Analysis of Energy Spectra of Fast Ion in the Large Helical Device

Hidetoshi NISHIMURA, Mitsutaka ISOBE[1)], Masaki NISHIURA[1)], Mamiko SASAO, Akio KOMORI[1)] and LHD experimental group

Department of Quantum Science and Energy Engineering, Tohoku University, Sendai 980-8579, Japan
[1)]*National Institute for Fusion Science, Toki 509-5292, Japan*

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On the basis of kinetic description, the analytical formula is derived as an evaluation formula of the energy spectra of fast ion produced by neutral beam injection (NBI) heating. The effects of particle loss due to the loss cone are newly considered in the kinetic equation. The distribution function is derived by integrating the kinetic transport equation as possible as analytical methods. Thus it will be able to reduce a computational time for the systematic analysis of the energy spectra of fast ion. The validity of the derived solution is appraised by comparing with the energy spectra experimentally measured by natural diamond detector (NDD) in the Large Helical Device (LHD). Good agreements in their behavior are obtained.

Keywords: fast ion, orbit loss, neutral beam injection heating, Large Helical Device, natural diamond detector

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1. Introduction

It is important to understand the behavior of fast ion in confinement magnetic field for future fusion reactors, because a condition for self-ignition plasma strongly depends on the confinement capability of fast ion originated from nuclear fusion reaction. They are also inevitable issues for the alleviation of heat load to the first wall. In toroidal helical devices, due to the three dimensional complicated structure of the confinement magnetic field, the orbits of fast ion are much more complicated than those in tokamak devices. It is important to understand their behavior for fusion reactors of toroidal helical type. As the simulated experiment of the MeV energy ions generated by the nuclear fusion reactions, fast ions are mainly produced by injecting neutral beam (NB).

Currently, transport phenomenon of fast ion originated from NB is mainly reproduced by a simulation calculation with Monte-Carlo method such as GNET code [1]. The calculation is possible to reflect the exact information of system (such as a geometrical structure) too complex to describe by analytical methods, however it takes more computational time. Meanwhile, their analytical formulation had also been done, relatively simple such as the classical slowing down distribution f_c given as

$$f_c = \frac{S}{4\pi}\frac{\tau_s}{v^3 + v_c^3} \tag{1}$$

are used in general. Here, S is the particle production rate of NB in the velocity space, τ_s the slowing down time, v_c the critical velocity. This distribution function reflects only classical slowing down effects due to the Coulomb interaction. The analytical formula including not only the collisional drag but also pitch angle scattering and particle loss due to the charge exchange reaction were also obtained [2, 3]. However their objects are tokamak devices, the effect of particle orbit on the complicated magnetic field in toroidal helical systems are not made consideration. Then, the analytical formula of distribution function for fast ion including the effect of particle orbits is derived in section 2. The main advantages to use this method will be able to reduce the computational time than the other methods, thus systematic analyses are possible. In section 3, calculated results of the energy spectra based on the derived solution are compared with those experimentally measured by neutral particle analyzer (NPA) based on NDD [4, 5] installed on LHD for a specific condition. In section 4, summary are presented.

2. Formulation

In this section, the analytical formula including the effects of orbit loss is derived as an evaluation formula of the energy spectra of fast ions produced by NBI heating. A description of the phenomena requires use of the kinetic transport equation. Regarding the collision processes as Markovian, the equation as a master equation describing the distribution function f of fast ions produced by injecting NBs is given as

$$\begin{aligned}\tau_s\frac{\partial f}{\partial t} =& \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^3+v_c^3\right)f + \frac{Z_{eff}v_c^3}{2v^3}\frac{\partial}{\partial\xi}\left(1-\xi^2\right)\frac{\partial f}{\partial\xi}\\ &+\tau_s\sum_n\frac{S_n}{v^2}\delta\left(v-v_n\right)\delta\left(\xi-\xi_n\right) - \frac{\tau_s}{\tau_{CX}}f\\ &-H\left(\lambda_a-\lambda\right)H\left(1-k_0^2\right)f\\ &-H\left(\lambda-\lambda_b\right)H\left(k_0^2-1\right)f\end{aligned} \tag{2}$$

author's e-mail: nishimura.hidetoshi@LHD.nifs.ac.jp

The first two terms in the right hand side are Coulomb collision terms, which describe a friction and a pitch angle scattering in the velocity space, respectively. Here, v is the absolute value of particle velocity, Z_{eff} the effective charge of the background plasma, and ξ the directional cosine to the direction of the magnetic field. Background plasma are assumed in the equilibrium state, thus their distribution function in the velocity space is canonical and their terms are linearized. The effects of energy diffusion are neglected for simplicity. The third term is a particle source term originating from NBI heating. Here, S_n, v_n, and ξ_n is the source rate, the velocity, and directional cosine of the NB particles, respectively, and the summation respect to n is executed over the number of NBs. The velocity broadening and the angle spread of NBs are neglected. However their effects can be considered in principle, because the responses to the non-impulsive source are completely described by the solution f of the above differential equation. The fourth term is particle loss term due to the charge exchange reaction, where τ_{CX} is a characteristic time of the variation of the distribution function in their reaction processes. The last two terms are particle loss terms due to the loss cone. Here, H is a Heaviside unit function, and their variables, λ is a pitch-angle-like parameter which uniquely corresponds to a pitch angle of the fast ion and k_0^2 is a parameter which characterizes the drift orbits of the charged particles, defined as,

$$\lambda \equiv \mu B/E \quad \text{and} \quad k_0^2 \equiv \frac{E + \mu B - e\Phi}{2\mu B}, \tag{3}$$

respectively. Here μ is the magnetic moment, B the magnetic field strength, E the total energy of the particle, e the particle charge, and Φ the electrostatic potential. The case of $0 < k_0^2 < 1$ corresponds to trapped particles, the case of $1 < k_0^2$ corresponds to un-trapped particles. λ_a and λ_b in (1) are the boundary values (whether or not particles are lost) of λ corresponding to trapped particle and un-trapped particle, respectively, same as in [6]. The description of orbital particle loss term using Heaviside unit function postulates that particles are lost at the moment that the particles satisfy a loss condition. The particle trajectories for the determination of the loss condition in the magnetic field are qualitatively evaluated by Lorentz orbit code [7], which numerically integrates kinetic equation given as

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = ev \times B, \tag{4}$$

where m is a particle mass. Here, set the source term as

$$S\,(v, \xi, t) = \sum_n \frac{S_n}{v^2}\delta\,(v - v_n)\,\delta\,(\xi - \xi_n). \tag{5}$$

the distribution function f and the source function S are expanded by using the terms of Legendre polynomial P_l which constitute complete orthogonal basis [8], then

$$f\,(v, \xi, t) = \sum_{l=0}^{\infty} f_l\,(v, t)\,P_l\,(\xi) \tag{6}$$

and

$$S\,(v, \xi, t) = \sum_{l=0}^{\infty} S_l\,(v, t)\,P_l\,(\xi). \tag{7}$$

In order to obtain the quasi-steady state solution, neglecting the time dependence of the distribution function f and the source function S, integrating the kinetic transport equation over the velocity, the analytical solution of the differential equation is derived as

$$f\,(v, \xi) = \sum_{l=0}^{\infty}\left(l + \frac{1}{2}\right)\frac{\tau_s}{v^3 + v_c^3}\sum_n S_n \sigma_l\,(v, \xi; v_n, \xi_n) \times P_l\,(\xi)\exp\left(-\tau_s\int_v^{v_n}\frac{\alpha\,(v')}{v'\tau_{CX}}dv'\right) \tag{8}$$

with

$$\sigma_l\,(v, \xi; v_n, \xi_n) = P_l\,(\xi_n)\,\frac{\alpha\,(v_n)^{\gamma(v,\xi)-\beta_l}}{\alpha\,(v)^{\gamma(v,\xi)-\beta_l}}H\,(v_n - v)\,, \tag{9}$$

$$\alpha\,(v) = \frac{v^3}{v^3 + v_c^3}, \quad \beta_l = \frac{l\,(l+1)\,Z_{eff}}{6}, \tag{10}$$

and

$$\gamma\,(v, \xi) = \frac{\tau_s}{3}\left[H\,(\lambda_a - \lambda_0)\,H\left(1 - k_0^2\right) + H\,(\lambda_0 - \lambda_b)\,H\left(k_0^2 - 1\right)\right]. \tag{11}$$

Here, because the effect of energy diffusion is neglected, the integration constants for each Legendre coefficients are determined to satisfy the condition for no ions having larger velocity than the maximal value of beam injection velocity v_n. In the limit of full particle confinement, $\gamma = 0$, this distribution function is consistent with that in [3]. At the energy range where the dependence of a charge exchange cross section on energy can be neglected, the integral respect to velocity v' is executed and the distribution function is determined by completely analytical methods.

3. Evaluation of Derived Solution

In this section, the validity of the solution derived in section 2 is appraised by comparing with the energy spectra of fast ion experimentally measured by NDD installed on LHD for a specific condition. In general, the position of the magnetic axis R_{ax} can be controlled with external vertical magnetic field, the shifts from about 3.4 to 4.1 m are possible in LHD [9]. It is numerically shown that the orbit of fast ion with relatively large pitch angle depends on the magnetic field configuration [10]. Thus, in order to evaluate the difference of orbit loss of fast ions depending on the magnetic field configuration, fast ions with relatively large pitch angle are chosen as a target of this comparison and typical three cases of magnetic field configuration (R_{ax} = 3.53, 3.6, 3.75 m) are used. The schematic view of the sightline of NDD on the side cross sectional view of LHD for the case of R_{ax} = 3.6 m (left) and the distribution of pitch angle along the sightline of NDD (right) are shown in Fig. 1. In this figure, Z stands for the height, $Z = 0$ corresponds to the equatorial plane of LHD. The blue elliptic cross sections are magnetic flux surfaces in the magnetic field configuration. On the determination of loss boundaries, λ_a and λ_b, the classification of fast ion orbits depending on the pitch angle and the initial starting points of major radius R calculated by Saida [7] using Lorentz orbit code for R_{ax} = 3.53, 3.6, 3.75 m are shown in Fig. 2. The red region represents orbit loss region of fast ions, the other regions represent confinement regions. The blue and yellow region corresponds to ripple trapped particles, the green region corresponds to passing particles. In this calculation the effects of the passing particles on the orbit loss are neglected at all.

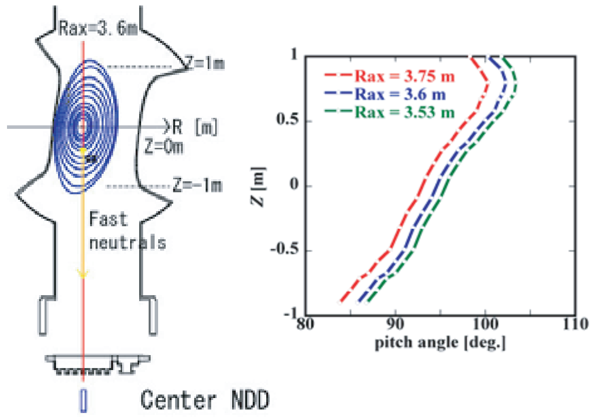

Fig. 1 The schematic view of the sightline of natural diamond detector (NDD) on the side cross sectional view of Large Helical Device (LHD) for the case of R_{ax} = 3.6 m (left), and the distribution of pitch angle along the sightline of NDD for R_{ax} = 3.53, 3.6, 3.75 m. (right). The blue elliptic cross sections are magnetic flux surfaces.

Main experimental discharge conditions used in this comparison are below. The strength of magnetic field is 2.5 T. The plasma is sustained in quasi-steady state with tangentially injecting Co-NB and counter-NB, their injection power are 2.3 and 1.6 MW, respectively, and their acceleration energy is about 150 keV. The (line averaged) electron density is 1-2 $\times 10^{19}$ m^{-3}. The ion temperature at the center of plasma is 2-3 keV.

The comparison of the energy spectra of fast ions between the calculation and the experimental measurements for R_{ax} = 3.53, 3.6, 3.75 m are shown in Fig. 3. The black line with symbols represents experimentally measured energy spectra (time-integrated during 0.2 s) of fast ions corrected by charge exchange reaction rate. Here, the attenuation of fast neutral particle flux due to the reionization reactions with neutral particles is neglected. The others represent numerically calculated energy spectra, which are obtained by line integrating the distribution function in the pitch angle along the sightline of NDD shown in Fig. 1. In the calculation the effective charge of the background plasma Z_{eff} assumed to be 1.2 for hydrogen majority plasma. The blue and red line corresponds to the energy spectra with and without the orbit loss, respectively. In the energy range below 20 keV background noise is dominant, both calculated energy spectra are normalized at the energy of about 20 keV, only the behavior of the energy spectra is compared. This is mainly because source rate of NBs is not exactly obtained. The calculation results with orbit loss for each case of R_{ax} = 3.53, 3.6, 3.75 m are in good agreement with experimentally measured them. It is found from the comparison of the energy spectra without orbit loss that the difference of sight-line with R_{ax} shifts constitutes a major cause for the difference of total flux. The deviation of

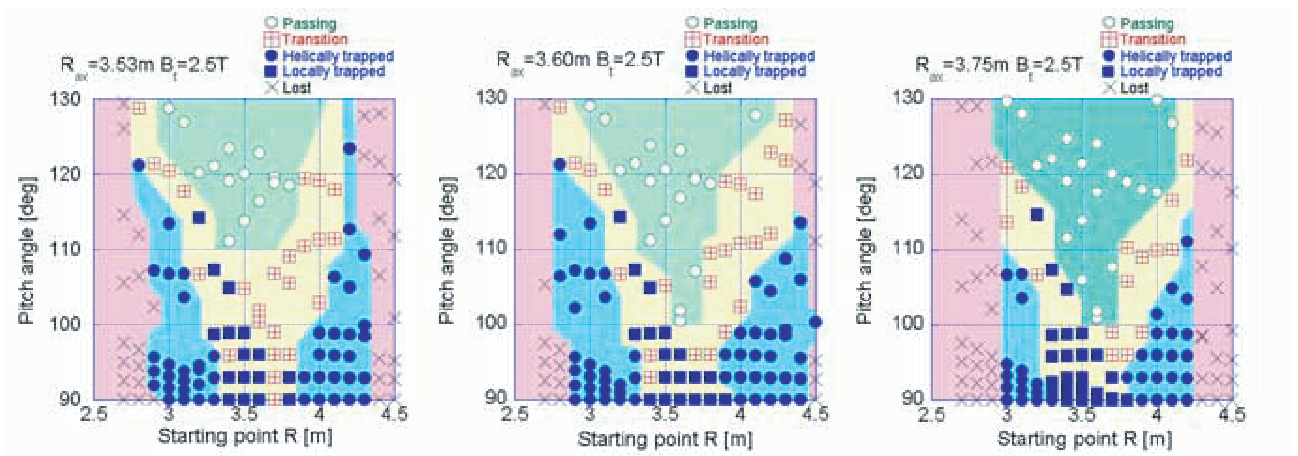

Fig. 2 The classification of fast ion orbits depending on the pitch angle and the initial starting points R calculated by Lorentz orbit code for each cases of magnetic axis R_{ax} = 3.53, 3.6, 3.75 m in LHD [7]. The red region represents orbital loss region of fast ions, the other regions represent orbital confinement regions. The blue and yellow region corresponds to ripple trapped particles, the green region corresponds to un-trapped particles.

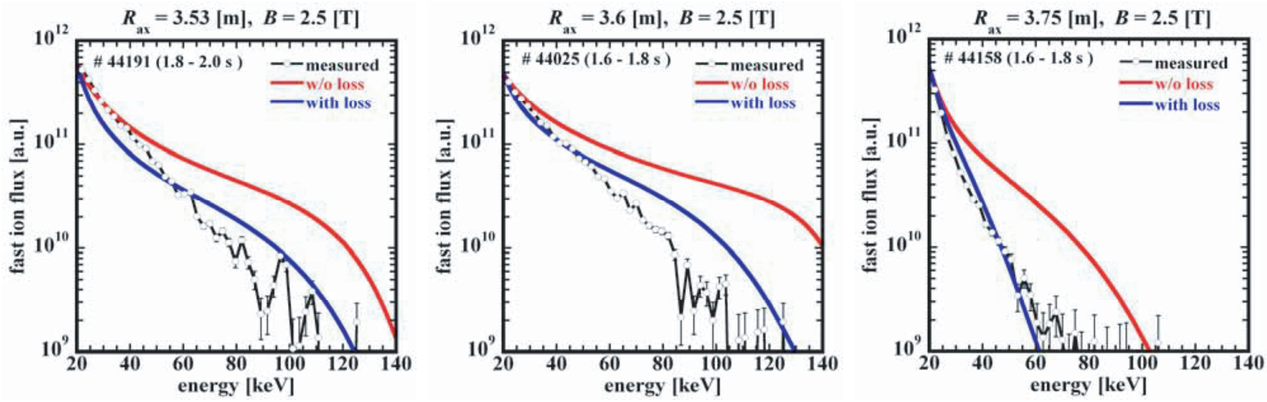

Fig. 3 The comparison of the energy spectra of fast ions between the calculation based on the distribution function and those experimentally measured by NDD for each cases of R_{ax} = 3.53, 3.6, 3.75 m. The black line with symbols represents experimentally measured energy spectra of fast ions. The others represent numerically calculated energy spectra, the blue and red line corresponds to the energy spectra with and without the effect of the orbital particle loss. In the energy region below 20 keV, the background noise is dominant.

the energy spectrum with orbit loss from the experiment at relatively high energy region for the case of R_{ax} = 3.53, 3.6 m is though to be resulted from the effect unconsidered here such as the energy diffusion and/or the attenuation factor. The reason is that for R_{ax} = 3.75 m the effect of particle loss on the energy spectra is dominant, thus the other unconsidered effects are not observed explicitly, but for R_{ax} = 3.53, 3.6 m the effect of particle loss is relatively small, thus the other effects can not be neglected. On the effect of the energy diffusion as a possible cause of the deviations, the energy diffusion terms may be introduced into kinetic transport equation as a perturbation, because they are relatively small as well as the deviations.

4. Summary

The effect of particle orbit can not be neglected in toroidal helical systems with three dimensional complicated structure of magnetic field for thought in considering the energy spectrum of fast ion associated with the transport phenomenon. To overcome this issue, the orbit loss terms are introduced into kinetic transport equation as a master equation. By integrating the equation as possible as analytical methods, the distribution function of fast ion produced by NBIs including the effects of orbital loss is derived. In order to evaluate the validity, the energy spectra of fast ion calculated from the derived distribution function are compared with those experimentally measured by NDD in LHD for a specific condition. Good agreements in their behavior are obtained for each case of R_{ax} = 3.53, 3.6, 3.75 m with orbit loss. This analytical formula will be able to reduce the computational time, less than 10 min. typically, and then systematic analysis of the energy spectra of fast ion will be possible.

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