Investigation of Trigger Mechanism in the Explosive Nonlinear Growth of the Double Tearing Mode

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Magnetic reconnection dynamics due to the nonlinearly destabilized double tearing mode (DTM) is simulated, focusing on the nonlinear growth phase in the framework of reduced resistive magnetohydrodynamics (MHD). The nonlinearly explosive growth of the DTM accompanying fast magnetic reconnection is found to result from a secondary instability, the mechanism of which consists of the sequential unstable modulation due to two- dimensional distortion of magnetic islands and modification of the nonlinear current profile. The trigger dynamics of the nonlinear growth phase is illustrated via the investigation of the evolution of both the kinetic and magnetic energies of the secondary instability.

Keywords: magnetic reconnection, double tearing mode, abrupt growth, modulational instability, current corrugation

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1. Introduction

Reversed shear of the q-profile in tokamaks is believed to be a key to high plasma confinement and an important issue for the success of fusion energy. The reason is that this configuration is associated with the formation of internal transport barriers, leading to better confinement of the core. However, it has been experimentally observed that the reversed shear configuration can also excite magnetohydrodynamic (MHD) instabilities and disrupt the plasma [1]. Thus, a detailed understanding of these instabilities is necessary. The double tearing mode (DTM) is one of the associated MHD instabilities and consists of two co-existing rational surfaces where reconnection occurs, forming magnetic islands. A feature of the DTM is that those islands can interact with each other, enhancing the dynamics of the mode. Thus, theories applied to a single tearing mode configuration cannot explain the nonlinear processes of the DTM. Pritchett et al. proposed an analytical resolution for its linear evolution [2]: they found that for a large distance between the rational surfaces, the instability scales as a tearing instability ($\gamma \sim \eta^{3/5}$), whereas when the rational surfaces are close to each other, the instability behaves linearly as a kink instability ($\gamma \sim \eta^{1/3}$) (Fig. 1). In recent years, nonlinear evolution of the DTM depending on the distance between the rational surfaces has also been systematically studied [3–7]. Interestingly, the intermediate coupling defined for $\eta^{1/3} \leq \gamma \leq \eta^{3/5}$ presents a nonlinear evolution that is very different from tearing- or kink-type evolution. After a linear growth, the mode enters a slow down regime in which the configuration continues to evolve but on a much longer scale than the linear scale. However, instead of saturating, the mode suddenly enters a fast growth regime. Ishii et al. [3, 4], who investigated the onset of the rapid growth, explained this dynamics by triangular deformation of the magnetic structure and the resulting current point formation. If the localized current amplitude reaches a certain level, fast growth is triggered. They concluded that the observed structure-driven nonlinear destabilization was a direct consequence of the accelerated magnetic deformation, with much less effect from the plasma flow. Wang et al. [6, 7] revisited the fast growth of the DTM by reexamining the flow, in particular the sheared flows that form during the fast growth regime. Their study claimed that the fast growth consists of driven reconnection with a small dependency on the resistivity ($\gamma \sim \eta^{1/5}$). Despite these extensive studies, the physical mechanisms of such explosive growth of the DTM, and specifically its trigger, have not been fully understood. Here, we reconsider the problem of the fast growth by

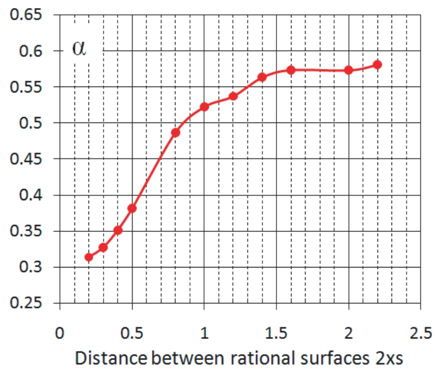

Fig. 1 Evolution of α ($\gamma \sim \eta^{\alpha}$) as a function of the distance between the two rational surfaces x_s.

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introducing a modulational-type instability analysis under the two-dimensionally deformed magnetic structures, similar to the study of the generation of zonal flow in microturbulence [8], in which the equilibrium with existing magnetic islands is considered. We reveal that the system exhibits some instability with a large growth rate. Such a secondary instability is considered to be the origin of the nonlinear destabilization.

2. Configuration and Numerical Method

The DTM is numerically studied via the resolution of the reduced MHD equations, assuming incompressibility. The two-field equations

$$\partial_t \psi + [\phi, \psi] = \eta \nabla^2 \psi, \tag{1}$$

$$\partial_t \nabla^2 \phi + \left[\phi, \nabla^2 \phi\right] = \left[\psi, \nabla^2 \psi\right], \tag{2}$$

express the time evolution of the field variables ψ and ϕ, where ψ is the magnetic flux function, and ϕ is the stream function. They are related to the magnetic field and velocity field, respectively, by $\vec{B} = \vec{e}_z \times \nabla_\perp \psi + B_z \vec{e}_z$ and $\vec{V} = \vec{e}_z \times \nabla_\perp \phi$, where $\vec{e}_z$ is the unit vector in the z-direction.

The geometry used to solve those equations is a typical slab one; the finite difference method is used in the x-direction, whereas we have Fourier expansion in the y-direction. No equilibrium flow is supposed ($\phi_0 = 0$), and the equilibrium field configuration is the same as in [2]:

$$B_{0y}(x) = 1 - (1 - B_c) \cosh^{-1}(\zeta x). \tag{3}$$

Setting $\pm x_s$ as the positions of the two rational surfaces, $B_{0y}(x_s) = 0$ yields $\zeta x_s = \cosh[1/(1 - B_c)]$, and the constant B_c is chosen such that $B_{0y}(x_s) = \pi/2$. The times are normalized to the Alfvèn transit times τ_A and the lengths to a unit length a.

There is a uniform space grid in the x-direction with a total mesh number of 2048 for a box size $L_x = 10$. In the y-direction, the box size is $2\pi L_y = 2\pi \times 1.2$, and the total mode number for the present calculation is $m_{tot} = 5$. In this case, five modes are sufficient enough to produce the fast growth rate [4]. Finally, the rational surfaces are separated by a distance $2x_s = 1.60$, and the resistivity is $\eta = 10^{-4}$.

3. Secondary Instability Analysis

The time evolutions of the magnetic and kinetic energies for the above parameters are shown in Fig. 2. This simulation shows a typical three-stage evolution. From $t = 0$ to $t \sim 550\tau_A$, the modes evolve exponentially. Only the mode $m = 1$ is linearly unstable, whereas the other modes are nonlinearly driven. Around $t \sim 550\tau_A$, the modes slow down, in a way similar to the case of the Rutherford regime of a single tearing instability. The magnetic flux evolves algebraically in time, while the velocity flux evolution is almost quasi-steady. Then, around $t \sim 1200\tau_A$, the kinetic energy increases abruptly, followed by the magnetic energy around $t \sim 1300\tau_A$. The magnetic mode $m = 0$ also increases in the final stage, so that it saturates at a higher energy level than that of $m = 1$. This situation corresponds to a global reconnection of the field lines in which the magnetic islands completely disappear and the magnetic field has the same orientation everywhere.

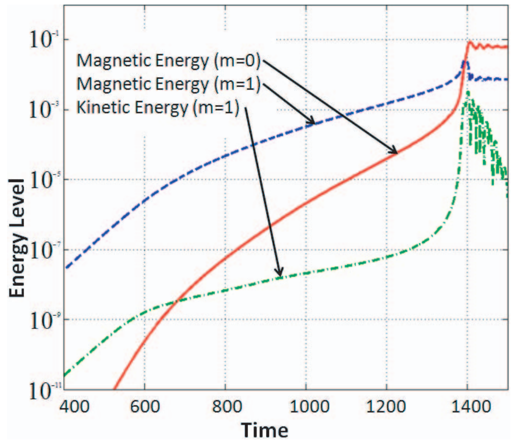

Fig. 2 Time evolution of the magnetic and kinetic energies for Fourier mode numbers 0 and 1.

To investigate the mechanisms leading to the fast growth, we use a new method that is similar to a secondary instability analysis. The reason for this investigation is as follows: after the linear growth, the instability enters the Rutherford regime where the dominant process is current diffusion (inertia is negligible) [9]. After this stage, the small magnetic island of a single tearing mode is expected to saturate at a critical width. However, with intermediate coupling of the DTM, the magnetic islands continue to grow on their respective tearing layers until they reach a size at which they deform each other (Fig. 3). Therefore, we investigate the possibility that the deformation of islands is the origin of a new instability.

To conduct such a study, we propose to examine the destabilization mechanism in the presence of two magnetic islands. The new quasi-steady equilibrium is now defined as a configuration with islands. It includes the previous equilibrium magnetic flux ψ_0 combined with the harmonics of the eigenfunction $\tilde{\psi}$ necessary to deform the original magnetic structure. The equilibrium functions are now expressed as $\psi = \psi_0 + \psi_{isl}$ and $\phi = \phi_{isl}$ where ψ_{isl}, ϕ_{isl} refers to the magnetic island harmonics.

Returning to the nonlinear simulation of the DTM, the trigger mechanism can be investigated by considering magnetic islands similar to those obtained in the previous simulation:

$$\psi_{isl}(x, y) = \psi(x, y, t = t_1), \tag{4}$$

$$\phi_{isl}(x, y) = \phi(x, y, t = t_1), \tag{5}$$

with t_1 representing different times in the nonlinear DTM calculation (Figs. 2 and 3). Here, an important hypothesis is considered. In the nonlinear simulation of the DTM, the magnetic islands and associated flows are indeed still

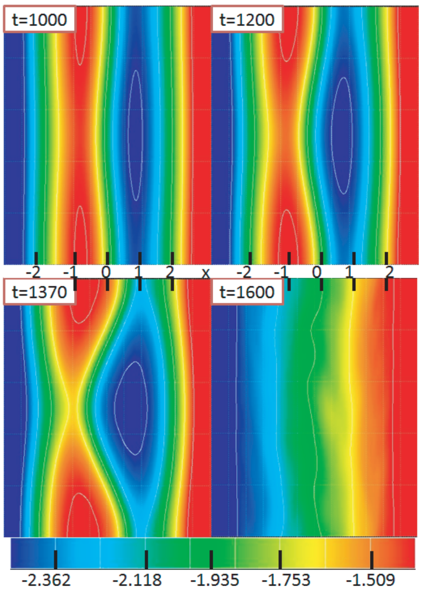

Fig. 3 Magnetic islands at different times (contour plots of the flux, colors indicate intensity).

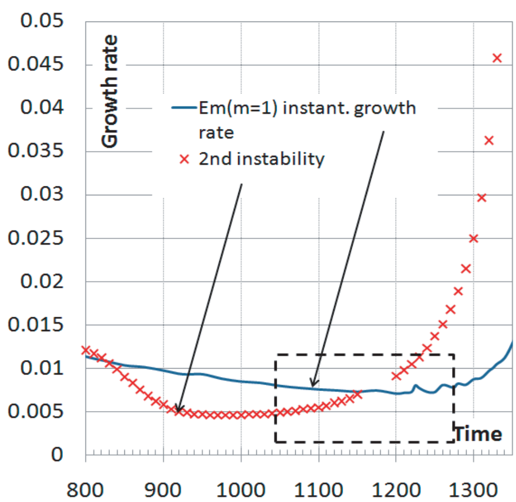

Fig. 4 Growth rate evolution for different initial times corresponding to different magnetic island sizes.

evolving in time. However, the time scale of their evolution is assumed to be much longer than that of the secondary instability (the magnetic flux is evolving on an algebraic scale, and the flows are considered to be quasi- steady). The assumption of static islands is therefore valid regarding the development of a new instability that leads to the abrupt growth. However, this assumption has some limitations. One can consider static islands only as long as the magnetic structure itself is not evolving on a faster time scale than the algebraic one; we therefore limit our study to times up to $t_1 \sim 1300\tau_A$ (square area in Fig. 4).

The two-field equation system in the linear analysis is now replaced with one that includes the two-dimensional (2D) deformation,

$$\partial_t \tilde{\psi} = -\left[\tilde{\phi}, \psi_0 + \psi_{t_1}\right] - \left[\phi_{t_1}, \tilde{\psi}\right] + \eta \nabla^2 \tilde{\psi}, \tag{6}$$

$$\partial_t \nabla^2 \tilde{\phi} = -\left[\tilde{\phi}, \nabla^2 \phi_{t_1}\right] - \left[\phi_{t_1}, \nabla^2 \tilde{\phi}\right] + \left[\tilde{\psi}, \nabla^2 \psi_0 + \nabla^2 \psi_{t_1}\right] + \left[\psi_0 + \psi_{t_1}, \nabla^2 \tilde{\psi}\right], \tag{7}$$

where $\tilde{\psi}$ and $\tilde{\phi}$ denote secondary perturbations.

The new equilibrium has x and y dependencies: islands can now deform the poloidal magnetic field $B_{0y}(x)$ due to their poloidal structure and also generate a radial component $B_x(y)$. Note that although this new simulation is considered in a linear framework, the modes are still coupled via the Poisson brackets. The set of equations above is numerically solved, yielding the linear growth rate of the secondary instability.

4. Numerical Results of the Secondary Instability

The linear growth of the secondary instability for different island sizes is plotted in Fig. 4 (red crosses). However, on the x-coordinate, instead of the size of the islands, we have indicated different times of the nonlinear DTM calculation for which we have considered the corresponding island size. For comparison, the instantaneous growth rate of the magnetic flux in the nonlinear simulation is also plotted (plain blue line), showing the growth rate of the secondary instability evolving on a much faster time scale around the trigger at $t \sim 1200\tau_A$. Two phases appear in this graph. In the first ($t < 1100\tau_A$), the linear growth rate of the new instability decreases and thin magnetic islands start to appear. Then, from $t \sim 1100\tau_A$, the growth rate starts to increase considerably, which corresponds to a time with larger magnetic islands. In other words, large, deformed magnetic islands have a greater destabilizing effect than thin magnetic islands. Let us examine these features. Before $t \sim 1100\tau_A$, the modes have entered the Rutherford regime, and the current profile has started to flatten due to quasi-linear effects, as shown in Fig. 5 (where the equilibrium current J_{eq} and the radial current associated with the mode $m = 0$, δJ_0, have been plotted), with the appearance of thin magnetic islands. As long as the current gradient at the rational surface (free energy) reduces due to the flattening effect of δJ_0, the growth rate of the second instability decreases (important flattening at $t \sim 900\tau_A$). Then, however, the growth rate starts to increase, which is expected to be evidence of a secondary instability. This result can be understood in terms of two effects. First, the modified current equilibrium (Fig. 6) shows that the flattening effect is replaced by corrugations with finite radial mode numbers, which can affect the dynamics of the tearing mode, as shown in [10]. As the mode $m = 0$ continues to grow, the associated current δJ_0 resulting from the nonlinear coupling of modes shows a corrugated radial component due to the generation of secondary instability, and those strong nonlinearities affect the equilibrium current. This can be related to studies of modulational insta-

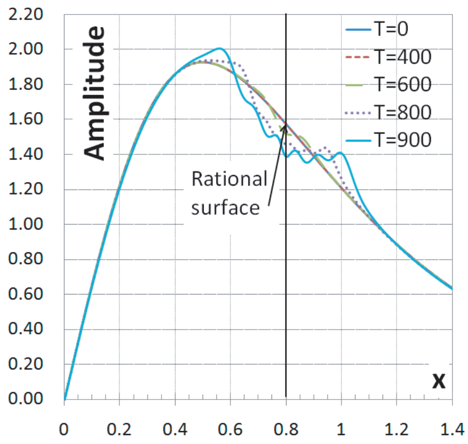

Fig. 5 Current equilibrium modification by δJ_0 ($t < 1000\tau_A$).

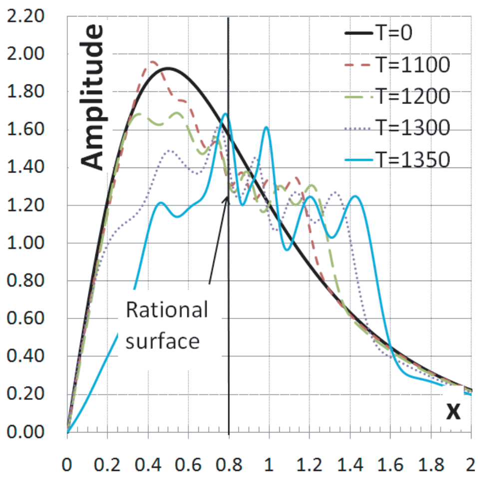

Fig. 6 Current equilibrium modification by δJ_0 ($t > 1000\tau_A$).

bility from background turbulence in the presence of large structures, leading to the generation of zonal flows [11]; here, the role of the large structure is played by the magnetic islands on each tearing layer. Furthermore, Fig. 3, in which we present 2D flux structures showing magnetic islands with very different sizes at different times, illustrates that the strong bending of the magnetic field lines by the islands pushing each other can critically affect the growth of the secondary instability. This effect directly drives the fast growth of the flow via the Maxwell stress ($[\psi, \nabla^2\psi]$) in the equation of the vorticity, this term being directly linked to magnetic structure.

5. Conclusion

The nonlinear destabilization of the DTM has been investigated via numerical simulations of the two-field equations, in which the equilibrium background has been defined with existing magnetic islands. It has been shown that 2D effects can destabilize subsequent processes. During the Rutherford regime, the background current is changed slightly by the nonlinear evolution of the perturbations, which flattens the current around the rational surfaces, and therefore reduces the free energy source (given by the current gradient). However, once nonlinearities become significant, the current is changed so that corrugation effects may become important, and thus this latter destabilizes consequent tearing modes. At the same time, 2D structure can also affect mode developments. The subsequent interplay between these current profile corrugations and 2D deformation of magnetic structure is expected to be the original trigger of the abrupt growth of the DTM. Further investigations are planned that will investigate the process through which such modulation can nonlinearly affect the mode development.

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