test case, the initial curve is extremely short on purpose (a ball-like object in the left of the beam tip in panel (b)). The short magnetic field line is conveyed by the flow in the Earth's core due to the frozen-in nature (panel (c)), and at the same time, it is stretched as shown in the sequence of the panels (d), (e), and the followings. It is amazing that the initially short magnetic field line is rapidly stretched, twisted, and folded, to form a rather coherent helical structure in the core. This intensive field line stretching indicates that the flow is a vital magnetic field generator, i.e., a dynamo. The helical structure of the magnetic field line is obvious in the final stage of the snapshot sequence; See panels (k) and (l) in which helical field lines are viewed from different view angles. The helical structure of the magnetic field line reflects the helical flow structure in the Earth's core. It is known that helical flow is one of the key

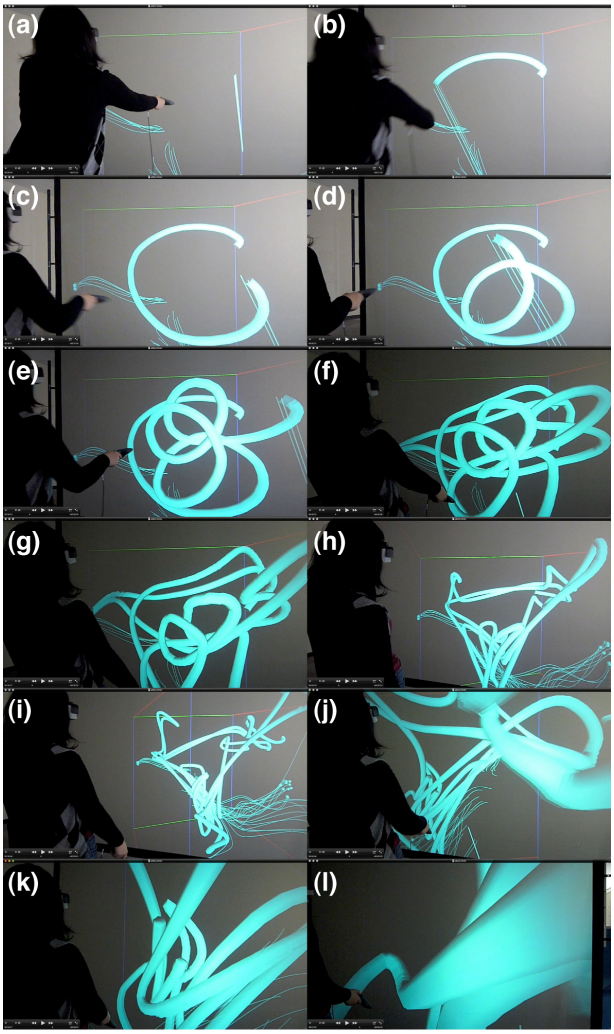

Fig. 6 Visualization of an ABC-flow with the VR timeline method in CAVE. (a) to (e) show 3-D drawing phase of the initial curve. (f) to (l) are snapshot sequence of the time development of the frozen-in field line or the timeline.

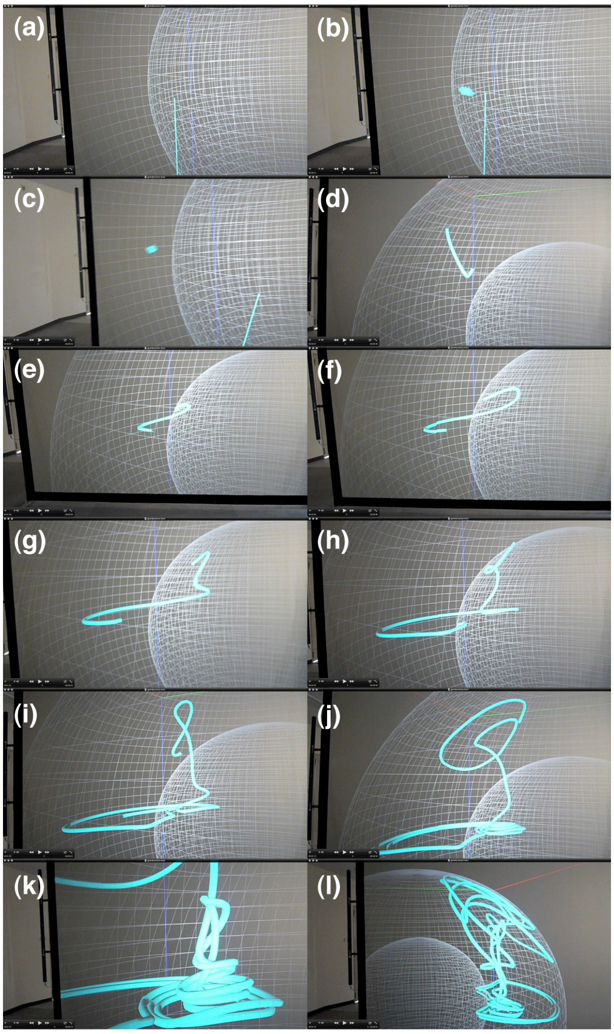

Fig. 7 Another application snapshots of the timeline method in VR. The flow data is obtained by a supercomputer simulation of an MHD convection flow in Earth's interior or the core. Time development of a magnetic field line—stretching, twisting, and folding process—in the Earth's core is visualized by the VR timeline. Initial magnetic field line that was very short in the initial condition (b) is rapidly stretched and it eventually becomes a long magnetic field line.

ingredients of the MHD dynamo process [16].

The applications of the VR timeline method in Figs. 6 and 7 indicate that this visualization method can be a powerful tool to analyze the vector fields of the flow and the magnetic field obtained by large scale computer simulations.

4. Summary

We have developed a VR timeline visualization method for scientific visualization of frozen-in vector fields, such as magnetic field in the ideal MHD and vorticity in the inviscid fluid.

Since the timeline method is essentially a 3-D visualization method, its seeding and perception require real time, interactive, and immersive view. The VR technology provides an ideal environment for them.

We have implemented the VR timeline method in VFIVE visualization framework for the CAVE-type VR systems. Applying the timeline method to an analytically defined flow (ABC flow) and numerically obtained flow (geodynamo simulation flow), we conclude that this method can be a powerful research tool for the fluid dynamics and MHD.

As expected, we have found that the VR timeline method is especially effective to analyze the stretching process the flow.

Our application experiences of this method have told us that local flow information just around the stretching point is also important. The rotation component around the stretching line is, for example, missed. We plan to improve this by adding some "toggles" or "crosses" around a timeline to visualize local flow information there.

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