Search for Very-High-β MHD Stable Quasi-Isodynamic Configurations

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Quasi-isodynamic configurations offer the possibility of very good energetic particle confinement. They seem to offer the possibility of achieving very high plasma β, too.

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1. Introduction

Quasi-isodynamic [1] (*qi*) configurations have been previously found by computational optimization with high stability β limit, good neoclassical confinement properties and excellent fast particle collisionless confinement for configurations with poloidally closed contours of the strength of the magnetic field B [2, 3]. It was shown analytically [3] that the secondary parallel current density in *qi* configurations remains contained within each plasma field period, namely, between the cross-sections with maximal magnetic field strength B. In the *qi* configurations considered earlier, the divergence of the current density perpendicular to the magnetic field lines changes sign only once along the magnetic field within one field period. From this it follows that the parallel current density cannot change sign along the magnetic field within one period. Thus, because of the vanishing net parallel current, the parallel current density exhibits a dipole component which impairs MHD stability at very high β in the *qi* situation considered here for configurations with shallow magnetic well in the associated vacuum magnetic field and the compatibility of very high β with small neoclassical ripple. The latter was experienced in a previous study of a very-high-β case [4] with a sizable ripple; tests showed it to be unlikely that this ripple could be reduced by one order of magnitude. An elimination of the dipole component of the current density should make the magnetic structure more stiff with respect to changes in β so that the attainability of a small ripple should be less strongly coupled to the β value. The search for possible ways to diminish this current density in quasi-isodynamic configurations was the subject of [5].

For this search a two-staged approach had been taken. In a model investigation it was clarified that quasi-isodynamicity is compatible with vanishing dipole current density in a more elaborate structure of the topography of B exploiting the possibility of detailed toroidal design of B in 3d configurations; then a configurational investigation established a geometry realizing the essential features of this model and is seen in Fig. 1, Fig. 2 (left) and Fig. 3 (left) where the geometry together with the strength of B on the boundary, the contours of $j_{||}/B$ and cross-sections of the flux surfaces, respectively, are shown. The goal of the

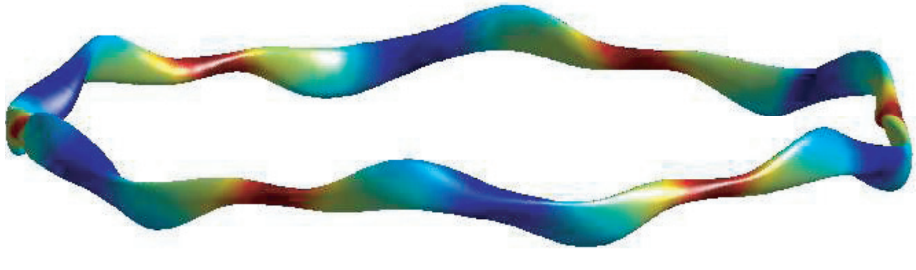

Fig. 1 Boundary magnetic surface showing the magnetic topography of [5]. The color varies from red (highest field strength) to blue (lowest field strength).

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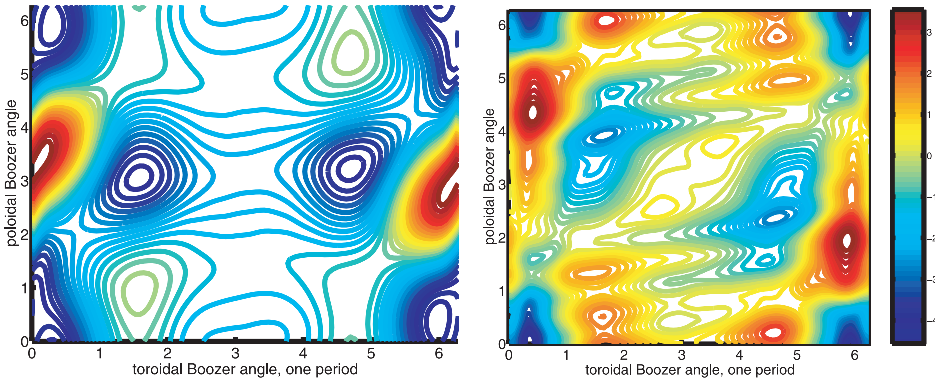

Fig. 2 Left: the structure of the parallel current density ($j_\parallel/B$) of the configuration of Fig. 1 showing $j_{\parallel 1,0} = 0$; Right: the structure of the parallel current density ($j_\parallel/B$) of the configuration of Fig. 4. The color varies from red (highest value of $j_\parallel/B$) to blue (lowest value of $j_\parallel/B$).

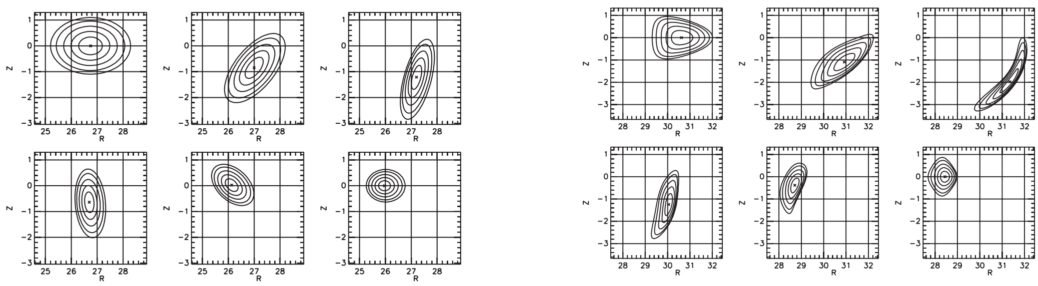

Fig. 3 Cross sections of magnetic surfaces of the configurations in Fig. 1 (left) and Fig. 4 (right) along half a period beginning with the minimum of B and ending at the maximum of B.

present work is to demonstrate the existence of very-high-β MHD stable configurations within this configurational class.

This goal has been achieved by stellarator optimization as will be described in detail in the next section. The following parameters were chosen for this optimization. The number of periods of that configuration is $N_p = 6$, the aspect ratio about 30 and its rotational transform per period about 1/6. These choices are kept here and their revision depends on further investigations part of which is subject of this work in which it is investigated whether MHD-stable equilibria of this type of configuration exist.

Since the aim of this research is very-high-β MHD stability, the β value has been chosen to be $\langle\beta\rangle \approx 0.2$. Whether a further increase of β would be meaningful depends on detailed further investigations of e.g. the behavior of energetic-particle collisionless confinement in view of the significant reduction of the strength of B in the core of the plasma. The pressure profile was also kept fixed and chosen to be $p = p_o(1 - 1.7324\,s + 0.7324 s^2)$ with s the label of the normalized toroidal flux.

The procedure of optimization has been analogous to [3] with two modifications. i) Preliminarily, the ballooning-stability requirement only concerned symmetric ballooning modes. ii) As in [5] — for keeping the dipole component of the secondary current small — the requirement of a small $m = 1, n = 0$ component of $\sqrt{g}$ in magnetic coordinates. The numerical values of the constraints used are given together with the detailed results.

2. Current result at $\langle\beta\rangle \approx 0.2$

The configuration of Figs. 1 and 3 (left) had been obtained at zero β. It exhibits a significant magnetic hill so that one of the essential ingredients of its optimization towards high β has been the transition to a vacuum field magnetic well as a prerequisit for MHD stability. As seen from Fig. 1, the starting point of the optimization is characterized by a roughly hexagonal plasma shape, i.e. by as little curvature as compatible with the straight sections encompassing the maxima of B. Since a magnetic well necessitates plasma curvature and higher order poloidal shaping (triangularity, indentation, ...), it is plausible that the local plasma column curvature had to increase. This is seen in Fig. 4, but most clearly in Fig. 5 (while not prominent in Fig. 6). The subsequent achievement of high β (≈ 0.20) is accompanied by two characteristic features. As in the initial condition, the Fourier components (in magnetic coordinates) of B and the volume element $\sqrt{g}$ corresponding to axisymmetric curvature are very small, simulating an aspect ratio of several hundred (see Fig. 7), which shows that this constraint is compatible with very-high-β MHD stability.

Also, since the strong poloidal as well as toroidal

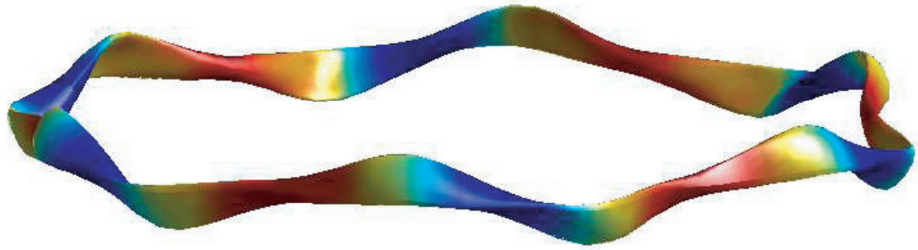

Fig. 4 Boundary magnetic surface showing the magnetic topography of the stable configuration.

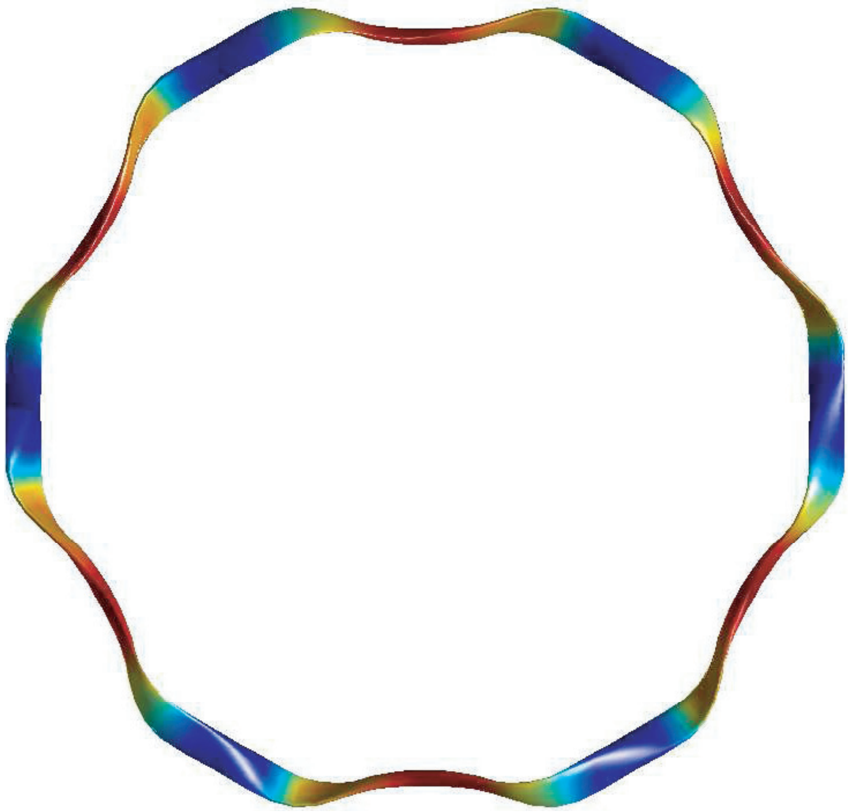

Fig. 5 Top view of the configuration shown in Fig. 4.

Fig. 6 Equatorial view of the configuration shown in Fig. 4.

shaping, see Fig. 3 (right), drives higher order Fourier components of $\sqrt{g}$, the optimization needed to exploit (and strictly observe) a window in rotational transform value, here chosen to be $6/7 < \iota < 6/6$. While introducing curvature is necessary for stability, see Fig. 8, ballooning instability will limit it. Here, preliminarily, this instability was evaluated in the following way. Solutions of the ballooning equation, see, e.g. [6], indicate instability by exhibiting two zeros. Considering solutions F passing through symmetry points with normalization $F(0) = 1$ and $dF/d\phi = 0$ then only involves monitoring the first zero encountered for demonstrating instability, see e.g. [7]. Shifting this first zero by stellarator optimization to $\phi > 2$, i.e. beyond two periods of the configuration, indicates absence of strong

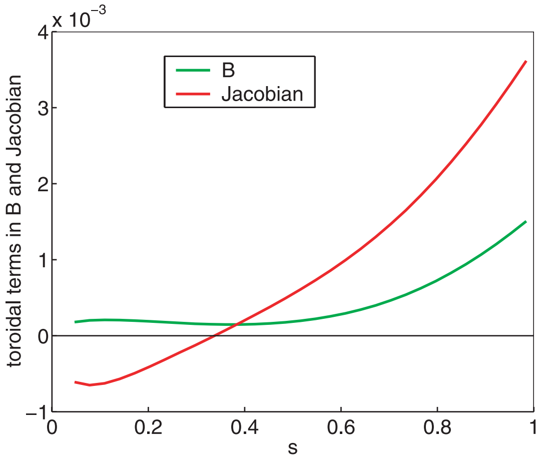

Fig. 7 Normalized Fourier coefficients in magnetic coordinates corresponding to toroidal curvature: for $B_{1,0}$ of B, $-B_{1,0}/B_{0,0}(0)$, and for $\sqrt{g}_{1,0}$ of $\sqrt{g}$, $\sqrt{g}_{1,0}/\sqrt{g}_{0,0}$, the constraint for the numerical value of the latter quantity was 5×10^{-3}.

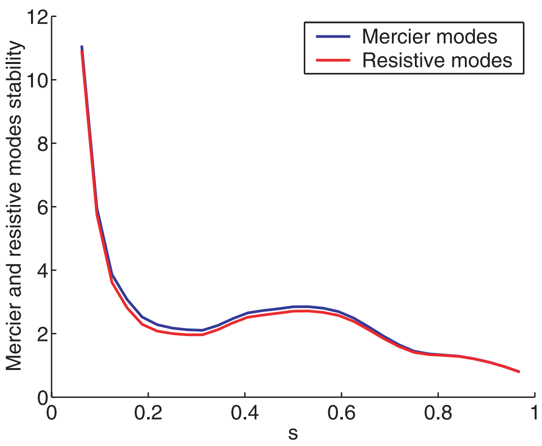

Fig. 8 Mercier and resistive-interchange stability criteria; the expressions plotted are proportional to $-D_I\Lambda^2(-D_R\Lambda^2)$ in the notation of [8], so that positive values indicate stability.

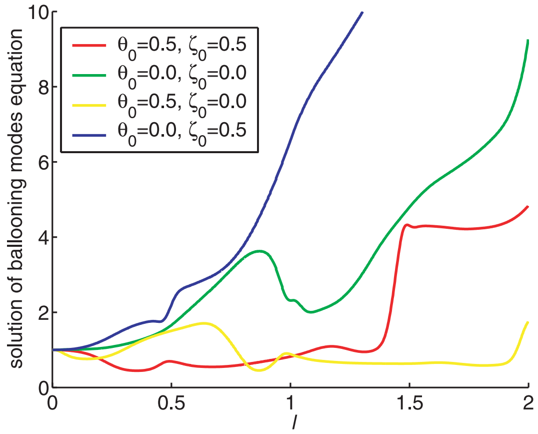

Fig. 9 Solutions of the ballooning equation — with the initial conditions being unity and having zero derivatives at symmetry points — as functions of the toroidal magnetic cordinate along fieldlines covering two periods and passing through the four symmetry points on the flux surface at $s = 0.3$.

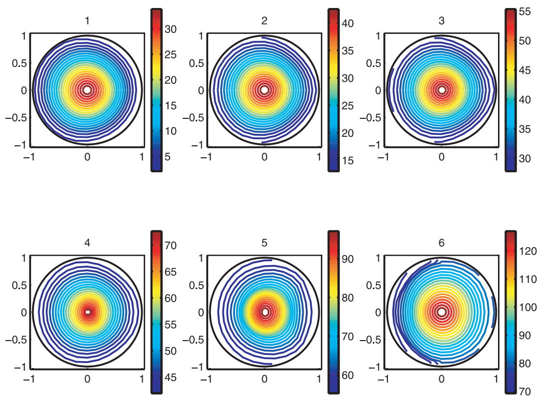

Fig. 10 Contours of $\mathcal{J}$ in the configuration obtained for different values of B, $B_{\rm ref}$, at which trapped particles are reflected; 1 : near the minimum of B, 6 : near the maximum of B.

ballooning instability for the field line passing through the symmetry point chosen. Evaluation at the four symmetry points of the field period and flux surface label $s \approx 0.3$ indicates the absence of strong-ballooning instability, see Fig. 9, because of the significant coverage of the magnetic surface selected. This analysis needs to be completed following the procedure used in [3], in particular because of the gap in marginal β found there between local and non-local ballooning analysis.

Some features observed in earlier stable configurations are prominent in the configuration obtained here, too: the triangular shape of the flux-surface cross-sections at the minimum of B and indentation in the range of strongest curvature. It remains to be investigated whether the higher-order poloidal and toroidal shaping found, e.g. the structure of $j_{\|}/B$ in Fig. 2 (right) and the quadrangularity at the maximum of B in Fig. 3 (right), is really necessary to achieve MHD stability. Another aspect is the accessibiliy of such a high-beta equilibrium starting from small beta. For a first information on this problem a series of smaller beta values has been studied for the same plasma boundary. It showed only small changes in rotational transform — this being one of the key necessities for accessibility — in keeping with the strongly reduced parallel current density, eg. about 10% change of iota between the high-beta case and a case at about a quarter of the beta value.

The neoclassical physics properties have not in detail been part of this high-β optimization; they should be benign in view of the contours of the second adiabatic invariant $\mathcal{J}$, see Fig. 10, for which the following constraint has been used: all $\mathcal{J}$ contours crossing a magnetic surface at $s \approx 3/4$ should be closed inside the plasma volume for all values $B_{\rm refl}$, i.e. values of B at which particles are reflected. The detailed investigation of the neoclassical physics properties and, eventually, optimization remains to be done in order to complete this case study of a very-high-β configuration, which, to our knowledge, is the first stellarator equilibrium in which a significant decoupling of the very-high-β properties and the neoclassical transport may be possible.

3. Summary

In the context of quasi-isodynamic stellarators with poloidally closed contours of the magnetic field strength it is investigated whether very-high-β MHD-stable equilibria exist. With a previously introduced new structure of a period as a starting point, equilibria are found which are Mercier, resistive-interchange and strong ballooning (symmetric) stable at $\langle\beta\rangle \approx 0.2$. Further work will concern further MHD-stability analysis and the details of the neoclassical physics properties of this type of configuration.

4. Acknowledgment

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