Comparative Study of Magnetic Field Configurations of LHD and CHS based on the Boundary Shape Analysis[*]

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In order to understand the confinement characteristics of different types of stellarators from the common point of view, characteristics of the geometric shape of the last closed magnetic surfaces are analyzed in terms of the Fourier modes. Magnetic configurations of LHD and CHS are compared for their different operational modes with shifted magnetic axis positions. Total number of Fourier modes is reduced for their outward shifted configurations to find out the essential modes for creating magnetic well. It is shown that the helical axis structure is important for the control of magnetic field configuration even for the planar-axis stellarators (heliotrons) of LHD and CHS.

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1. Inroduction

Recent scientific achievements in LHD experiments [1] are very important in the toroidal confinement research for fusion energy developments. Stable high beta plasmas of 5% averaged beta and very high density plasmas above 10^{20} m^{-3} range with 3 tesla magnetic field are strongly supporting data for the design of alternative concepts to tokamak-type reactors. In the present situations of stellarator experiments such as no new large-scale experiment has been initiated in the last decade, LHD has been a leading helical experiment and will continue to producing important data alone for stellarator research. Such condition raises the problem that the physical discussions of the experimental data are made within a very limited scope of the magnetic configurations, that is, for only small range of configurations realized in LHD device.

LHD has a unique helical magnetic configuration with a planar magnetic axis while other stellarators in the world have non-planar magnetic axis configurations [2] except a couple of old devices in Russia. In order to improve the understandings of confinement properties obtained in LHD experiments to the more general understandings of stellarator confinement, we need to incorporate discussions of confinement in different magnetic configurations. Although we hope to achieve finally comprehensive understandings for all stellarators, the first step should be comparisons of experimental data with other planar-axis stellarators. From such a point of view, CHS device is an appropriate target because it is in the same group of planar-axis heliotrons as LHD and we have lots of experimental data obtained in the past, which we can compare with LHD data.

When we explain device characteristics of planar-axis stellarators, we simply give helical mode number, helical pitch parameters, a number of toroidal periods, pitch modulation parameters, etc., all of which are just helical coil winding parameters. Since the magnetic configurations are created with a combination of helical coil and poloidal coil magnetic field, real characteristics of magnetic configurations are not described with these numbers. In this paper, we compare configurations of LHD and CHS in terms of boundary shapes of the last closed magnetic surfaces (LCMS), which should be sufficient to give all physical characteristics of magnetic configurations when the plasma pressure and the plasma current profiles are given. For comparison of two devices, vacuum magnetic field configurations are discussed in this paper.

2. Device Parameters

Nominal values of helical coils for both devices are listed in Table 1. Because the position of the plasma boundary is determined by the location of the ergodic layer, the important characteristics of magnetic surfaces, e.g., the aspect ratio and the rotational transform are not directly given from those numbers. However there is one important number α^* which gives modulation of helical coil winding law as the following formula:

$$\theta = \frac{N}{\ell}\phi + \alpha^* \sin\left(\frac{N}{\ell}\phi\right),$$

where θ and ϕ are the poloidal and toroidal angles along the helical coil winding guide line respectively. Such a modulation of the helical coil winding law makes the in-

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Table 1   Device parameters for LHD and CHS.

	LHD	CHS
Helical mode: ℓ	2	2
Number of toroidal periods: N	10	8
Major radius: R	3.9 m	1.0 m
Helical coil winding radius: Rh	0.975 m	0.313m
Pitch parameter: γ_c	1.25	1.25
Pitch modulation: α^*	0.1	0.3

Fig. 1   Magnetic surfaces of LHD and CHS for symmetric positions.

clination of the helical coil more vertical on the outboard side of the torus. The effect of this coil shape modulation on the magnetic surface shape will be discussed in the following sections. Another apparent difference between helical coil shapes of LHD and CHS is the polarity of helicities, although it does not give any change of physical confinement characteristics.

Sets of poloidal coils are very similar for both devices. Three pairs of upper and lower circular coils with different major radii are installed as a poloidal coil assembly with flexibility of vertical field control and plasma shaping.

3. Boundary Shape of LCMS

Big advantage of stellarator research compared to tokamaks is a large flexibility of available magnetic field configurations in three dimensional (3-D) space. However this flexibility is strongly limited for individual devices because it is very hard, after manufacturing devices, to change shapes of main helical coils (modular coils), which are the source of 3-D magnetic field structure. The remaining flexibility is the axisymmetric poloidal field control given by tuning currents in poloidal coils. Fortunately this small remaining knob for the magnetic field control gives a large flexibility of the device to change the confinement characteristics.

It has been well known that the favorable drift orbits of confined particles are created for one of the magnetic configurations of LHD (inward shifted configuration) with the strong vertical field pushing the magnetic surfaces toward the torus center. On the other hand, the favorable MHD stability is produced with the magnetic well formation in the outward shifted configuration. Since the position of the magnetic axis is more sensitive to the vertical field than the position of the boundary, the position of the magnetic axis in the major radius R_{ax} is used in LHD and CHS as a parameter to distinguish these different configurations. Since we are comparing two devices on the common physical aspect, we normalize R_{ax} with the device major radius R given in Table 1, which gives a non-dimensional number R_0. We will analyze the difference of boundary shapes for configurations of different values of R_0 and compare these features for two devices of LHD and CHS.
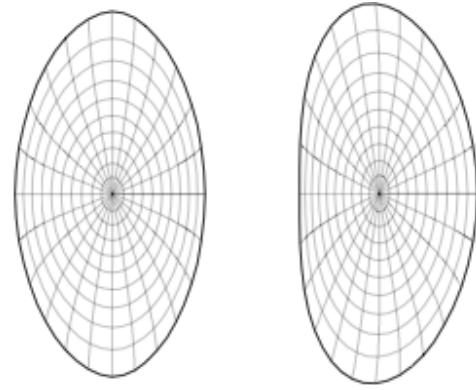
For analyzing the boundary shape, we use the Fourier decomposition of the 3-D torus boundary used in the VMEC equilibrium solver [3], which is expressed as following formulas:

$$R(\theta, \phi) = \sum \text{rbc}(m, n) \cdot \cos(m\theta - n\phi),$$
$$Z(\theta, \phi) = \sum \text{zbs}(m, n) \cdot \sin(m\theta - n\phi),$$

θ and ϕ are two (poloidal and toroidal) angle parameters mapped on the boundary surface (θ is specially defined as to minimize the range of mode spectra). R and Z are values of coordinate of each point of the boundary surface (with angle parameters of θ and ϕ) in a cylindrical coordinate system for torus configuration. rbc and zbs are the two-dimensional Fourier coefficients for the cosine and sine expansions of R and Z, respectively.

As a reference configuration among various shifted ones, we take a configuration with the magnetic axis position almost centered in the magnetic surface (we call here 'symmetric' for no shift neither inward or outward). For LHD, it is $R_0 = 0.961$ configuration ($R_{ax} = 3.75$ m), while for CHS, it is $R_0 = 0.921$ configuration ($R_{ax} = 0.921$ m). The difference of R_0 values comes from different helical coil pitch modulation factor α^*. Vacuum magnetic surfaces of two devices at the toroidal position of vertically elongated cross section are shown in Fig. 1. Because we are interested in the shape of the magnetic surfaces, the difference of size (about 3 times different) is neglected.

In the equilibrium calculations based on the boundary shape, more than 100 Fourier modes are normally included. However, when we discuss the basic confinement characteristics of configurations, a relatively small number of dominant modes are important. Figure 2 shows a distribution of amplitudes of Fourier modes of LCMS for $R_0 = 0.961$ configuration of LHD. Since the variation of amplitude is large, amplitudes are plotted in the logarithmic scale and abstract values are used eliminating sign of values. Except limited number of dominant modes, amplitudes of many modes are smaller than those modes by more than one order of magnitude.
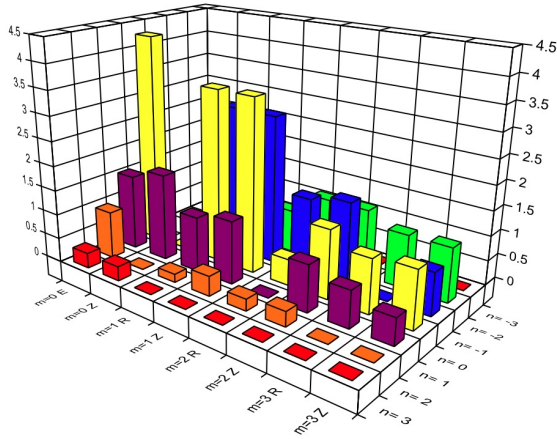
Fig. 2 Distribution of Fourier modes of boundary shape of LHD symmetric configuration. The $\log_{10}$ values of absolute amplitudes are plotted.

Table 2 Comparison of dominant Fourier modes of LCMS shape of LHD and CHS. Amplitudes are normalized to the component rbc(0, 0) corresponding to major radius.

Fourier mode	LHD	CHS
rbc(0, 0)	1.0000	1.0000
rbc(0, 1)	0.0018	-0.0062
zbs(0, 1)	0.0030	0.0099
rbc(1, -1)	-0.0511	0.0027
zbs(1, -1)	0.0514	0.0040
rbc(1, 0)	0.1651	0.2010
zbs(1, 0)	0.1666	0.2234
rbc(1, 1)	-0.0007	-0.0636
zbs(1, 1)	0.0010	0.0681
rbc(2, -1)	-0.0018	-0.0009
zbs(2, -1)	0.0026	-0.0001
rbc(2, 0)	-0.0002	0.0008
zbs(2, 0)	-0.0013	0.0036
rbc(2, 1)	0.0000	0.0078
zbs(2, 1)	0.0005	0.0058
rbc(2, 2)	-0.0001	0.0022
zbs(2, 2)	-0.0001	0.0020

We compare LHD and CHS magnetic configuration based on those dominant modes listed in Table 2. Mode amplitudes are normalized to the component rbc(0, 0) which corresponds to the major radius. Components of rbc(1, 0) and zbs(1, 0) give (normalized) minor radius and their ratio shows toroidally averaged ellipticity. The small difference of ellipticity is the result of different selection of poloidal current settings in LHD and CHS experiment. Helical structure is given by the components of rbc(1, −1) and zbs(1, −1) for LHD and rbc(1, 1) and zbs(1, 1) for CHS since the polarity of helicity is opposite. The difference of magnetic surface shape in Fig. 1 comes from the relatively larger amplitudes of CHS for rbc(2, n) and zbs(2, n) com-

Table 3 Comparison of minimum number of Fourier modes of modified $R_0 = 0.995$ and $R_0 = 0.921$ configuration.

Fourier mode	Modified $R_0 = 0.995$	$R_0 = 0.921$
rbc(0, 0)	1.0000	1.0000
rbc(0, 1)	-0.0259	-0.0062
zbs(0, 1)	0.0304	0.0099
rbc(1, -1)	0.0001	0.0027
zbs(1, -1)	0.0022	0.0040
rbc(1, 0)	0.2010	0.2010
zbs(1, 0)	0.2234	0.2234
rbc(1, 1)	-0.0661	-0.0636
zbs(1, 1)	0.0674	0.0681
rbc(2, 1)	0.0015	0.0078
zbs(2, 1)	-0.0002	-0.0058

ponents, which is the effect of different pitch modulation factor α^* for the helical windings.

4. Creation of Magnetic Well in CHS

For high beta equilibrium, magnetic well is created by the Shafranov shift in LHD and CHS. Magnetic well is also created for vacuum configuration when the magnetic axis is shifted outward. By analyzing Fourier modes, we investigate what type of boundary shape modification is responsible for the creation of the magnetic well. For the first step of analysis, the outward shifted configuration ($R_0 = 0.995$) is modified to have the same major radius and the aspect ratio to the symmetric one ($R_0 = 0.921$) for the purpose of excluding the real geometric effect and focusing on only the boundary shape. This modification is done by replacing the rbc(0, 0), rbc(1, 0) and zbs(1, 0) components of $R_0 = 0.995$ configuration with those of $R_0 = 0.921$.

During the modification is made, we should be careful to modify other non-axisymmetric components in proportion to the change of aspect ratio because the effect of these modes, for example on the profile of rotational transform, depends directly on the aspect ratio. Through such a modification of Fourier components, the magnetic well and the rotational transform profile could be kept almost unchanged. Then we reduce the number of Fourier modes to find out the minimum number of modes necessary to create the magnetic well. Table 3 shows the comparison of limited number of Fourier modes for the modified $R_0 = 0.995$ configuration and $R_0 = 0.921$ configuration.

It is obvious that the larger Fourier modes of rbc(0, 1) and zbs(0, 1) for modified $R_0 = 0.995$ configuration are the dominant differences between two configurations. Actually if we eliminate modes with smaller amplitudes than these modes (including them) having only 5 dominant modes, modified $R_0 = 0.995$ configuration lose magnetic well. The configuration formed by 7 modes with rbc(0, 1) and zbs(0, 1) components recovers the magnetic well. In
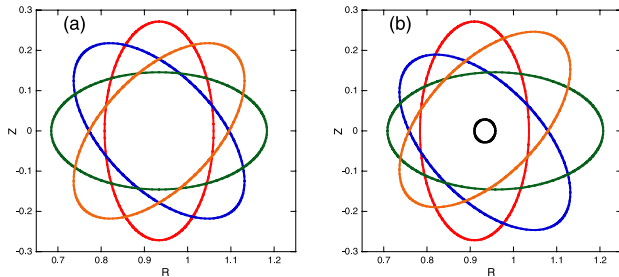
Fig. 3   Cross sections of boundary shape at four different toroidal positions: (a) $R_0 = 0.995$ configuration without helical axis components rbc(1, 0) and zbs(1, 0). (b) the same with helical axis components. The helical excursion of magnetic axis is also plotted in (b) (a small circle at the center).

terms of boundary shape, Fourier modes (0, 1) represent helical axis structure or, in other word, non-planar axis structure. Figure 3 shows the comparison of shape of cross sections of LCMS at four toroidal positions for two modified $R_0 = 0.995$ configurations with and without helical axis components. Figure 3 (b) shows also the excursion of magnetic axis.

It is generally understood that the helical axis structure of stellarators is favorable to create the magnetic well while the straight helical configuration is intrinsically magnetic hill. It should be noted that the toroidal phase of helical structure is important [4]. In Fig. 3 (b), the horizontally elongated cross section is located relatively rightward shifted to the vertically elongated one. If it is oppositely located (leftward shifted) with the opposite sign of helical structure Fourier mode, the magnetic well is not created.

5. Creation of Magnetic Well in LHD

In LHD case, a similar procedure was taken for the outward shifted configuration with $R_0 = 1.000$ to reduce the number of Fourier modes keeping the existance of the vacuum magnetic well. As well as CHS case, when the Fourier modes are reduced to 5 components, the magnetic well disappears. Then we add the helical axis structure by recovering rbc(0, 1) and zbs(0, 1) modes. This modification gives very shallow magnetic well but the deep well is not created. The difference in LHD case compared with CHS is that rbc(2, 1) and zbs(2, 1) modes are necessary to recover magnetic well for outward shifted case. In terms of boundary shape, these modes give D shape to the cross section. Because the toroidal mode number of these components is not vanishing ($n = 1$), the orientation of D shape is rotating along the toroidal angle. The D shape similar
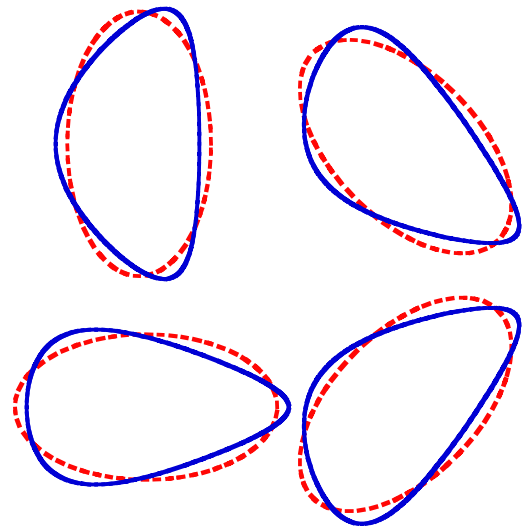

Fig. 4   Cross sections of LCMS of modified LHD outward shifted configuration for four toroidal positions. Dotted lines are cross sections without (2, 1) components and solid lines are the ones including them. Amplitude of (2, 1) components are artificially enhanced by 5 times for easier recognition of the modification of shapes in small figures.

to tokamak case is given at the toroidal angle where the cross section of LCMS is horizontally elongated. Figure 4 shows the effect of (2, 1) mode on the boundary shape at four toroidal positions by comparing cross sections without and with these Fourier modes.

6. Conclusion

Vacuum magnetic configurations of LHD and CHS are analyzed based on their shapes of the last closed magnetic surfaces for different magnetic axis positions. Although there is a difference of pitch modulation parameter α^* between them, basic characteristics of inward and outward shifted configurations are similar for normalized parameter R_0. For outward shifted configurations, helical axis structure is important one for creating the magnetic well in CHS while the rotating D shape components are necessary in LHD as well as the helical axis components. The analysis of the boundary shape for inward shifted configurations will be reported in a separate paper.

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