# Study of Neoclassical Transport in LHD Plasmas by Applying the DCOM/NNW Neoclassical Transport Database

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In helical systems, neoclassical transport is one of the important issues in addition to anomalous transport, because of a strong temperature dependency of heat conductivity and an important role in the radial electric field determination. Therefore, the development of a reliable tool for the neoclassical transport analysis is necessary for the transport analysis in Large Helical Device (LHD). We have developed a neoclassical transport database for LHD plasmas, DCOM/NNW, where mono-energetic diffusion coefficients are evaluated by the Monte Carlo method, and the diffusion coefficient database is constructed by a neural network technique. The input parameters of the database are the collision frequency, radial electric field, minor radius, and configuration parameters ($R_{\mathrm{axis}}$, beta value, etc). In this paper, database construction including the plasma beta is investigated. A relatively large Shafranov shift occurs in the finite beta LHD plasma, and the magnetic field configuration becomes complex leading to rapid increase in the number of the Fourier modes in Boozer coordinates. DCOM/NNW can evaluate neoclassical transport accurately even in such a configuration with a large number of Fourier modes. The developed DCOM/NNW database is applied to a finite-beta LHD plasma, and the plasma parameter dependences of neoclassical transport coefficients and the ambipolar radial electric field are investigated.



Keywords: LHD, neoclassical transport, neural network, Monte Carlo method

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## 1. Introduction

In helical systems, neoclassical transport is one of the important issues for sustaining high-temperature plasma. In particular, in the long-mean-free-path (LMFP) regime, the neoclassical transport coefficient increases as collision frequency decreases ($1/\nu$ regime). Therefore, neoclassical transport plays an important role as well as anomalous transport by plasma turbulence. Many studies have been performed to evaluate the neoclassical transport coefficient analytically and numerically in helical systems. Among the studies, the drift kinetic equation solver (DKES) [1, 2] code has been commonly used for experimental data analyses [3, 4] and theoretical predictions [5]. However, in the LMFP regime, especially with finite beta, a large number of Fourier modes of the magnetic field must be used for determining the distribution function and a convergence problem occurs.

On the other hand, the neoclassical transport coefficient has also been evaluated using the Monte Carlo method directly following particle orbits, where the mono-energetic diffusion coefficients are estimated by the radial diffusion of test particles [6–8]. This method has a good property in the LMFP regime except for its long calculation time. Thus, we have developed a Monte Carlo simulation code, the Diffusion Coefficient Calculator by the Monte Carlo Method (DCOM) code [9], which is optimized in performance in the vector computer.

For evaluating the neoclassical diffusion coefficient of thermal plasmas, we must consider energy convolutions. Therefore, it is necessary to interpolate discrete data by the DCOM. In a non-axisymmetric system, the diffusion coefficient shows complex behavior and strongly depends on collision frequency and radial electric field (e.g., $1/\nu$, $\sqrt{\nu}$, and $\nu$ regimes). The interpolation based on a traditional analytical theory has a problem with connected regions between two regimes.

Recently, a technique using a neural network (NNW) [10] has been developed as a good method for the construction of a database using limited data. In particular, a neural network having one or more hidden layers, called the multi-layer perceptron, has high fitting abilities for nonlinear phenomena. The NNW is applied in the study of fusion plasma, and is used in the construction of the NNW database for neoclassical transport in TJ-II plasmas [11].

Therefore, we apply the NNW method to the fitting of the diffusion coefficient of the LHD, which shows a complex behavior in several collisional regimes, i.e., $\nu$, $\sqrt{\nu}$, $1/\nu$, plateaus, and P-S regimes. We used a multilayer perceptron NNW with only one hidden layer, which is generally known as MLP1. The neoclassical transport database, DCOM/NNW [12], has been constructed using the input

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parameters $r/a$, $\nu^*$, and $G$, and $D^*$ can be obtained as an output of the NNW, where $\nu^*$ is the normalized collision frequency, $G$ is the normalized radial electric field, and $D^*$ is the normalized diffusion coefficient. We have constructed six database for the six different LHD configurations; the magnetic axis shift in the major radius direction between $R_{\text{axis}} = 3.45$ m and $R_{\text{axis}} = 3.90$ m.

For the construction of this database, we considered only vacuum plasmas and did not consider finite-beta plasmas. However, a relatively large Shafranov shift occurs in finite beta LHD plasmas, and the magnetic field configuration becomes complex leading to a large increase in the number of Fourier modes to express the mod-B structure in the Boozer coordinates.

In order to analyze the transport of experimental plasma with a finite beta, we have to consider the finite beta. Therefore, we apply the effect of the finite beta to the neoclassical database, DCOM/NNW. We evaluate the neoclassical transport coefficients and the ambipolar radial electric field using an improved DCOM/NNW.

## 2. Construction of Neoclassical Transport Database Including Finite Beta Effect

We calculate an equilibrium magnetic field with finite beta values at the plasma center, $\beta_0 = 1$, 2, and 3%, by VMEC [13,14]. In this study, we assumed the plasma density, $n = n_0 \times \left\{1 - (r/a)^8\right\}$, and the plasma temperature, $T = T_0 \times \left\{1 - (r/a)^2\right\}$. The Fourier spectrum of the magnetic field $B_{m,n}$ in the Boozer coordinate,

$$B = \sum_{m,n} B_{m,n}(\psi)\cos(m\theta - n\zeta), \tag{1}$$

as a function of the normalized minor radius $r/a$ in the $R_{\text{axis}} = 3.75$ m configuration with $\beta_0 = 0$ and 3% is shown in Fig. 1. Here, $B_{0,0}$ is the amplitude of the (0, 0) component of a magnetic field strength at the magnetic axis, where ($m$, $n$) represents the Fourier component of the magnetic field with the poloidal number $m$ and the toroidal number $n$. It is shown that the dominant magnetic field components in the LHD with $\beta_0 = 0$ % plasma are (2, 10) and (1, 0); (2, 10) corresponds to the main helical mode and (1, 0) is the toroidicity. Figure 1 (b) shows that the amplitudes of (2, 10) and (1, 0) component decrease and many other components increase by the Shaflanov shift in the finite beta, especially in the edge plasma, and the magnetic field configuration changes to a more complex one.

In this study, we calculate a mono-energetic diffusion coefficient $D$ using the DCOM code. By employing the DCOM code, we can calculate the diffusion coefficient without the convergence problem even if we assume many Fourier modes of the magnetic field. We use 50 Fourier modes to evaluate the magnetic field in this study. To obtain mono-energetic diffusion coefficients, we assume the energy of the test particle as $1.0 \times 10^{-3}$eV. The magnetic

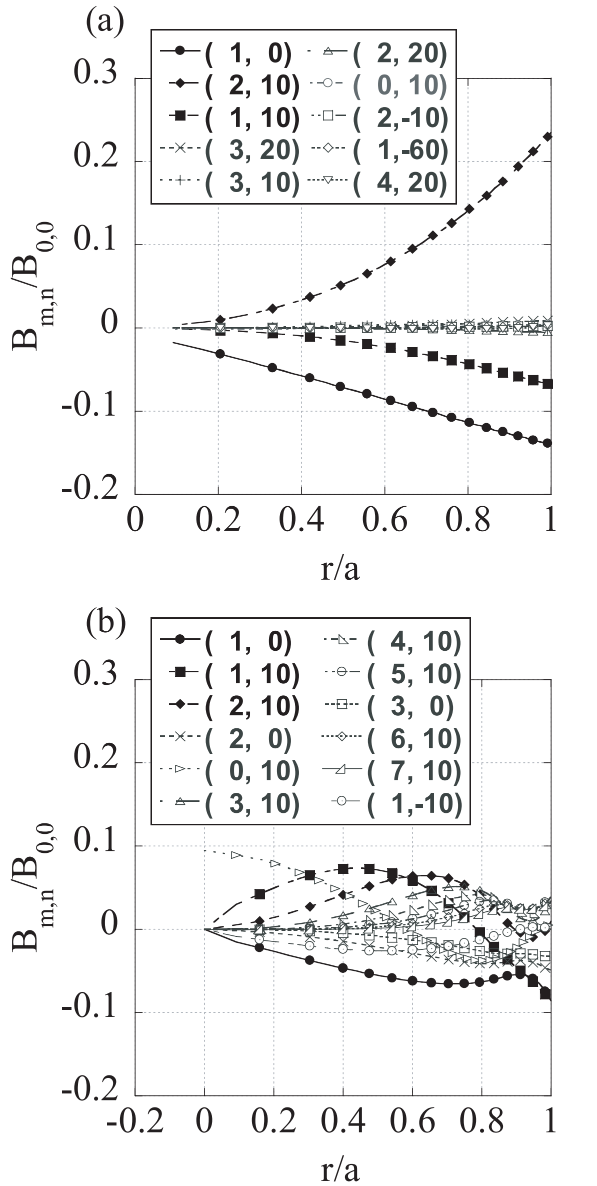


Fig. 1 Amplitude of dominant $B_{m,n}$ component normalized by $B_{0,0}$ at $r/a = 0$ as function of normalized minor radius r/a with (a) $\beta_0 = 0$ % and (b) $\beta_0 = 3$ %.

field is set to 3 T at the magnetic axis. The test particles are monitored for several collision times until the diffusion coefficient is converged. Figure 2 shows the normalized diffusion coefficient $D^*$ at $r/a = 0.5$ in $R_{\text{axis}} = 3.75$ m, which is calculated using the DCOM code as a function of the normalized collision frequency $\nu^*$ without the radial electric field. Here, we normalize the collision frequency by $\upsilon\iota/R$ and the diffusion coefficient by the tokamak plateau value in the mono-energetic case, $D_{\text{P}} = (\pi/16)(\upsilon^3/\iota R_{\text{axis}}\omega_{\text{c}}^2)$, where $\upsilon$, $\iota$, and $\omega_c$ are the velocity of the test particles, the rotational transform, and the cyclotron frequency of the test particles, respectively. It is found that the normalized diffusion coefficients rise rapidly as the central plasma beta values increases from 0 to 3 %.

In Fig. 3, the diffusion coefficient at $r/a = 0.5$ is shown as a function of $\beta_0$ in each regime. We can see that the dif-

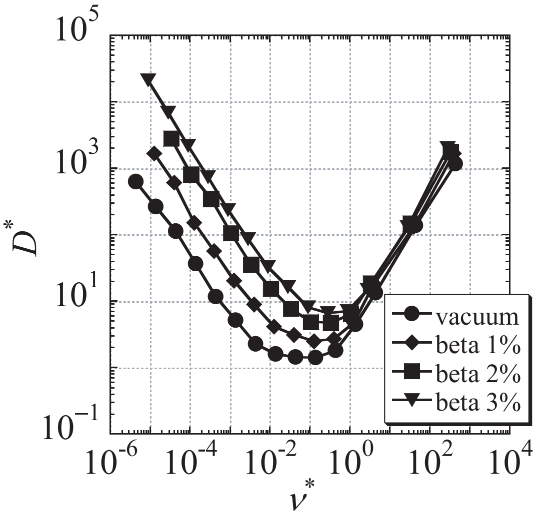


Fig. 2 Normalized mono-energetic diffusion coefficients as function of normalized collision frequency without radial electric field at $r/a = 0.5$ in $R_{\rm axis} = 3.75$ m.

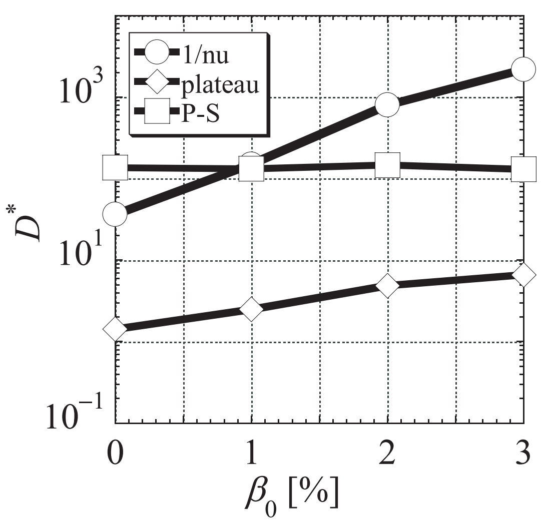


Fig. 3 Normalized diffusion coefficient as function of beta value, $\beta_0$, without radial electric field at $r/a = 0.5$ in $R_{\rm axis} = 3.75$ m.

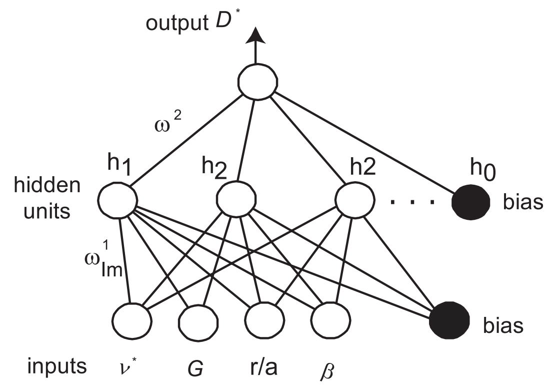


Fig. 4 Schematic view of MLP1 neural network used in calculations of $D^*$.

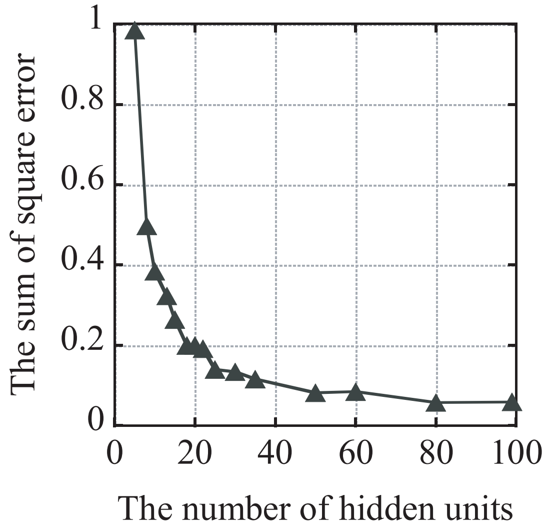


Fig. 5 The sum of relative error as function of number of hidden units of NNW.

fusion coefficients increase exponentially with $\beta_0$ in the $1/\nu$ regime ($\nu^* \approx 1 \times 10^{-4}$). It is found that the plateau values ($\nu^* \approx 1 \times 10^{-1}$) also increase by the beta value. On the other hand, in the P-S regime ($\nu^* \approx 3 \times 10^{1}$), the diffusion coefficients are almost independent of the value of $\beta_0$. Because the collision time is long enough to receive the contribution of the complicated magnetic field, the diffusion coefficient increases in the $1/\nu$ and plateau regimes. Thus, it is necessary to consider the effect of the beta value to evaluate accurately the transport in high-temperature plasmas, in which the collision frequency is in the $1/\nu$ or the plateau regimes.

We add the beta values to the inputs of DCOM/NNW, which is the neoclassical transport database using the NNW. We consider a multilayer perceptron (MLP) NNW model which is the most widely used. We use the MLP1 model having only one hidden layer as shown in Fig. 4. The operation of this NNW is expressed as

$$y\,(x_1,\, x_2,\, ...,\, x_l) = f\left(\sum_{m=0}^{m} w_m^2 f\left(\sum_{l=0}^{l} w_{lm}^1 x_l\right)\right), \qquad (2)$$

where $w_{lm}^1$ is the weight that connects the input layer and the hidden layer $h_m$, and $w_m^2$ is the weight that connects the hidden layer $h_m$ and the output layer. We consider two LHD configurations, $R_{\rm axis} = 3.60$ m and $R_{\rm axis} = 3.75$ m. We adjust the weights of NNW using the computational results of DCOM, which are called as training data. We calculate the diffusion coefficient changing the plasma beta $\beta_0 = 0$, 1, 2, and 3 % to obtain the training data. In this study, we used 2688 training data for $R_{\rm axis} = 3.75$ m configuration using the DCOM with 11 normalized minor radius, $0.1 \le r/a \le 0.9$; 14 normalized collision frequencies, $3.16 \times 10^{-6} \le \nu^* \le 1.00 \times 10^{3}$; 9 normalized radial electric fields, $0.00 \le G \le 3.16 \times 10^{-1}$; and 4 beta values, $0.0\,\% \le \beta_0 \le 3.0\%$. Also, 1777 training data calculated by DCOM are used for $R_{\text{axis}} = 3.60$ m configuration with 11 normalized minor radius, $0.1 \le r/a \le 0.9$; 14 normalized collision frequencies, $3.16\times10^{-6} \le \nu^* \le 1.00\times10^{3}$; 7 normalized radial electric fields, $0.00 \le G \le 3.16\times10^{-1}$; and 4 beta values, $0.0\,\% \le \beta_0 \le 3.0\,\%$.

The accuracy of the NNW depends on the number of hidden units. In Fig. 5, it is shown the relative error between training data and the outputs of NNW decrease as the number of hidden units increases. However, if we increase the number of hidden units too much, overlearning occurs, which gives poor predictions for a new parameter data although the error is very small.

In this study, the number of hidden unit is set as 15, and the mean relative error between DCOM results and outputs of NNW database is about 12.3 % in $R_{\text{axis}} = 3.75$ m and 15.7 % in $R_{\text{axis}} = 3.60$ m.

We can obtain $D^*$ using the improved NNW database for arbitrary $\beta_0$ in addition to $\nu^*$, $G$, and $r/a$. Figure 6 shows the contour plot of $D^*$ obtained using the newly NNW database at $r/a = 0.5$, $G = 0.0$ in $R_{\text{axis}} = 3.75$ m. The horizontal axis indicates $\nu^*$ and the vertical axis indicates $\beta_0$. We determined that $D^*$ does not depend on the beta value in P-S regime. Also, $D^*$ increases in $1/\nu$ and plateau regimes as $\beta_0$ increases. It is also seen that the plateau regime narrows as $\beta_0$ increases.

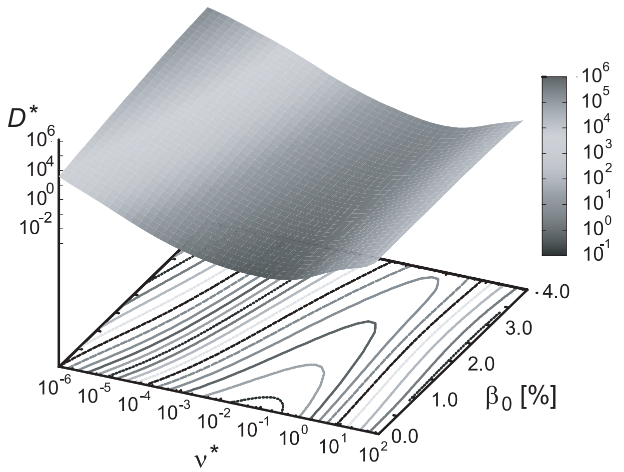


Fig. 6 The normalized mono-energetic diffusion coefficients, $D^*$ as a function $D^*(\nu^*, G, r/a, \beta_0)$ in $R_{\text{axis}} = 3.75$ m, where $G$ and $r/a$ are constant ($G = 0.0$ and $r/a = 0.5$), which are outputs of DCOM/NNW.

## 3. Neoclassical Transport Analysis using the DCOM/NNW

The diffusion coefficient with a Maxwellian energy distribution can be evaluated using the normalized mono-energetic diffusion coefficient $D^*(\nu^*, r/a, G, \beta_0)$ as

$$D_j^s = \frac{4}{\sqrt{\pi}} \int D^s(v)\left(\frac{v}{v_{\text{th}}}\right)^{2j} \exp\left[-\left(\frac{v}{v_{\text{th}}}\right)^2\right] \frac{\mathrm{d}v}{v_{\text{th}}}, \tag{3}$$

where

$$D^s(v) = D_{\text{p}}^s D^*(\nu^*,\, r/a,\, G, \beta_0)\left(\frac{v}{v_{\text{th}}}\right)^3, \tag{4}$$

with $s$ = e, i (for electrons or ions), and $j$ = 1, 2, 3 (related to the particle diffusion coefficient by the density gradient, the particle (thermal) diffusion coefficient by the temperature (density) gradient, and the thermal diffusion coefficient by the temperature gradient, respectively) [15].

The radial electric field $E_{\text{r}}$ can be estimated using the ambipolar condition of the neoclassical transport flux in the non-axisymmetric configuration. The ambipolar condition is given by $J_{\text{r}} \equiv e\Gamma_{\text{e}} - Z_{\text{i}}e\Gamma_{\text{i}} = 0$, where $\Gamma_{\text{e}}$ and $\Gamma_{\text{i}}$ are the electron and ion neoclassical fluxes, and an additional anomalous contribution is assumed to be intrinsically ambipolar. The particle flux $\Gamma_s$ and energy flux $Q_s$ are given by

$$\Gamma_s = -nD_1^s\left\{\frac{1}{n}\frac{\partial n}{\partial r} - \frac{q_s E_{\text{r}}}{T_s} + \left(\frac{D_2^s}{D_1^s} - \frac{3}{2}\right)\frac{1}{T_s}\frac{\partial T_s}{\partial r}\right\}, \tag{5}$$

$$Q_s = -nT_s D_2^s\left\{\frac{1}{n}\frac{\partial n}{\partial r} - \frac{q_s E_{\text{r}}}{T_s} + \left(\frac{D_3^s}{D_2^s} - \frac{3}{2}\right)\frac{1}{T_s}\frac{\partial T_s}{\partial r}\right\}, \tag{6}$$

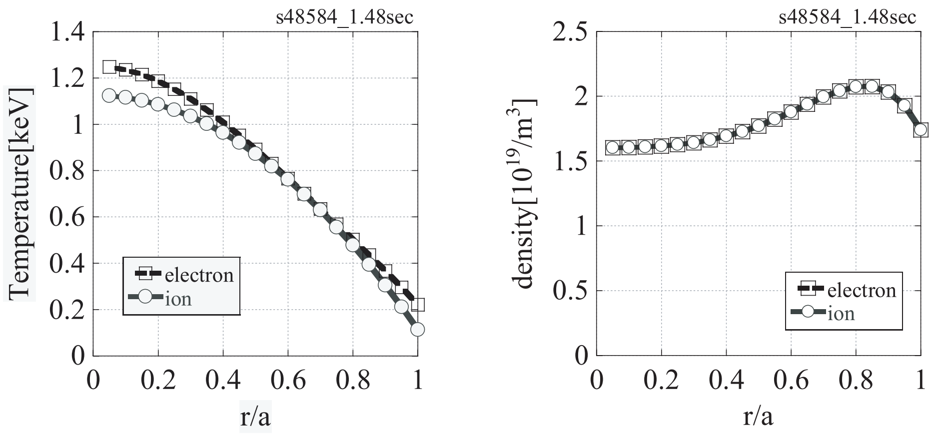


Fig. 7 Profile of plasma temperature and particle density ($n_{\text{e}} = n_{\text{i}}$).

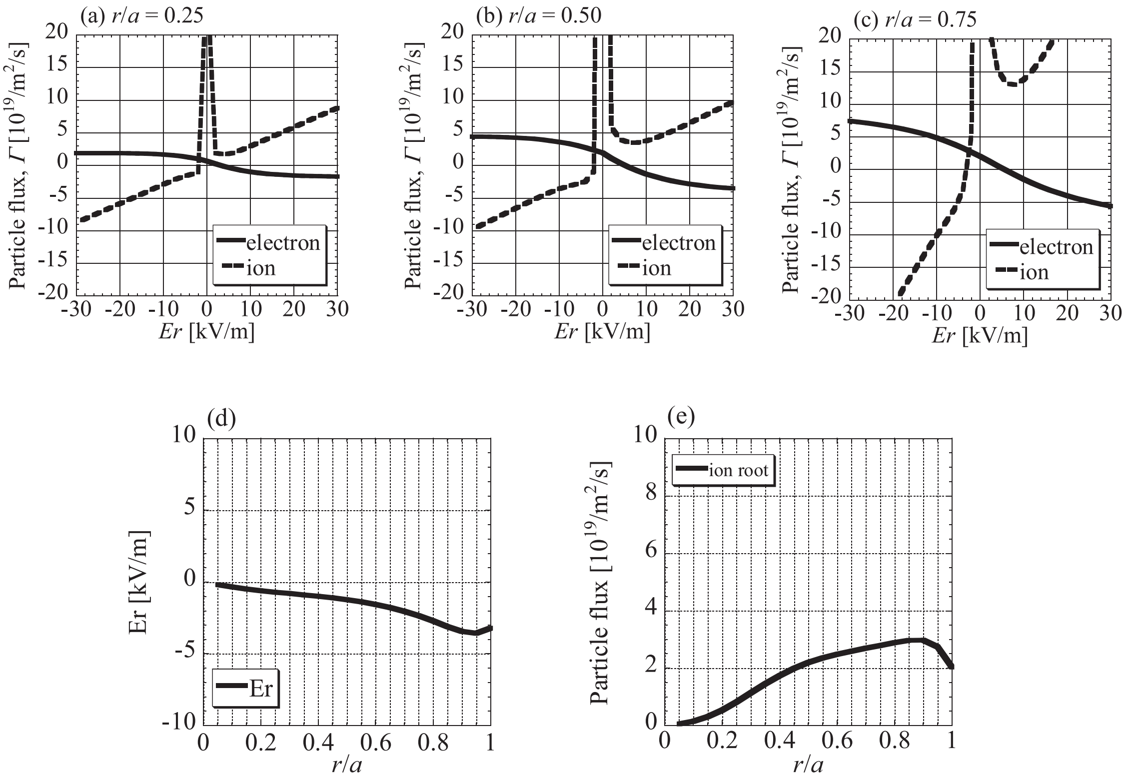


Fig. 8 Particle fluxes $\Gamma$ as a function of electric field applying the DCOM/NNW with $\beta_0$ = 0.72 %; (a) $r/a$ = 0.25, (b) $r/a$ = 0.5, and (c) $r/a$ = 0.75. Radial electric field (d) and particle flux (e) in the ambipolar condition.

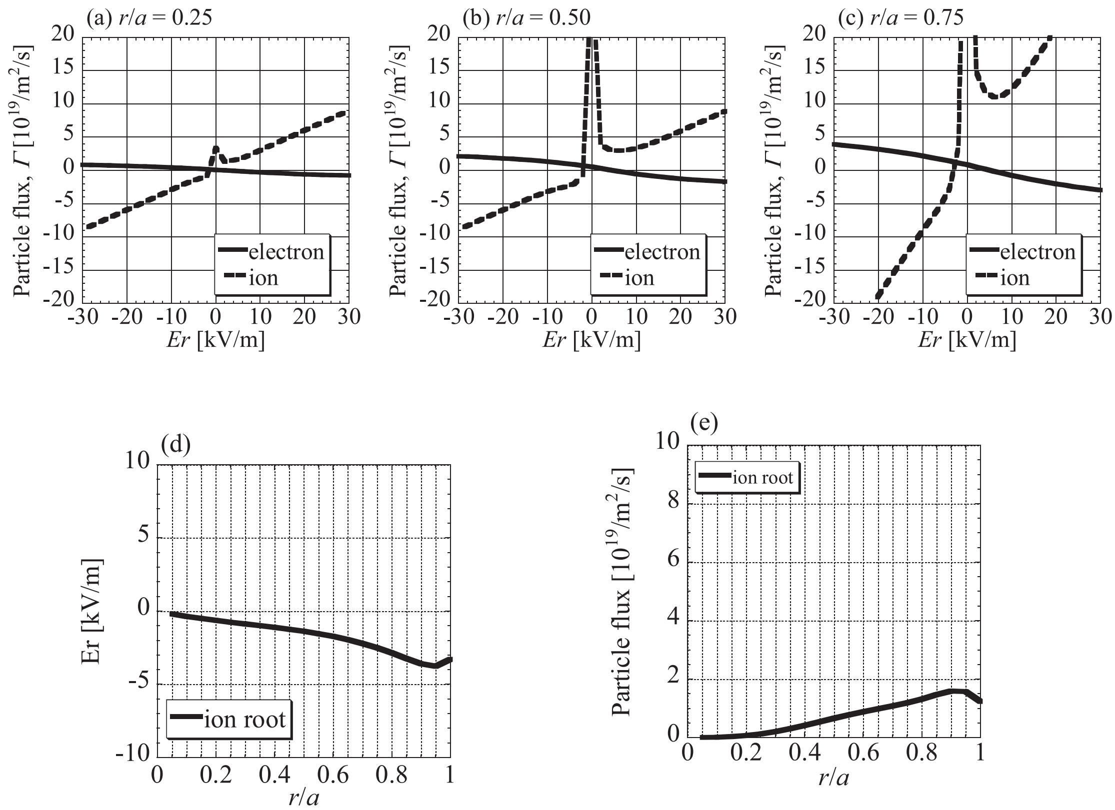


Fig. 9 Particle fluxes $\Gamma$ as a function of electric field applying the DCOM/NNW with $\beta_0$ = 0.00 %; (a) $r/a$ = 0.25, (b) $r/a$ = 0.5, and (c) $r/a$ = 0.75. Radial electric field (d) and particle flux (e) in the ambipolar condition.

with $s$ = e for electrons and $s$ = i for ions, and $q_e = -e$, $q_i = Z_i e$. Applying the constructed neoclassical transport database, DCOM/NNW, the ambipolar radial electric field and the neoclassical (particle and energy) fluxes can be evaluated for any density and temperature profile.

We study the neoclassical transport by DCOM/NNW including the finite beta effect. In this analysis, we consider hydrogen plasma and $R_{\mathrm{axis}} = 3.75\,\mathrm{m}$. The assumed temperature and density profiles of electrons and ions are shown in Fig. 7 and the magnetic field strength is set to $B_0 = 1.5\,\mathrm{T}$ (#48584, $t = 1.48\,\mathrm{s}$). Assuming these plasma parameters, the particle flux $\Gamma_s$ is calculated using Eq. (5).

The neoclassical particle flux and the ambipolar radial electric field in the $R_{\mathrm{axis}} = 3.75\,\mathrm{m}$ configuration using the DCOM/NNW are shown in Fig. 8. In this calculation, the beta value is assumed as $\beta_0 = 0.72\,\%$. In Figs. 8 (a)-(c), the particle fluxes at $r/a$ = 0.25, 0.5, and 0.75 are shown as a function of $E_r$. Figure 8 (d) shows the ambipolar radial electric field and Fig. 8 (e) is the particle flux as a function of $r/a$. We find only one root (ion root) with $\beta_0 = 0.72\,\%$.

In order to confirm the effect of the finite beta, we evaluate neoclassical transport and the ambipolar radial electric field assuming the zero-beta plasma. Figure 9 shows the obtained results using $\beta_0 = 0\,\%$ as the input parameter of the DCOM/NNW. The temperature and density profiles are the same as in Fig. 7. We can see similar profiles of the electric fields in the $\beta_0 = 0.0$ and $0.72\,\%$ cases. However, it is found that the particle flux $\Gamma$ with $\beta_0 = 0.0$ and $0.72\,\%$ are greatly different, as shown in Figs. 8 (e) and 9 (e). In the case of $\beta_0 = 0.72\,\%$, $\Gamma$ increases to about three times that of $\beta_0 = 0.0\,\%$.

## 4. Summary

Neoclassical transport has been studied applying the neoclassical transport database, DCOM/NNW. We have extended DCOM/NNW to include the effect of configuration changes due to the finite beta effect. In the finite beta, the magnetic field becomes complex, and the number of necessary Fourier mode increases. Using the extended DCOM/NNW, we have estimated neoclassical transport more accurately than previously. It is found that the particle flux increases about three times as the plasma beta increases from 0 to 0.72 % at the center. This result indicates that the inclusion of finite beta effect is necessary for an accurate evaluation of neoclassical transport.


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