On Fractal Properties of Equipotentials over a Real Rough Surface Faced to Plasma in Fusion Devices

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We consider a sheath region bounded by a corrugated surface of material conductor and a flat boundary held to a constant voltage bias. The real profile of the film deposited from plasma on a limiter in a fusion device was used in numerical solving of the Poisson's equation to find a profile of electrostatic potential. The rough surface influences the equipotential lines over the surface. We characterized a shape of equipotential lines by a fractal dimension. The long-range correlation in the potential field is imposed by the non-trivial fractal structure of the surface. Dust particles bounced in such irregular potential field can accelerate due to the Fermi acceleration.

Keywords: plasma-surface interaction, sheath layer, dust motion, fractal boundary, deposited film, fusion device, Fermi acceleration

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The electric field close to a material surface contacted with plasma in a fusion device influences the property of edge plasma and a motion of small dust particles. The models of edge plasma consider typically a smooth shape of a material surface bounded the edge plasma. Contrary to this, real surfaces in fusion devices are corrugated. An intensive erosion of plasma-facing materials leads to a formation of amorphous films of irregular shape [1] on the plasma-facing components (vacuum chamber, divertor plates and limiters). The analysis [1] of a local fracture surface has shown that topography of the film surface is not a trivial stochastic variation of heights. Film surfaces have a stochastic topography and a hierarchy of granularity on the scale from ∼10 nanometers to ∼100 micrometers [1] with non-trivial self-similarity of the structure. Such property means a long-range correlation and power laws which govern electric field scaling and dynamics of dust particles moved in the electric field. An irregular potential field over a corrugated surface can drive quite non-regular dynamics of a small dust particle [2] leading to the flights toward the core plasma of the fusion device. So, an implementation of a real surface relief in the problem can help to reveal an important property of the plasma-wall interaction. The problem is close to a problem of the Laplacian fields considered transport phenomena over rough surfaces [3,4]. Analytical solutions of the Lapalacian fields were considered just in cases of highly symmetric boundary conditions. When the surface presents irregularities the problem becomes more difficult. The Laplacian potential around a linearly self-similar Koch tree is studied numerically [3–5]. The Koch tree is a fractal with a trivial self-similarity. Contrary to this, real surfaces obey more complicated fractal structure [1] with a long-range correlation that could contribute a specific scaling property of a potential field over the surface.

We have investigated the problem of characterizing the electric potential in the region bounded by a real rough profile and a flat boundary (Fig. 1). The mathematical formulation of the problem [6] is to consider the linearized Poisson equation in the sheath layer for the potential φ:

$$\Delta\varphi = \varphi/\lambda_D{}^2 , \tag{1}$$

λ_D–Debye length. The standard variational formulation of

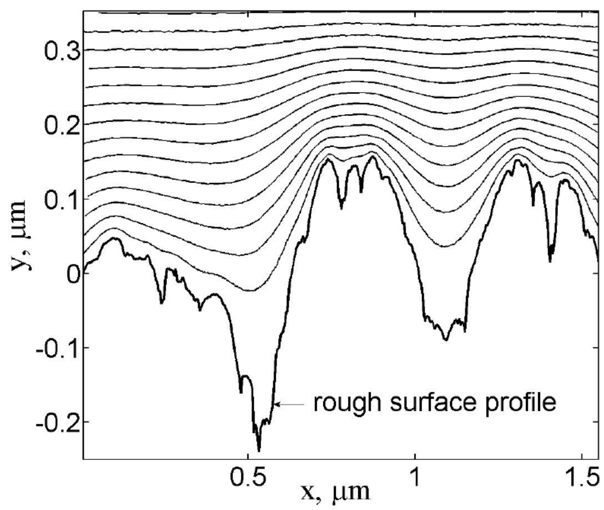

Fig. 1 Equipotential curves over a real surface relief [1] with the Dirichlet boundary condition. The lines separate regions of linear decreasing potential: $\varphi = 1$ at the top and $\varphi = 0$ at the bottom on the surface.

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a finite element method was used to solve the problem. A triangular mesh was obtained from a Delaunay-Voronoy tessellation and the P1-Lagrange interpolation [5, 7]. The obtained linear system is solved by using the Cholesky method [7] and the MATLAB toolbox software [8]. The values of the potential at the corrugated surface ($\varphi = 0$) and the flat opposite boundary ($\varphi = 1$) are kept fixed. The potential at the lateral vertical lines bounded the domain, are also fixed with a linear variation between the values of the potentials at the rough curve and at the flat boundary. After the integration of Eq. 1 we obtain a set of equipotential lines by the numerical spline interpolation, Fig. 1. The roughness of the equipotential lines is gradually changed toward the top bound. Due to the scaling invariance of the problem (based on the linearized Poisson Eq. 1), such irregular structure can be projected to other domain scale, e.g. of Debye length one. We quantify an irregular shape of equipotential lines by a fractal dimension. By using MATLAB toolbox software, we estimate the Hurst exponent [1], H, of the lines which characterizes an irregularity. The rough surface relief and equipotential lines are characterized by $H > 1/2$ that means the presence of memory and long-range correlation. A fractal dimension D of a curve relates with the Hurst exponent as $D = 2 - H$. The plot of the fractal dimension vs. a minimal distance to the surface is shown in Fig. 2. The fractal dimension drops from a value related to the surface dimension $D = 1.15$ to 1 (Euclidian dimension of a smooth line). Uncertainty in Fig. 2 is of ~2-5% from a limited data statistics on the Hurst exponent estimation. We emphasize that a drop of the fractal dimension is not rapid one: there is no exponential decreasing as was discussed in the case of the

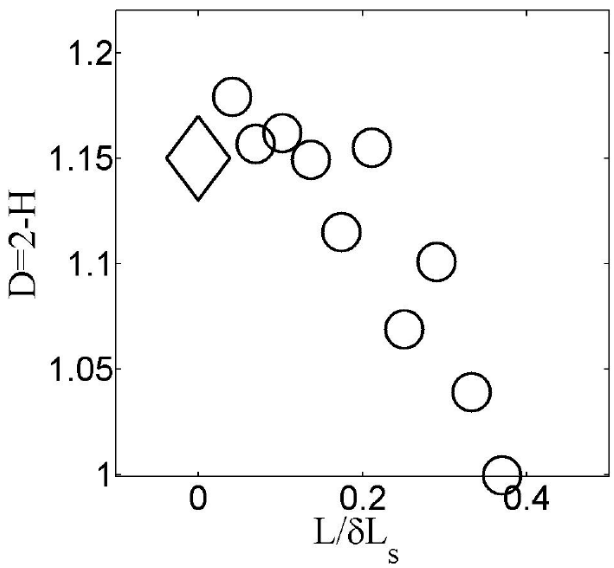

Fig. 2 Fractal dimension of equipotential lines (circles) vs. a minimal distance from the surface normalized by a maximal scale of surface irregularity $\delta L_s = y_{max} - y_{min}$. Diamond – fractal dimension of the real surface profile.

Koch tree bound (trivial self-similarity) [5]. It means that the long-range correlation in the potential field is strictly imposed by the non-trivial fractal structure of the surface with $H > 1/2$. Such imposing should be considered in the problem of the edge plasma turbulence: a stochastic boundary condition can contribute random perturbations.

A roughness of the equipotentials influences dust particle dynamics. Billiards [9] are very convenient models of dust particle motion over a fractal surface in the fusion device. In accordance with the boundary geometry, dynamics of the billiard particle can be, depending on the initial conditions, regular or completely chaotic. This problem is related to the unbounded increase of energy in periodically forced Hamiltonian systems and known as the Fermi acceleration. It has been shown that in the case of a quite smooth boundary perturbation the growth of the particle velocity is bounded. Otherwise, the velocity can increase indefinitely. Numerical and analytical investigations of the perturbed billiard [9] have shown that the billiard has strong chaotic properties. Particle velocity dependence as a function of the number N of collisions with the boundaries has approximately the square-root behaviour $\upsilon(N) \sim \upsilon_0 N^{1/2}$. A sheath region is bounded between a rough surface and plasma with turbulent oscillations of electric field [1]. This property simulates the Fermi acceleration problem. Taking typical dust velocity $\upsilon_0 \sim$ 10-100 m/s, a length scale of a near-wall potential well $l \sim 500\,\mu$m [2], then a time scale of a dust particle oscillation in such well per one period is of $\sim 10^{-5}$-10^{-4} s. When particles move in the potential field with a fractal structure it is expected statistical distribution of their trajectories. Some of dust particles are expected to be bounced many times in the potential well with a fractal structure (Fig. 1). Assuming such dust particle will be confined in the same domain (a billiard-like problem) for a time of 100-1000 seconds, the number of bouncing oscillations could be of $N \sim 10^6$-10^8 (of course, it may be eliminated by some other mechanisms). From this, we may roughly estimate an increasing of the velocity $\upsilon(N) \sim \upsilon_0 N^{1/2} \sim (10^3$-$10^4)\upsilon_0 \sim$ 10-1000 km/s. In the fusion reactor it may lead to a significant influx of dust particles toward the core plasma region. To estimate such flux, the statistical distribution of the dust trajectories in the irregular potential field (e.g., Fig. 1) should be considered. This result provides new insight into practical applications, e.g., estimation of dust influx toward edge and core plasma in fusion devices.

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[1] V.P. Budaev, Physica A **382**, 359 (2007).
[2] S. Krasheninnikov *et al.*, Phys. Plasmas **11**, 3141 (2004).
[3] C.J.G. Evertszt and B. Mandelbrot, J. Phys. A Math. Gen. **21**, 1781 (1992).
[4] B. Sapoval, Phys. Rev. Lett. **73**, 3314 (1994).
[5] H. de O. Dias Filho *et al.*, Physica A **342**, 388 (2004).

[6] M. Lieberman and A. Lichtenberg, *Principles of plasma discharges and materials processing* (John Wiley & Sons, New York, 1994).
[7] O.C. Zienkiewicz and R.L. Taylor, *The Finite Element Method*, Vol. 1 - 3 (Butterworth-Heinemann, Oxford, 2000).
[8] *Partial Differential Equation Toolbox User's Guide: For Use with MATLAB* (The MathWorks, Inc., 1997).
[9] A. Loskutov and A. Ryabov, J. Stat. Phys. **108**, 995 (2002).