Numerical Investigation on Accuracy Improvement of Permanent Magnet Method for Measuring j_C in High-Temperature Superconducting Film*)

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Possibility of the accuracy improvement in the permanent magnet method for measuring the critical current density of a high-temperature superconducting (HTS) thin film has been investigated numerically. To this end, a numerical code has been developed for analyzing the shielding current density in an HTS sample. By using the code, the permanent magnet method has been reproduced numerically. The results of computations show that, by using the magnet strength B_F as large as possible, the high accuracy can be assured with little effort. Furthermore, in order to improve the measurement accuracy of the critical current density near the film edge, it is necessary to use the magnet with the smallest radius.

Keywords: current measurement, high-temperature superconductor, simulation, thin film

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1. Introduction

As is well known, flywheel systems and fusion reactor systems have been developed with high-temperature superconductors (HTSs), and HTS materials have various characteristic parameters. Especially, a critical current density j_C is one of the most important parameters of the HTS. Therefore, it is necessary to accurately measure the critical current density j_C in an HTS sample.

The standard four-probe method has been generally used for measuring a critical current density j_C of the HTS sample. However, this method may leads not only to the destruction of a sample surface but also to the degradation of superconducting characteristics. For this reason, a noncontact method has been so for desired for measuring the critical current density. Currently, the popular method is the inductive method proposed by Claassen *et al*. [1].

As a novel noncontact method for measuring j_C, Ohshima *et al*. proposed the permanent magnet method for measuring the critical current density j_C in a HTS thin film [2, 3]. While moving a permanent magnet placed above an HTS thin film, they measure an electromagnetic force acting on the film. Consequently, they found that the maximum repulsive force F_M is roughly proportional to the critical current density j_C. This means that j_C can be estimated by measuring F_M. Recently, j_C-distributions are evaluated by using this method [4].

In order to numerically reproduce the permanent magnet method, a numerical code has been developed by analyzing the time evolution of a shielding current density in the HTS thin film [5]. By using the code, the results of computations indicate that the measurement accuracy is degraded near the HTS film edge. However, it is found that the maximum repulsive force is roughly proportional to the critical current density j_C [5] near the film edge. In conclusion, the critical current density j_C near the film edge can be estimated from the proportionality constant determined in advance from the resulting j_C-F_M line. However, the determination of proportionality constant wastes time because it is necessary to calculate the proportionality constants as a function of the magnet position. Therefore, the accuracy improvement is desired near the film edge.

The purpose of the present study is to investigate the possibility of the accuracy improvement by changing the operating conditions in the permanent magnet method.

2. Governing Equations

In Fig. 1, we show a schematic view of a permanent magnet method. A cylindrical permanent magnet of the radius r_m and the height h_m is placed above a square-shaped HTS thin film of the length a and the thickness b. Furthermore, we adopt the Cartesian coordinate system $\langle O : \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z \rangle$, where z-axis is the thickness direction, and the origin O is the centroid of an HTS film.

A distance L between a magnet bottom and a film surface is controlled as follows:

1. From $L = L_{max}$ to $L = L_{min}$, the magnet is moved to

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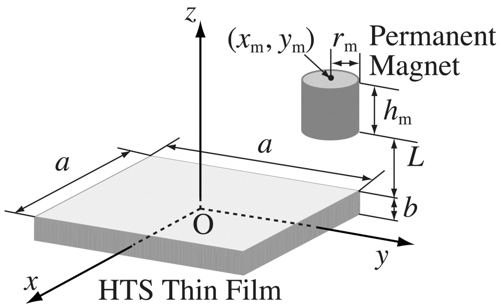

Fig. 1 A schematic view of a permanent magnet method.

the film at the constant speed $v = (L_{max} - L_{min})/\tau_0$. Here, τ_0 is a constant.

2. From $L = L_{min}$ to $L = L_{max}$, the magnet is moved away from the film at the same speed v.

For determining a magnet strength, we adopt a magnetic flux density B_F at for the case with $L = L_{min}$ and $(x, y, z) = (0, 0, b/2)$. In addition, the xy-coordinates the magnet center is defined by (x_m, y_m).

As usual, we assume that the thin-layer approximation [6]: since the thickness of the HTS is sufficiency thin, a shielding current density can hardly flow in the thickness direction. In the following, an HTS film cross-section passing through z = const. and its boundary are denoted by Ω and $\partial\Omega$, respectively.

Under the above assumptions, a shielding current density in an HTS is written as $\boldsymbol{j} = (2/b)\nabla S \times \boldsymbol{e}_z$, and the behavior of the scalar function $S(\boldsymbol{x}, t)$ is governed by the following integro-differential equation [6]:

$$\mu_0 \frac{\partial}{\partial t}\left[\int_\Omega d^2\boldsymbol{x}' Q\left(|\boldsymbol{x} - \boldsymbol{x}'|\right) S\,(\boldsymbol{x}', t) + \frac{2}{b} S(x, t)\right] + \frac{\partial}{\partial t}\langle \boldsymbol{B}\cdot\boldsymbol{e}_z\rangle + (\nabla\times\boldsymbol{E})\cdot\boldsymbol{e}_z = 0. \quad (1)$$

Here, $\boldsymbol{x}$ is defined by $\boldsymbol{x} \equiv x\boldsymbol{e}_x + y\boldsymbol{e}_y$. In addition, am angle brackets $\langle\ \rangle$ are an average operator over the thickness of the HTS, and it is given by

$$\langle f\rangle \equiv \frac{1}{b}\int_{-b/2}^{b/2} f dz. \quad (2)$$

Incidentally, the explicit form of a function $Q(\boldsymbol{x}, \boldsymbol{x}')$ is described in Ref. [6].

The shielding current density $\boldsymbol{j}$ is closely related to the electric field $\boldsymbol{E}$. The relation is expressed by the J-E constitute equation: $\boldsymbol{E} = E(|\boldsymbol{j}|)[\boldsymbol{j}/|\boldsymbol{j}|]$. As a function $E(j)$, we use the power law: $E(j) = E_C[j/j_C]^N$, where E_C is a critical electric field and N is a constant.

For the initial and boundary conditions to (1), we assume $S = 0$ at $t = 0$ and $S = 0$ on $\partial\Omega$. By solving the initial-boundary problem of (1), we can obtain the time evolution of a shielding current density. Throughout the present study, the physical and geometric parameters are fixed as follows: $a = 40$ mm, $b = 200$ nm, $y_m = 0$ mm, $h_m = 3$ mm, $\tau_0 = 39$ s, $L_{max} = 20$ mm, $L_{min} = 0.5$ mm, $E_C = 0.1$ mV/m, and $N = 20$.

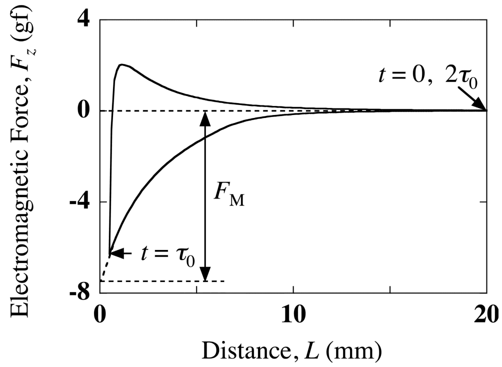

Fig. 2 Dependence of the electromagnetic force F_z on the distance L for the case with $x_m = 0$ mm, $r_m = 2.5$ mm, $B_F = 0.2$ T, and $j_C = 4.5$ MA/cm^2.

3. Numerical Results

In order to simulate the permanent magnet method, a numerical code has been developed for solving the initial-boundary problem of (1). The code can be executed by specifying an assumed critical current density j_C. In the present study, we use the assumed value, 0.1 MA/cm^2 $\leq j_C \leq$ 10 MA/cm^2, where the assumed values of j_C derives from Ohshima's experimental results [2, 3].

In Fig. 2, we show the dependence of the electromagnetic force F_z on the distance L. We see from this figure that the F_z has a hysteresis curve. A repulsive force is acting on the film for $0 \leq t \leq \tau_0$, whereas an attractive force occurs for $\tau_0 < t \leq 2\tau_0$. By extrapolating the repulsive force, we can evaluate the electromagnetic force at $L = 0$ mm (see Fig. 2). Hereafter, this force is called the maximum repulsive force, and it is denotes by F_M.

3.1 Influence of magnet strength on accuracy

Let us first investigate the influence of the relation between the critical current density j_C and the maximum repulsive force F_M for the various strength B_F. In Fig. 3, we indicate that the dependence of the critical current density j_C on the maximum repulsive force F_M. We see from this figure that, when the magnet strength is $B_F = 0.01$ T and 0.03 T, the proportional relations between j_C and F_M do not hold for the case with a high j_C. The reason is as follows: in order to hold the proportional relation between j_C and F_M, an inequality $\underset{\boldsymbol{x}\in\Omega}{\mathrm{Max}}\,|\boldsymbol{j}(\boldsymbol{x})|/j_C \geq 1$ must be satisfied numerically. In other words, the magnetic shielding of the HTS is broken down and, simultaneously, the HTS has the normal conducting state. The results of the computations show that, for $B_F = 0.01$ T and 0.03 T, the inequality is not satisfied. On the other hand, j_C is almost proportional to F_M for the case with $j_C \lesssim 3.0$ MA/cm^2 for each B_F. As a result, for $j_C \lesssim 3.0$ MA/cm^2, a proportionality constant can be obtained even when the magnet strength B_F is low.

In the present study, a proportionality constant is determined from j_C-F_M line for $j_C \leq 1.0$ MA/cm^2 by using the least-square method. Therefore, an estimated formula

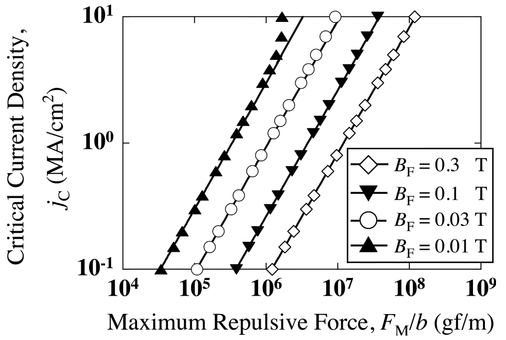

Fig. 3 Dependence of the critical current density j_C on the maximum repulsive force F_M for the case with $x_m = 0$ mm and $r_m = 2.5$ mm. Here, the solid lines show the estimated value of the critical current density j_C^*.

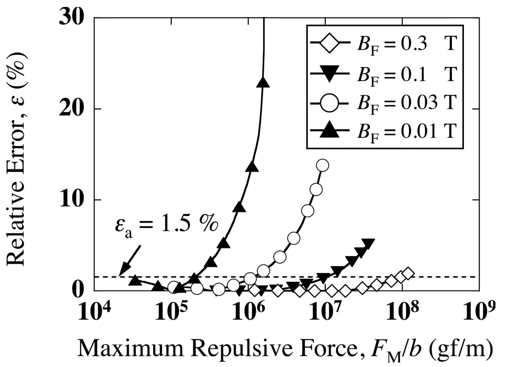

Fig. 4 Dependence of the relative error ε on the maximum repulsive force F_M for the case with $x_m = 0$ mm and $r_m = 2.5$ mm.

of the critical current density j_C can be defined as

$$j_C^* \equiv K(B_F, r_m)(\mathrm{MA\cdot cm^{-1}\cdot gf})(F_M/b), \tag{3}$$

where K is the proportionality constant.

Next, in order to quantitatively examine the difference between j_C^* and j_C, we define a relative error: $\varepsilon \equiv |j_C^* - j_C|/|j_C|$. In Fig. 4, we show that the dependence of the relative error ε on the maximum repulsive force F_M. We see from this figure that the relative error ε increases with F_M/b. In particular, the relative error ε drastically increases with decreasing B_F. From these results, we assume that there exists the limit of the j_C-measurement for each B_F.

Finally, we investigate the limit of the maximum repulsive force F_M. For this purpose, we use the acceptable error $\varepsilon_a = 1.5\,\%$ (see Fig. 4) and, subsequently, we determine the limit of the maximum repulsive force F_M^* from ε_a. In Fig. 5, we indicate the limit of the maximum repulsive force F_M^* on the magnet strength B_F. From this result, this figure can serve as a guide to the measurement accuracy j_C in the permanent magnet method. In fact, if a measured F_M for a certain B_F satisfy the inequality $F_M \leq F_M^*$, the accuracy with $\varepsilon \leq 1.5\,\%$ is assured. For the case with $F_M > F_M^*$, it is only necessary to remeasure F_M by changing the strength B_F. We conclude that, by using the magnet strength B_F as large as possible, the high accuracy can be assured with little effort. However, it is found that, by changing the strength B_F, the accuracy improvement near the film edge cannot be achieved.

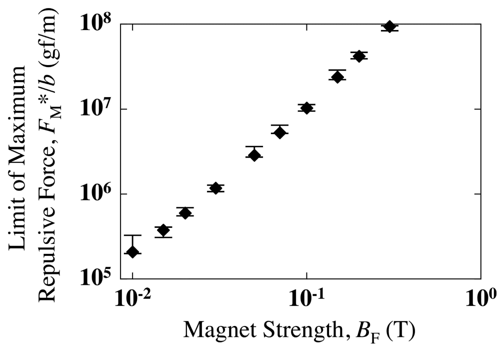

Fig. 5 Dependence of the limit of maximum repulsive force F_M^* on the magnet strength B_F for the case with $x_m = 0$ mm and $r_m = 2.5$ mm.

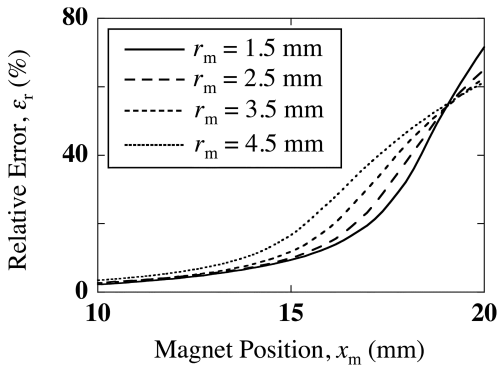

Fig. 6 Dependence of the relative error ε_r on the magnet position x_m.

3.2 Accuracy improvement

In this subsection, we investigate whether the radius r_m of the magnet affect the measurement accuracy of the critical current density j_C. To this end, we assume that an HTS film has a uniform j_C-distribution, and j_C and B_F is fixed as $j_C = 4.5$ MA/cm^2 and $B_F = 0.3$ T, respectively. Therefore, the proportional relation between j_C and F_M is given by

$$j_C^N = \alpha(r_m)(\mathrm{MA\cdot cm^{-1}\cdot gf})(F_M/b). \tag{4}$$

For the parameters in the present study, the proportionality constants are $\alpha\,(1.5\,\mathrm{mm}) = 1.82 \times 10^{-8}$; $\alpha\,(2.5\,\mathrm{mm}) = 8.23 \times 10^{-8}$; $\alpha\,(3.5\,\mathrm{mm}) = 4.64 \times 10^{-9}$; $\alpha\,(4.5\,\mathrm{mm}) = 2.91 \times 10^{-9}$. By substituting the resulting maximum repulsive force F_M into (4), an estimated value of the critical current density is determined.

Let us investigate the influence of the radius r_m on the accuracy. As a criterion of the accuracy, we define a relative error: $\varepsilon_r \equiv |j_C^N - j_C|/|j_C|$. In Fig. 6, we show the dependence of the relative error ε_r on the magnet position x_m for the various radius r_m. This figure indicates that the relative

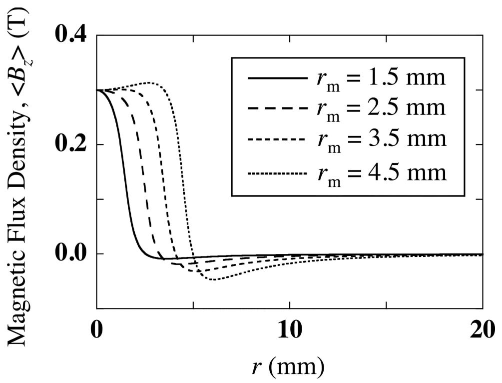

Fig. 7 The spatial distributions of the magnetic flux density $\langle B_z \rangle$ for the case with $x_m = 0$ mm.

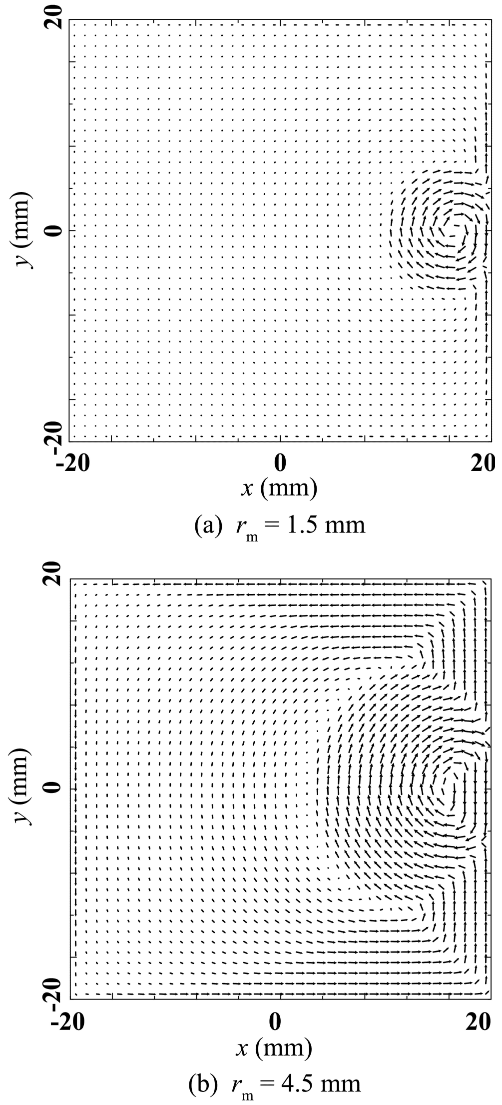

Fig. 8 The spatial distributions of the shielding current density for the case with x_m = 17 mm.

error ε_r increases with the magnet position x_m. Moreover, ε_r decreases with r_m for $x_m \lesssim 19$ mm (e.g. for the case with $x_m = 17$ mm, the relative error is $\varepsilon_r = 19.7$ % for r_m = 1.5 mm; $\varepsilon_r = 36.5$ % for $r_m = 4.5$ mm). Consequently, the measurement accuracy of j_C near the film edge improves by using a smaller radius r_m.

Let us numerically investigate why the measurement accuracy of j_C degrades remarkably. In Fig. 7, we show the spatial distributions of the magnetic flux density $\langle B_z \rangle$ for the various radius r_m. Here, r denotes the r-coordinate for the cylindrical coordinate (r, θ, z). This figure shows that the applied area of the magnetic flux density $\langle B_z \rangle$ decreases with the radius r_m. As a result, the area of the shielding current density decreases with radius r_m (see Fig. 8). This figure shows that, for r_m = 1.5 mm, the distribution the shielding current density almost has axisymmetric. On the other hand, for r_m = 4.5 mm, $\boldsymbol{j}$ has the strongly nonaxisymmetric distribution, because an HTS edge significantly affects the distribution of $\boldsymbol{j}$. Consequently, in order to enhance the measurement accuracy of j_C near the film edge, it is necessary to use the magnet with the smallest radius.

4. Conclusion

We investigate the possibility of the accuracy improvement by changing the operating conditions. Conclusions obtained in the present study are summarized as follows:

1. By using the magnet strength B_F as large as possible, the high accuracy can be assured with little effort. As a result, it may be possible to reduce the time for measuring the critical current density j_C.
2. In order to enhance the measurement accuracy of j_C near the film edge, it is necessary to use the magnet with the smallest radius.

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