Study of Radial Diffusion of Energetic Ions by High-*m* Magnetic Perturbations Using DCOM Code*)

Satohiro IZUMI and Sadayoshi MURAKAMI
Kyoto University, Kyoto 606–8501, Japan

(Received 19 January 2011 / Accepted 19 September 2011)

The radial diffusion of energetic ions is studied in the presence of high-*m* magnetic perturbation using DCOM code. Diffusion coefficients, D, are evaluated varying ion energy and strength of magnetic perturbation. The diffusion coefficient dependency of the magnetic perturbation strength is determined to be influenced by the ion energy. In addition, the ion energy dependence of D shows that $D \propto E^{1/2}$ in lower energy (~10 keV) and $D \propto E^{3/2}$ in higher energy (>10 keV) states.

Keywords: energetic ion, microscopic electromagnetic perturbation, anomalous diffusion, DCOM code

DOI: 10.1585/pfr.6.2403143

1. Introduction

A good confinement of energetic particles is required in magnetic confinement fusion reactors. Because α-particle energy is used to sustain high-temperature fusion plasma, the loss of energetic α-particles may cause serious damage to the first wall of a fusion reactor.

Recently confinement experiments of energetic ions at ASDEX-U and DIII-D suggest the existence of anomalous diffusion of off-axis injected beam particles [1, 2], and the measured beam current profile does not agree with that predicted from classical theory. It has been suggested that the microscopic electromagnetic fluctuations caused by background plasma turbulence may cause anomalous particle diffusion. However, it has long been considered that microscopic electromagnetic fluctuations do not affect the diffusion of energetic particles.

Hauff et al. [3] have investigated energetic particle behavior in a turbulent electromagnetic field obtained by a plasma micro turbulence code, GENE. Interestingly, it have been shown that the energetic particle diffusion alters near the turbulent field. In the simulation, the particle energy dependence of the diffusion coefficient has been determined, and the following results have been obtained. In the case of electrostatic fluctuation, $D \propto E^{-1}$ (passive particle) and $D \propto E^{-3/2}$ (trapped particle), in the case of magnetic fluctuation, $D \propto E^{-1/2}$ (passive particle) and D of the trapped particle is independent of E, where D is diffusion coefficient and E is particle energy.

In this study, we use DCOM (Diffusion COefficient evaluation by Monte-carlo method) code, which can calculate radial diffusion of particles by the Monte Carlo method, to study the effect of microscopic electromagnetic fluctuations on radial diffusion of energetic particles. In a tokamak configuration, whereby the major radius R is 3.6 m, the minor radius a is 0.6 m and the magnetic field at magnetic axis B is 3.0 T and the species of the test ions is α particle, we follow the particle trajectory in the presence of magnetic fluctuations and calculate the second cumulant C_2. In addition we estimate the diffusion coefficient D from the obtained C_2 and investigate the dependence of the diffusion coefficient on the particle energy.

2. Simulation Model

We assume that the magnetic field has the following form with equilibrium magnetic field $\boldsymbol{B}$ and magnetic perturbation $\delta\boldsymbol{B}$

$$\boldsymbol{B}_{\mathrm{t}} = \boldsymbol{B} + \delta\boldsymbol{B} \tag{1}$$

$$\delta\boldsymbol{B} = \nabla \times b\boldsymbol{B} \tag{2}$$

The drift motion equation of the guiding centers in the above magnetic field had been derived as [4]

$$\begin{aligned}\frac{d\psi}{dt} &= -\frac{\delta}{\gamma}\left(J\frac{\partial B}{\partial\theta} - I\frac{\partial B}{\partial\zeta}\right) \\ &\quad + \frac{e^2B^2}{\gamma m}\rho_{\parallel}\left(J\frac{\partial b}{\partial\theta} - I\frac{\partial b}{\partial\zeta}\right)\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{d\theta}{dt} &= \left(\delta\frac{\partial B}{\partial\psi} + e\frac{\partial\Phi}{\partial\psi}\right)\frac{J}{\gamma} \\ &\quad - \frac{e^2B^2}{\gamma m}\rho_{\parallel}\left(\rho_{\mathrm{c}}J' + \frac{\partial b}{\partial\psi}J - \iota\right)\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{d\zeta}{dt} &= -\left(\delta\frac{\partial B}{\partial\psi} + e\frac{\partial\Phi}{\partial\psi}\right)\frac{I}{\gamma} \\ &\quad + \frac{e^2B^2}{\gamma m}\rho_{\parallel}\left(\rho_{\mathrm{c}}I' + \frac{\partial b}{\partial\psi}I + 1\right)\end{aligned} \tag{5}$$

$$\frac{d\rho_{\parallel}}{dt} = -\frac{\delta}{\gamma}\left[\left(\iota - \rho_{\mathrm{c}}J' - \frac{\partial b}{\partial\psi}\right)\frac{\partial B}{\partial\theta} + \left(1 + \rho_{\mathrm{c}}I'\frac{\partial b}{\partial\psi}\right)\frac{\partial B}{\partial\zeta}\right]$$

author's e-mail: izumi_s@p-grp.nucleng.kyoto-u.ac.jp
*) This article is based on the presentation at the 20th International Toki Conference (ITC20).

$$-\left[\left(\delta\frac{\partial B}{\partial\psi}+e\frac{\partial\Phi}{\partial\psi}\right)\frac{J}{\gamma}\frac{\partial b}{\partial\theta}-\left(\delta\frac{\partial B}{\partial\psi}+e\frac{\partial\Phi}{\partial\psi}\right)\frac{I}{\gamma}\frac{\partial b}{\partial\zeta}\right] \tag{6}$$

$$\gamma = e(J + \iota + \rho_c(JI' - IJ'))$$

$$\delta = \mu + \frac{e^2B}{m}\rho_{\parallel}^2 \qquad \rho_c = \rho_{\parallel} + b$$

where ψ is magnetic surface function, θ is poloidal angle, ζ is toroidal angle, $\rho_{\parallel} = v_{\parallel}(m/eB)$ is Larmor radius with $v_{\parallel}$ as parallel velocity along the magnetic field lines, I is poloidal current, J is toroidal current, e is charge, m is mass of particle, Φ is electrostatic potential, ι is rotational transform, and μ is magnetic moment.

We represent the magnetic perturbation b in Eq. (2) using a Fourier series as

$$b(\psi,\theta,\zeta) = \sum_{m,n} b_{mn}(\psi)\cos(m\theta - n\zeta + \zeta_{mn}) \tag{7}$$

$$b_{mn}(\psi)/a = s_{mn}\exp\left(-\frac{(\psi-\psi_{mn})^2}{\Delta\psi^2}\right) \tag{8}$$

where ζ_{mn} is phase, a is minor radius, s_{mn} is strength of magnetic perturbation, ψ_{mn} is center of magnetic perturbation, and $\Delta\psi$ is width of magnetic perturbation.

To quantitatively estimate particle spreading, we use the second cumulant C_2 defined as

$$C_2 = \langle (r - \langle r\rangle)^2\rangle \tag{9}$$

where r is the position of particles and $\langle\rangle$ is ensemble average. In a non-uniform magnetic field with magnetic fluctuations, the time evolution of C_2 is classified using correlation time of the particle orbit Δt and characteristic time of the system T_L:

$$C_2(t) \propto \begin{cases} t^2 & : t \leq \Delta t \\ t^1 & : \Delta t \leq t \leq T_L \\ t^0 & : T_L \leq t \end{cases} \tag{10}$$

The region in which C_2 is proportional to t^2 is known as the ballistic phase, whereby particle orbits follow a dynamics law where C_2 is proportional to t^1, particle spreading is known as the diffusive phase. The diffusion coefficient D is described as

$$D = \frac{1}{2}\frac{dC_2}{dt} \tag{11}$$

The proportion of C_2 to t^0 indicates that particles have reached a steady state.

This study investigates the behavior of energetic ions, therefore we neglect the influence of the electron potential Φ the effects of which would be negligible. In addition we ignore the Coulomb collision effect because its time scale would be very long compared with that for radial diffusion by magnetic turbulence. The species of the test ions is α particle. To show statistical accuracy, the dependence of D on the number of test ions, N, is investigated. We estimate D when N is varied from $N = 2.0\times10^3$ to $N = 1.2\times10^4$ (Fig. 1). Figure 1 shows that D converges in the range from $N = 2.0\times10^3$ to $N = 1.2\times10^4$. Thus, in the following simulations, $N = 1.0\times10^4$. In this range, the relative error of D to $N = 1.0\times10^4$ reaches about 1%. Test ions are distributed at the radial position $r/a = 0.72$ at $t = 0$, where a is minor radius. We assume a tokamak magnetic configuration: Major radius R is 3.6 m, minor radius a is 0.6 m, and magnetic field at magnetic axis B is 3.0 T. We assume that the magnetic perturbation has 30 high-m Fourier modes, whereby poloidal mode m = 200 and toroidal mode n = -118 ∼ -147. In addition $\Delta\psi/\psi_a = 0.30$, and $\psi_{mn}/\psi_a = 0.52$; ψ_a is the outermost magnetic surface. In this condition, the center of magnetic perturbation almost corresponds to the radial initial position of the particles. Each ζ_{mn} of Fourier modes is chosen randomly. The strength of magnetic perturbation s_{mn} of all Fourier modes are same: $s_{mn} = s$. In the DCOM [5–7], Eq. (3)–(6) are integrated by the sixth-order Runge-Kutta method. For example, in the case of larger s ($s = 1.0\times10^{-4}$) and E = 3keV, the relative error of energy is about 3.0×10^{-8}% at the end of the calculation.

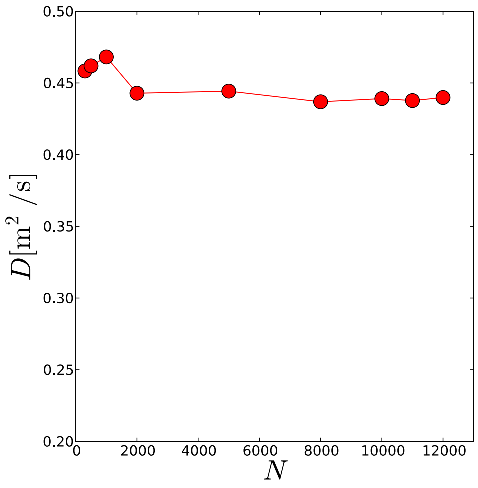

Fig. 1 Plot of the test particle number, N, dependency of the estimated diffusion coefficient.

3. Simulation Results

Figure 2 shows the time evolution of the second cumulant C_2 in which the magnetic perturbation strength s is varied from 0 to 1.0×10^{-4}. Particle energy, E, is assumed to be E = 3 keV. The magnetic field structure shows clearly nested magnetic surfaces in the case of $s = 0$. When the magnetic perturbation increase to $s = 5.0\times10^{-7}$, such magnetic surfaces start to break around the center of the magnetic perturbations, and a stochastic region can be seen from $r/a \sim 0.6$ to $r/a \sim 0.8$. The relation between s and the radial width of the stochastic region, W, is shown in Table 1. It is evident that the magnetic surfaces are broken throughout the region with further increase in s from 5.0×10^{-6}.

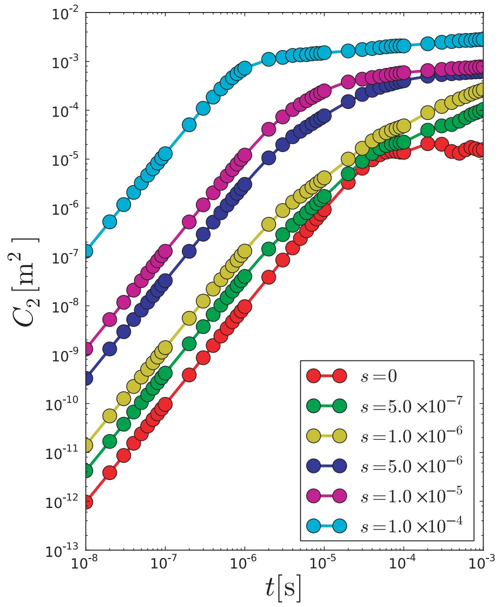

Fig. 2 Time evolutions of second cumulant C_2 of particles for various strengths of magnetic perturbation s is from 0 to 1.0×10^{-4}.

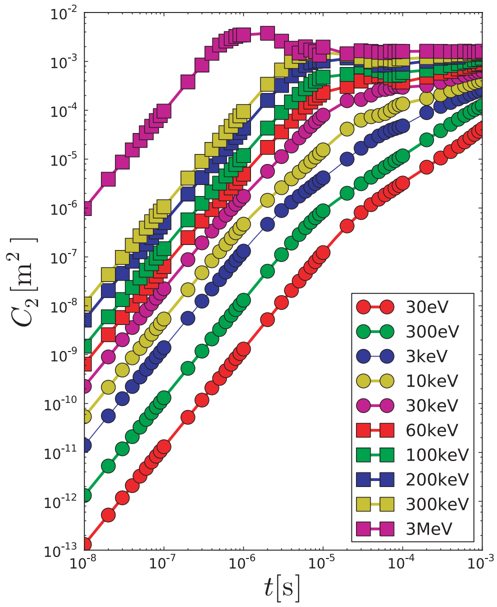

Fig. 4 Time evolutions for second cumulant C_2 of particles for various energy of particles. E is from 30 keV to 3 MeV.

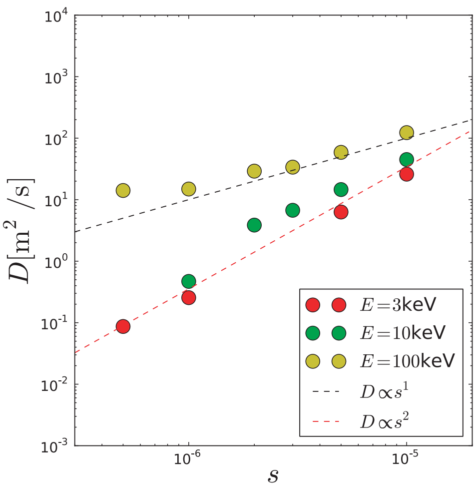

Fig. 3 Diffusion coefficients in the presence of magnetic perturbation D at particle energy $E = 3$ keV and $E = 10$ keV is proportional to s^2. On the contrary, D at $E = 100$ keV is proportional to s^1.

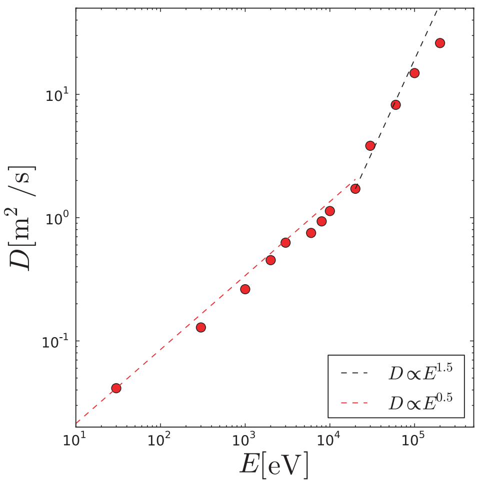

Fig. 5 Diffusion coefficient versus energy of particles. D is proportional to $E^{1/2}$(∼10 keV) and to $E^{3/2}$(10 keV∼).

In the case of $s = 0$, C_2 increases in proportion to t^2 and then becomes constant. The ballistic phase appears first, followed by the steady state. In the case of $0 < s \leq 1.0 \times 10^{-5}$, the region where C_2 is proportional to t^1 appears, we can confirm that particles diffuse in a radial direction due to magnetic turbulence during the diffusion phase. In the case of larger s ($s = 1.0 \times 10^{-4}$), the diffusive phase does not appear. Figure 3 shows the variant of diffusion coefficient D calculated by Eq. (11) due to s. We can see that D is proportional to s^2 when particle energy is 3 keV and 10 keV, and is proportional to s^1 when particle energy increases to 100 keV. These results indicate that the s dependency is influenced by the particle energy and that s dependency is weak in the energetic particle. We next consider the relation between the saturated value of C_2 and the magnetic perturbation strength. Table 1 shows the change in C_2 as s increases. The relation $C_2 \sim W^2/12$ is observed in Ref. [7]; therefore we evaluate the value $\sqrt{C_2}/W$ through the change in s. Although we obtain about twice the difference among them, a clear relation is not evident due to differences in the modes of perturbation magnetic fields. The mode number is much higher in this paper than that of Ref. [7].

Figure 4 shows the time evolution of C_2 when particle energy E varies from 30 eV to 3 MeV with a fixed value

Table 1 Magnetic perturbation and width of stochastic region.

s	W	$C_2(t=10^{-3})$	$\sqrt{C_2}/W$
5.0×10^{-7}	$0.2a$	0.1041×10^{-3}	8.502×10^{-2}
1.0×10^{-6}	$0.4a$	0.2629×10^{-3}	6.756×10^{-2}
5.0×10^{-6}	a	0.6291×10^{-3}	4.180×10^{-2}
1.0×10^{-5}	a	0.7732×10^{-3}	4.634×10^{-2}
1.0×10^{-4}	a	0.2810×10^{-2}	8.835×10^{-2}

$s = 1.0\times10^{-6}$. In the lower energy case (~30 keV), the ballistic and diffusive phases are clear. However, in the intermediate energy case (60 keV~200 keV), the duration of the diffusive phase is short and the ions reach steady state rapidly. In the higher energy case (200 keV~3 MeV), the diffusive phase disappears. Figure 5 shows the variant of D due to particle energy E. D is proportional to $E^{1/2}$ in the lower case (~10 keV): D is constantly proportional to $E^{3/2}$ in the higher region (>10 keV).

4. Discussion

In [8], the diffusion coefficient D for the lower energy particle,

$$D \equiv \frac{(\Delta r)^2}{2\Delta t} \tag{12}$$

is estimated where Δt is the time at which particle orbit loses the dynamic correlation, and Δr is the radial displacement of particles at Δt conditions. In collisionless plasma, the approximate Δt is τ_c, where τ_c is the transit time of the passing particle given as $\tau_c \sim L_c/v_{\|}$ where $L_{\|}$ is connection length ($\sim \pi qR$). Δr is estimated as $\Delta r \sim (\delta B/B)L_{\|}$ where $L_{\|}$ is the correlation length of magnetic perturbation in the direction of equilibrium magnetic field and is approximately L_c. Because $\delta B/B \sim am_{\mathrm{pol}}s/r$ Δr becomes $\Delta r \sim (am_{\mathrm{pol}}s)/rL_{\|}$ where m_{pol} is the poloidal mode number. Finally Eq. (12) can be represented as

$$D \sim \frac{(\Delta r)^2}{2\tau_c} \sim \frac{v_{\|}}{2L_c}\left(\frac{am_{\mathrm{pol}}}{r}L_{\|}\right)^2 s^2. \tag{13}$$

Substituting $v_{\|} \sim v = \sqrt{2E/m}$ in Eq. (13), the dependence on s and E of D can be obtained as

$$D \propto s^2, \qquad D \propto E^{1/2}. \tag{14}$$

The results of the lower energy cases agree with this dependence, while those of the higher energy cases do not. Such discrepancy is related to the orbit size of energetic particle.

The banana width of higher energy particles is much larger than that of lower energy particles. The banana width Δ_b is given as

$$\Delta_b \sim \left(\frac{R}{r}\right)^{1/2}\left(\frac{2\pi}{\iota}\right)\rho_\Omega, \qquad \rho_\Omega = \frac{mv_\perp}{qB}. \tag{15}$$

In the case of $E = 3$ keV and 100 keV, for example, normalized Δ_b/a by minor radius a is 1.8×10^{-2} and 1.0×10^{-1} respectively. thus the particle affected by different magnetic field perturbations during one bounce motion, which may change the diffusion process in the case of $E = 100$ keV. Further study is necessary to understand the alteration of parameter dependencies in the high-energy particles.

5. Conclusion

To study the anomalous transport mechanism of energetic particles, we have investigated the diffusion of energetic particles in the presence of high-m magnetic perturbations using DCOM. We have investigated the time evolution of the second cumulunt C_2 with specific magnetic perturbation strength s and estimated the diffusion coefficient D. We have determined that D is proportional to s^2 for the lower energy particle and to s^1 for the higher energy particle. Moreover, we have investigated the dependence of diffusion coefficient D on particle energy E: $D \propto E^{1/2}$ in lower energy (~10 keV) and $D \propto E^{3/2}$ in higher energy (10 keV~). The obtained results for the higher energy particle do not agree with the previous results [8], and the energy dependence as reported in [3] could not be obtained. In 200 keV ~ 3 MeV, the spreading of particles did not reach the diffusive phase; therefore we could not calculate the diffusion coefficient. Thus further study is necessary for the energetic ion diffusion by a new approach which can measure the non-diffusive process.

[1] S. Günter *et al.*, Nucl. Fusion **34**, 535 (2007).
[2] W. W. Heidbrink *et al.*, Phys. Rev. Lett. **103**, 175001 (2009).
[3] T. Hauff *et al.*, Phy. Rev. Lett. **102**, 075004 (2009).
[4] Roscoe B. White, *The Theory of TROIDALLY CONFINED PLASMA (Second Edition)* (Imperial College Press, 2001).
[5] A. Wakasa *et al.*, Jpn. J. Appl. Phys. **46**, 1157 (2007).
[6] A. Malucov *et al.*, Contrib. Plasma Phys. **43**, 198 (2003).
[7] A. Malucov *et al.*, Physica A **322**, 13 (2003).
[8] M. Okamoto and N. Nakazima, J. Plasma Fusion Res. **78**, 885 (2002).