Turbulence Evolution in Plasma Shear Flows

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The renormalized nonlinear analysis of the temporal evolution of drift-type modes in plasma shear flows is developed. The theory accounts for the effect of the tubulent motions of plasma on the saturation of the resistive drift instability. The nonlinear balance equation, which determines the saturation level of the resistive drift instability in shear flow is obtained. It was prowed that the "nonlinear effect of the enhanced decorrelation by shear flow" has nothing in common with process of the saturation. The same conclusion is applicable to all fluid models of plasma, obtained in drift approximation, in which all nonlinearities, other than E × B are ignored. The linear non-modal kinetic theory to the Vlasov-Poisson system is developed. This theory reveals the velocity shear in a non-modal time-dependent effect of the finite Larmor radius.

Keywords: plasma instability, shear flow, renormalized theory, saturation, non-modal kinetic theory

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1. Introduction

It was convincingly demonstrated that shear flows, both along and across the ambient magnetic field, are fundamental reality in fusion and space plasma. A great variety of experimental results are now available [1], which confirm a suppression of turbulence by $\boldsymbol{E}\times\boldsymbol{B}$ velocity shear as a key feature of the regimes of improved confinement [2] of plasma and formation of transport barriers in tokamaks. This phenomenon stimulated considerable theoretical interest to the problems of the plasma stbility and plasma turbulence in shear flow [3].

The Kelvin's method of shearing modes or a so-called non-modal approach appears to be very effective for the analytical investigations of the temporal evolution of numerous low frequency modes in plasma shear flows [4–8]. Using this method we find that under condition of strong velocity flow shear V_0' non-modal evolution of the drift-type perturbations dominates and supersedes the modal development of linear instabilities. Drift modes are suppressed before they can grow, preventing the development of nonlinear processes and imposing its own physical character on the dynamics. The suppression of the drift resistive instability in the case of sufficiently strong flow shear appears a non-modal process, during which the initial amplitude of the separate spatial Fourier mode of the perturbed electrostatic potential decreases with time as $(V_0't)^{-2}$. In Section II we present developed first renormalized nonlinear theory of the drift turbulence of the plasma shear flow. Here we pay the attention to flows with a "moderate" velocity shear (of the order of or greater than the instability growth rate, but less than the drift waves frequency). In this case nonlinear processes determine the saturation of the drift resistive instability before the development of the linear non-modal suppression [4–8] of instability. In Section III we present in first non-modal approach to kinetic theory of plasma shear flow. Conclusions are given in Section IV.

2. Renormalized Hydrodynamic Theory for Drift Modes in Plasma Shear Flow

We investigate the nonlinear evolution of drift modes in shear flow using the Hasegawa–Wakatani equations [9] for the dimensionless density $n = \tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations,

$$\rho_s^2\left(\frac{\partial}{\partial t} + V_0(x)\frac{\partial}{\partial y} - \frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)\nabla^2\phi$$
$$= a\frac{\partial^2}{\partial z^2}(n-\phi), \tag{1}$$

$$\left(\frac{\partial}{\partial t} + V_0(x)\frac{\partial}{\partial y} - \frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)n + v_{de}\frac{\partial\phi}{\partial y}$$
$$= a\frac{\partial^2}{\partial z^2}(n-\phi), \tag{2}$$

$\boldsymbol{V}_0(x) = V_0(x)\boldsymbol{e}_y$ is the velocity of the sheared flow, $a = T_e/n_0e^2\eta_\parallel$, $\eta_\parallel$ is the resistivity parallel to the homogeneous magnetic field $B\|\boldsymbol{z}$, ρ_s is the ion Larmor radius at electron temperature T_e, $v_{de} = cT_e/eBL_n$ is the diamagnetic drift velocity, $L_n^{-1} = -\mathrm{d}\ln n_{0e}(x)/\mathrm{d}x$. To make the next analysis simpler we consider in what follows the case of the spatially homogeneous velocity shear, for which $V_0(x) = V_0'x$. We transform Eqs. (1), (2) to new spatial vari-

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ables [4] ξ, η,

$$t = t, \quad \xi = x, \quad \eta = y - V_0' x t, \quad z = z.' \tag{3}$$

With these variables, Eqs. (1) and (2) have a form

$$\rho_s^2 \left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) \nabla^2 \phi = a \frac{\partial^2}{\partial z^2} (n - \phi), \tag{4}$$

$$\left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) n + v_{de} \frac{\partial \phi}{\partial \eta} = a \frac{\partial^2}{\partial z^2} (n - \phi). \tag{5}$$

The above calculations show that with variables ξ and η we exclude from Eqs. (1) and (2) the spatial inhomogeneity originated from shear flow, but the Laplacian operator ∇^2 now becomes time-dependent [4],

$$\nabla^2 = \frac{\partial^2}{\partial \eta^2} + \left(\frac{\partial}{\partial \xi} - V_0' t \frac{\partial}{\partial \eta} \right)^2, \tag{6}$$

leaving us with an initial value problem to solve [4–8]. The absence of spatial inhomogeneity in Eqs. (4), (5) gives an opportunity to investigate in linear approximation the temporal evolution of the separate spatial Fourier mode of the electrostatic potential ϕ. It is interesting to note that transformation (3) conserves the $\boldsymbol{E} \times \boldsymbol{B}$ convective nonlinear derivative in Eqs. (1) and (2) in the form similar to one in a plasma without any flows.

With new variables ξ_1, η_1, which are determined by the nonlinear relations

$$\xi_1 = \xi - \widetilde{\xi}(t) = \xi + \frac{cT_e}{eB} \int_{t_0}^{t} \frac{\partial \phi}{\partial \eta} dt_1,$$

$$\eta_1 = \eta - \widetilde{\eta}(t) = \eta - \frac{cT_e}{eB} \int_{t_0}^{t} \frac{\partial \phi}{\partial \xi} dt_1 \tag{7}$$

the convective nonlinearity in Eqs. (4) and (5) becomes of the higher order with respect to the potential ϕ. Omitting such nonlinearity, as well as small nonlinearity of the second order in the Laplacian, resulted from the transformation to nonlinearly determined variables ξ_1, η_1, we come to the linearized set of the equations (3) and (4) with solution

$$\phi(\xi_1, \eta_1, z, t) = \int dk_\perp \int dl \int dk_z \phi(k_\perp, l, k_z, t_0) \times g(k_\perp, l, k_z, t) e^{ik_\perp \xi_1 + il\eta_1 + ik_z z}. \tag{8}$$

where $\phi(k_\perp, l, 0)$ is the initial data, and $g(k_\perp, l, k_z, t)$ is the linearly unstable solution to system (4),(5) [4]. In times $t < \left(V_0' l \rho_s\right)^{-1}$ solution g is of a modal form [4], $g(k_\perp, l, k_z, t) = \exp(i\omega_d t + \gamma t)$ with frequency $\mathrm{Re}\,\omega = \omega_d = -l v_{de} / \left(1 + \rho_s^2 \left(k_\perp^2 + l^2\right)\right)$ and growth rate $\gamma = \mathrm{Im}\,\omega = \omega_d \rho_s^2 \left(k_\perp^2 + l^2\right) / a k_z^2 \left(1 + \rho_s^2 \left(k_\perp^2 + l^2\right)\right)$ of the resistive drift instability. With variables ξ and η this solution has a form

$$\phi(\xi, \eta, t) = \int dk_\perp \int dl \phi(k_\perp, l, 0) \times e^{i\omega_d t + \gamma t + ik_\perp \xi + il\eta - ik_\perp \widetilde{\xi}(t_1) - il\widetilde{\eta}(t_1)}. \tag{9}$$

Eq. (9) is in fact a nonlinear integral equation for potential ϕ, in which the effect of the total Fourier spectrum on any separate Fourier harmonic is accounted for. The functions $\tilde{\xi}(t)$ and $\tilde{\eta}(t)$ in the exponential of Eq. (9) involve through Eq. (7) integrals of ϕ, which in turn, involve in their exponentials the integrals (9) and so on. This form of solution, however, appears very useful for the development of the approximate renormalized solutions to Hasegawa-Wakatani system, which accounted for the effect of the turbulent motions of plasma on the saturation of the drift-resistive instability. Assuming that the displacements $\widetilde{\xi}(t)$, $\widetilde{\eta}(t)$ obey the Gaussian statistics with mean zero, we obtain with Markovian approximation the renormalized solution for the potential (9), in which the average effect of the random convection is accounted for,

$$\phi(\xi, \eta, t) = \int dk_\perp \int dl \phi(k_\perp, l, 0) \exp\left(i\omega_d t + \gamma t - \int_0^t d\hat{t}\, C(k_\perp, l, \hat{t}) + ik_\perp \xi + il\eta \right). \tag{10}$$

$C(k_\perp, l, \hat{t})$ in Eq. (10) is determined by the integral equation

$$C(k_\perp, l, t) = \frac{c^2 T_e^2}{e^2 B^2} \int dk_{1\perp} \int dl_1 |\phi(k_{1\perp}, l_1, t)|^2 \times |[\boldsymbol{k}_\perp \times \boldsymbol{k}_{1\perp}]|^2 \frac{C(k_{1\perp}, l_1, t)}{\omega_d^2(k_{1\perp}, l_1)}. \tag{11}$$

where $|\phi(k_{1\perp}, l_1, t)|^2 = |\phi(k_{1\perp}, l_1, 0)|^2 e^{2\gamma(k_{1\perp}, l_1)t}$. Because of the limited four-page format of the present publication the details of the renormalization procedure will be published in a separate paper.

The saturation of the instability occurs when $\partial (\phi(\xi, \eta, t))^2 / \partial t = 0$, i.e. when

$$\gamma(k_\perp, l) = C(k_\perp, l, t) \tag{12}$$

Inserting this equation into Eq. (11) we obtain the equation, which determines the level of the instability saturation. The sought-for value in that equation is a time t_{sat} at which the balance of the linear growth and nonlinear damping occurs for given initial disturbance $\phi(k_{1\perp}, l_1, 0)$ and dispersion. With obtained t_{sat} the saturation level will be equal to $|\phi(t_{sat})|^2 \simeq \int dk_{1\perp} \int dl_1 |\phi(k_{1\perp}, l_1, 0)|^2 e^{2\gamma(k_{1\perp}, l_1) t_{sat}}$. Also, the well known order of value estimate $\phi \sim 1/k_\perp L_n$ for the potential ϕ in the saturation state is obtained easily from Eqs. (11), (12). This level, however, is temporal. The evolution of the drift turbulence of plasma shear flow continues in times $t > \left(V_0' l \rho_s\right)^{-1}$ and is determined by the velocity shear induced non-modal effect of drift wave suppression [4] originated from the time dependence of the Laplacian (6). Due to this effect solution $\phi(k_\perp, l, k_z, t)$ obtains strongly non-modal form, $\phi(k_\perp, l, k_z, t) \simeq \left(l\rho_s V_0' t\right)^{-2}$ (for which, however, Markovian approximation, used above is not valid). The development of that non-modal suppression in times comparable to the inverse growth rate gives the condition $V_0' \geq \gamma / l\rho_s$, which may be considered as a rough estimate for the velocity shear rate, sufficient for the suppression of the instability in times γ^{-1}.

Obtained results show that the nonlinearity of the Hasegawa-Wakatani system of equations in variables ξ and η, with which frequency and growth rate are determined without spatially inhomogeneous Doppler shift and wave number is time independent, does not display any effects of the enhanced decorrelations provided by flow shear. In the laboratory frame of reference such spatial Fourier modes are observed as a sheared modes with time dependent component of the wave number $k_x = k_\perp - lV_0' t$ directed along the velocity shear,

$$\phi(\boldsymbol{r}, t) = \int \mathrm{d}k_\perp \int \mathrm{d}l \phi(k_\perp, l, 0) \times \mathrm{e}^{i(k_\perp - lV_0' t)x + ily + i\omega_\mathrm{d} t + \gamma t - ik_\perp \tilde{\xi}(t_1) - il\tilde{\eta}(t_1)}. \quad (13)$$

The displacements $\tilde{\xi}(t)$ and $\tilde{\eta}(t)$ are observed in the laboratory frame as the displacements $\tilde{x}(t) = \tilde{\xi}(t)$ and

$$\tilde{y}(t) = \int_{t_0}^{t} \tilde{v}_y(t_1)\,\mathrm{d}t_1 = \frac{cT_\mathrm{e}}{eB} \int_{t_0}^{t} \mathrm{d}t_1 \frac{\partial \phi}{\partial x} + \int_{t_0}^{t} \mathrm{d}t_1 \frac{\partial V_0(x, t_1)}{\partial x} \tilde{x}(t_1) \quad (14)$$

In the case of the stationary perturbations with $\gamma = 0$ for times $t \gg \tau_\mathrm{correlation} \sim (\omega_\mathrm{d} - lV_0' x)^{-1}$, the well known result [10] for the variance $\left\langle (\tilde{y}(t))^2 \right\rangle$

$$\left\langle (\tilde{y}(t))^2 \right\rangle = \frac{c^2 T_\mathrm{e}^2}{e^2 B^2} \mathrm{Re} \int \mathrm{d}k_\perp \int \mathrm{d}l \left|\phi(k_\perp, l, 0)\right|^2 \times \frac{C_1(k_\perp, l, \hat{t}, x)}{\left(\omega_\mathrm{d}(k_\perp, l) - lV_0' x\right)^2} \times \left(\frac{2}{3}\left(lV_0'\right)^2 t^3 - 2k_\perp l V_0' t^2 + 2k_\perp^2 t\right), \quad (15)$$

displays the effect of the anisotropic dispersion conditioned by flow shear, observed in the laboratory frame of reference - dispersion increases much faster along flow than in the direction of the flow shear. However, that effect has nothing in common with process of the instability saturation, which is determined by the balance equation (12), in which any effects of the flow shear are absent.

3. Nonmodal Kinetic Approach

The Hasegawa-Wakatani model (1), (2) as well as other fluid equations, obtained in drift approximation, are developed, as a rule, under conditions of cold ions, $T_\mathrm{i} \ll T_\mathrm{e}$, and long waves, $k_\perp \rho_\mathrm{i} \ll 1$. The secular growth of the wavenumber $k_x(t)$ in convective variables needs the development of the non-modal kinetic theory, which can properly treat the long-time evolution of the perturbations with arbitrary values of the $k_x(t)\rho_\mathrm{i}$, and may be applicable to plasmas with comparable ion and electron temperatures. The gyrokinetic theory for shear flows is developed as a rule [11, 12] in a coordinate frame that is shifted by $V_0(x)$, but unchanged in configuration space, leaving the inhomogeneous convective terms in Vlasov equation. The application of the transform (3) jointly with transformation of the velocity to convective set of reference as $\boldsymbol{v}_\alpha = \boldsymbol{v} - V_0' \xi t \boldsymbol{e}_y$ to Vlasov equation for the distribution function F_α of the α species ($\alpha = \mathrm{i}, \mathrm{e}$) of the shear flow of the magnetized plasma excludes the spatial inhomogeneity introduced by flow velocity. In new variables, Vlasov equation contains only velocity shear parameter, V_0', (instead of $V_0(x)$ in laboratory frame variables),

$$\frac{\partial F_\alpha}{\partial t} + v_{\alpha\xi} \frac{\partial F_\alpha}{\partial \xi} - \left(v_{\alpha\eta} - v_{\alpha\xi} V_0' t\right) \frac{\partial F_\alpha}{\partial \eta} + \omega_{\mathrm{c}\alpha} v_{\alpha\eta} \frac{\partial F_\alpha}{\partial v_{\alpha\xi}} - \left(\omega_{\mathrm{c}\alpha} + V_0'\right) v_{\alpha\xi} \frac{\partial F_\alpha}{\partial v_{\alpha\eta}} - \frac{e_\alpha}{m_\alpha}\left(\frac{\partial \varphi}{\partial \xi} - V_0' t \frac{\partial \varphi}{\partial \eta}\right) \frac{\partial F_\alpha}{\partial v_{\alpha\xi}} + v_{\alpha z} \frac{\partial F_\alpha}{\partial z_\alpha} - \frac{e_\alpha}{m_\alpha} \frac{\partial \varphi}{\partial \eta} \frac{\partial F_\alpha}{\partial v_{\alpha\eta}} - \frac{e_\alpha}{m_\alpha} \frac{\partial \varphi}{\partial z_\alpha} \frac{\partial F_\alpha}{\partial v_{\alpha z}} = 0. \quad (16)$$

Now the spatially homogeneous, but time dependent, Eq. (16) may be Fourier transformed over the variables ξ, η, z with wave numbers k_x, k_y and k_z. With $v_\xi = v_\perp \cos\phi$, $v_\eta = \sqrt{\mu} v_\perp \sin\phi$, $\phi = (\phi_1 - \sqrt{\mu}\omega_{c\alpha} t)$ and coordinates $X_\alpha = \xi + \frac{v_\perp}{\sqrt{\mu}\omega_{\mathrm{c}\alpha}} \sin\left(\phi_1 - \sqrt{\mu}\omega_{\mathrm{c}\alpha} t\right)$ and $Y_\alpha = \eta - \frac{v_\perp}{\mu\omega_{\mathrm{c}\alpha}} \cos\left(\phi_1 - \sqrt{\mu}\omega_{\mathrm{c}\alpha} t\right) - V_0' t \ (X_\alpha - \xi)$ of the leading center, the perturbation f_α of the distribution function F_α is easily calculated for any values of the velocity shear rate V_0' and is equal to

$$f_\alpha\left(t, v_\perp, \phi, v_z, k_x, k_y, k_z\right) = \frac{ie_\alpha}{m_\alpha} \sum_{n=-\infty}^{\infty} \sum_{n_1=-\infty}^{\infty} \times \int^{t} \mathrm{d}t_1 \varphi\left(t_1, k_x, k_y, k_z\right) \times \exp\left(-ik_z v_z (t - t_1) + in(\phi_1 - \sqrt{\mu}\omega_\mathrm{c} t - \theta(t)) - in_1(\phi_1 - \sqrt{\mu}\omega_\mathrm{c} t_1 - \theta(t))\right) \times J_\mathrm{n}\left(\frac{k_\perp(t) v_\perp}{\sqrt{\mu}\omega_\mathrm{c}}\right) J_{\mathrm{n}_1}\left(\frac{k_\perp(t_1) v_\perp}{\sqrt{\mu}\omega_\mathrm{c}}\right) \times \left[\frac{k_y}{\mu\omega_{\mathrm{c}\alpha}} \frac{\partial F_\alpha}{\partial X_\alpha} + \frac{\sqrt{\mu}\omega_\mathrm{c} n_1}{v_\perp} \frac{\partial F_\alpha}{\partial v_\perp} + k_{1z} \frac{\partial F_\alpha}{\partial v_z}\right] \quad (17)$$

where $k_\perp(t)^2 = \left(k_x - V_0' t k_y\right)^2 + k_y^2$ and $\mu = 1 + V_0'/\omega_\mathrm{c}$. The Poisson equation for the separate spatial Fourier harmonic

$\varphi(\boldsymbol{k}, t)$ gives the governing integral equation

$$\begin{aligned}
&\left[\left(k_x - V_0' t k_y\right)^2 + k_y^2 + k_z^2\right]\varphi(\boldsymbol{k}, t) \\
&= \sum_\alpha \frac{i}{\lambda_{\mathrm{D}\alpha}^2} \sum_{n=-\infty}^{\infty} \int_{t_0}^{t} \mathrm{d}t_1 \varphi(\boldsymbol{k}, t_1) \\
&\times I_{\mathrm{n}}\left(k_\perp(t) k_\perp(t_1)\rho_\alpha^2\right) \mathrm{e}^{-\frac{1}{2}\rho_\alpha^2\left(k_\perp^2(t) + k_\perp^2(t_1)\right)} \\
&\times \mathrm{e}^{-\frac{1}{2}k_z^2 v_{T\alpha}^2 (t-t_1)^2 - in\sqrt{\mu}\omega_{c\alpha}(t-t_1) - in(\theta(t) - \theta(t_1))} \\
&\times \left[\frac{k_y v_{\mathrm{d}\alpha}}{\sqrt{\eta}} - n\sqrt{\mu}\omega_{\mathrm{c}\alpha} + ik_z^2 v_{\mathrm{T}\alpha}^2 (t - t_1)\right] \\
&\quad + \sum_\alpha e_\alpha \delta n_\alpha(\boldsymbol{k}, t_0), \qquad (18)
\end{aligned}$$

which in fact presents minimal model of the linear kinetic theory of plasma shear flows, which incorporates the shear flow effects. It introduces new linear time-dependent effect of the finite Larmor radius modified by flow shear.

4. Conclusions

In this paper, we present our recent results of the investigations of the turbulence evolution in shear flows. We obtain that the nonlinear integral balance equation (12), which determines the level of drift turbulence in the steady state, does not include any effects associated with flow shear over times during which the solution (7) is valid. The effect of the anisotropic dispersion of plasma displacements, given by Eq. (15), presents the variance of the plasma displacements resulted from the perurbations of the ordinary modal form (7), *observed in the laboratory frame*. That prowes that the "nonlinear effect of the enhanced decorrelation by shear flow" considerd in Refs. [13,14] has nothing in common with process of the saturation or even quenching of the instabilities. This result may be extended to other fluid models of plasma. The fluid equations, obtained in drift approximation, in which all nonlinearities other than $E \times B$ nonlinearity are ignored, do not include among the nonlinear mechanisms of the instability saturation the process of the enhanced nonlinear decorrelation by velocity shear. It is important to note, that the same conclusions concerning "suppression of the instability by the enhanced nonlinear decorrelation by velocity shear" are completely applicable to stratified shear flows of incompressible fluids, where the nonlinear convective derivative

$$\boldsymbol{v} \cdot \nabla = \frac{\partial \psi}{\partial z}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial z} \qquad (19)$$

is presented in terms of the stream function $\psi = \boldsymbol{e}_y \psi$, with which $\boldsymbol{v} = \nabla \times \psi$ and where basic flow along axis x with velocity gradient along axis z is assumed. Eq. (19) also conserves its form under transformation (3).

The effect of the flow shear reveals in convective variables at time $t > \left(V_0' l \rho_s\right)^{-1}$ as a non-modal effect of drift wave suppression [4]. The development of this non-modal suppression in times comparable to the inverse growth rate occurs when $V_0' \geq \gamma / l\rho_s$. This condition may be considered as a crude estimate for the velocity shear rate, sufficient for the suppression of the instability in times γ^{-1}.

We find that in linear nonmodal kinetic theory of plasma shear flows, which is valid for arbitrary values of $k_\perp(t)\rho_\alpha$, velocity shear reveals in the integral equation (18) as the non-modal time-dependent effect of the finite Larmor radius. The investigations of Eq. (18), as well as weak nonlinear and renormalized extensions of that equation, will clarify the role of the shear flows in the suppression of the drift turbulence and reducing the anomalous transport .

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