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Formation of parallel electric field by trapped and energetic electrons in an oblique shock wave

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The effects of trapped and energetic electrons on parallel electric field, $E_\parallel = (\boldsymbol{E}\cdot\boldsymbol{B})/B$, in a magnetosonic shock wave propagating obliquely to an external magnetic field are studied by theory and particle simulations. The analytical expression for F, where F is the integral of $E_\parallel$ along $\boldsymbol{B}$ is obtained, including the number of trapped electros $n_{\rm t}$ as a factor. It is shown that as $n_{\rm t}$ increases, the magnitude of F increases. Theoretical analysis also suggests that the increase in F causes the electrons to be trapped deeper and accelerated to higher kinetic energies. These theoretical predictions are confirmed by relativistic electromagnetic particle simulations.

Keywords: Particle acceleration, collisionless shock, nonlinear wave, wave-particle interaction, particle simulation

1. Introduction

It has been found with theory and particle simulations [1] that prompt electron acceleration to ultrarelativistic energies with $\gamma > 100$, where γ is the Lorentz factor, can occur in a magnetosonic shock wave propagating obliquely to an external magnetic field with $|\Omega_e|/\omega_{\rm pe} \gtrsim 1$, where $\Omega_e(<0)$ and $\omega_{\rm pe}$ are the electron gyro and plasma frequencies, respectively. The acceleration is extremely strong when the propagation speed of the shock wave $v_{\rm sh}$ is close to $c\cos\theta$, where θ is the angle between the wave normal and magnetic field. In this acceleration mechanism, the electric field parallel to the magnetic field $E_\parallel$ in the oblique shock wave and its integral along the magnetic field, $F = -\int E_\parallel ds$, play essential roles. Some electrons can get trapped when a negative dip of F is formed in the end of main pulse. The electrons then oscillate in the main pulse region and their kinetic energies take maxima near the position of the peak of F. For this mechanism, a clear physical picture was given in Ref. [1] and a theory for the maximum energy was developed in Ref. [2] under the assumption that the wave is stationary. In these works, the effects of trapped electrons on wave evolution were not concerned, and the time variations of the electron maximum energy were not studied.

The simulations also demonstrated that once electrons are trapped, they cannot readily escape from the wave and are trapped deep in the main pulse region, which indicates that the number of trapped electrons increases continually with time [2]. In Ref. [3], the mechanism for the deep trapping was discussed. It was shown with theory and simulation that if $\partial F/\partial t > 0$ at particle positions, the parallel energies of the reflected electrons decrease, causing deep trapping. The reason for the increase of F is, however, unclear.

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In this paper, we study, with theory and long-time simulations, the feedback of the trapped and accelerated electrons on the shock wave. We develop a theory for the field strength including the number of the trapped electrons as a factor, and compare it with the simulations. It is found that the trapped electrons strengthen $E_\parallel$ and F and that because of this, the magnitude of F increases with time. These results lead to the conclusion that the electrons are trapped deeper and accelerated to higher kinetic energies owing to the electromagnetic fields that they produce themselves.

In Sec. 2, we present a theory for effects of trapped electrons on F. We then discuss how the change in F affects the motions of trapped electrons in Sec. 3. In Sec. 4, the theoretical prediction is confirmed by a one-dimensional (one space coordinate and three velocity components) relativistic electromagnetic particle simulation. Section 5 gives a summary of our work.

2. Theory for effects of trapped electrons on F

We theoretically study effects of trapped electrons on parallel electric field in an oblique shock wave. If the number of the trapped electrons $n_{\rm t}$ increases with time, the shock wave profile would change slowly with time. We assume that the shock wave profile at a time t can be described by a stationary solution for $n_{\rm t}$ at this time. This assumption would be valid if the characteristic time of the change in $n_{\rm t}$ is much longer than that of the shock wave propagation. This condition can be written as

$$n_{\rm t}/(dn_{\rm t}/dt) \gg \Delta_{\rm m}/v_{\rm sh}, \quad (1)$$

where $\Delta_{\rm m}$ is the width of the main pulse region, and $v_{\rm sh}$ is the propagation speed of the shock wave. Simulation results are consistent with Eq. (1), which will be shown in Sec. 4.

We consider a magnetosonic shock wave propagating in the x direction in a external magnetic field in the (x, z) plane,

$$\boldsymbol{B}_{l0} = B_{l0}(\cos\theta, 0, \sin\theta), \tag{2}$$

where the subscript l refers to the quantities in the laboratory frame and the subscript 0 indicates the far upstream region. We suppose that the wave is stationary. Then, in the wave frame, the time derivatives of the quantities are zero, $\partial/\partial t = 0$, and the Faraday's law gives the y and z components of the electric field as constants;

$$E_{wy} = E_{wy0} = -(v_{\rm sh}/c)\gamma_{\rm sh}B_{lz0}, \tag{3}$$

$$E_{wz} = E_{wz0} = 0, \tag{4}$$

where the subscript w denotes the wave frame and $\gamma_{\rm sh} = (1 - v_{\rm sh}^2/c^2)^{-1/2}$. For one-dimensional propagation with $\partial/\partial y = \partial/\partial z = 0$, the x component of the magnetic field is constant,

$$B_{wx} = B_{wx0} = B_{lx0}. \tag{5}$$

In the following, we analyze quantities in the wave frame, for which we omit the subscript w.

By virtue of Eqs. (3)–(5), we write the parallel electric field $E_\parallel$ in the wave frame as

$$E_\parallel = (E_x B_{x0} + E_{y0} B_y)/B, \tag{6}$$

from which the parallel potential F is given as

$$F = -\int ds E_\parallel = -\int_{x_0}^{x} dx E_\parallel B/B_{x0}, \tag{7}$$

where x_0 is a certain point in the far upstream region. This is also expressed, with the electric potential ϕ, as

$$F = \phi - \frac{E_{y0}}{B_{x0}} \int_{x_0}^{x} dx B_y. \tag{8}$$

For the case of no trapped electrons, the expression for F was given in Ref. [4]. For small amplitude waves in a cold plasma, F is given as $eF \sim \delta^2 m_{\rm i} v_{\rm A}^2$, where δ is the wave amplitude and $v_{\rm A}$ is the Alfven speed. For large-amplitude shock waves with $\delta \sim 1$, simulation values of F are consistent with the phenomenological relation,

$$eF \sim m_{\rm i} v_A^2 B_m/B_0, \tag{9}$$

where B_m is the maximum B.

We here consider the effect of trapped electrons on F. The trapped electrons generate B_y because they move along the magnetic field with parallel speed $v_\parallel \sim c$ in the main pulse region where $B_z \simeq B$. Assuming that the current of trapped electrons is given by $J_{\rm t} \sim (0, 0, -en_{\rm t}c)$, we can estimate, from the z component of Ampere's law, the magnitude of B_y produced by the trapped electrons as

$$B_y^{(\rm t)} \sim -\frac{4\pi}{c} J_{{\rm t}z}\Delta_{\rm h} \sim 4\pi e n_{\rm t}\Delta_{\rm h}, \tag{10}$$

where $\Delta_{\rm h}$ is the half width of the region where the trapped particles exist and the index (t) indicates the quantities produced by the trapped electrons.

Even if $n_{\rm t}$ is quite small, the contribution of the trapped electrons to B_y can be significant. In order to show this, we compare the magnitude of $B_y^{(\rm t)}$ and that of $B_y^{(0)}$, where the index (0) indicates the quantities produced by the transmitted electrons. The ratio of these values is given by

$$\frac{B_y^{(\rm t)}}{B_y^{(0)}} \sim \frac{J_{tz}}{J_{ez}} \sim \frac{n_{\rm t}c}{n_e v_{ez}}, \tag{11}$$

where the subscript e denotes the passing electrons. Because v_{ez} is much smaller than c, this ratio can be great. The contribution of trapped electrons to ϕ is, however, negligible if $n_{\rm t}$ is small, because $\phi^{(\rm t)}/\phi^{(0)} \sim n_{\rm t}/n_{\rm e}$.

We thus obtain, from Eqs. (8) and (10), $F^{(\rm t)}$ as

$$F^{(\rm t)} \sim -2\pi \frac{E_{y0}}{B_{x0}} n_{\rm t} e \Delta_{\rm h}^2. \tag{12}$$

If $v_{\rm sh} \sim c\cos\theta$, Eq. (12) gives

$$eF^{(\rm t)} \sim \frac{n_{\rm t}}{2n_{\rm e0}} \frac{\Delta_{\rm h}^2 \omega_{\rm pe}^2}{c^2} m_{\rm e}c^2 \tag{13}$$

This indicates that the magnitude of F increases with $n_{\rm t}$.

3. Electron motion in nonstationary F

Particle simulation demonstrates that once electrons are trapped in the main pulse region, they cannot readily escape from it and the number of trapped electrons $n_{\rm t}$ continually increases with time [2, 3]. Since Eq. (13) suggests that the magnitude of F increases with $n_{\rm t}$, we here suppose that F gradually grows with time in association with the increase of $n_{\rm t}$ and then discuss how the time change of F affects the motion of trapped electrons.

We assume that B_y and F change with time, while other quantities are stationary. From the y and z components of equation of motion, we have

$$v_z = -\frac{c}{eB_{x0}}\frac{dp_y}{dt} - \frac{c}{B_{x0}}E_{y0} + \frac{B_z}{B_{x0}}v_x, \tag{14}$$

$$v_y = \frac{c}{eB_{x0}}\frac{dp_z}{dt} + \frac{B_y}{B_{x0}}v_x. \tag{15}$$

Substituting Eqs. (14) and (15) into an energy equation for electrons,

$$m_e c^2 \frac{d\gamma}{dt} = -e\boldsymbol{E}\cdot\boldsymbol{v}, \tag{16}$$

we obtain

$$\frac{d}{dt}(\gamma h) = -e\frac{\boldsymbol{E}\cdot\boldsymbol{B}}{B_{x0}}v_x, \tag{17}$$

where h is defined as

$$h = m_e c^2\left(1 - \frac{v_z v_{\rm sh} B_{z0}}{c^2 B_{x0}}\right), \tag{18}$$

which indicates that h is positive if B_{z0}/B_{x0} is of order unity [1]. We introduce F defined as [3]

$$F(x,t) = -\int^{x}\frac{\boldsymbol{E}(x',t)\cdot\boldsymbol{B}(x',t)}{B_{x0}}dx'. \tag{19}$$

Using the relation

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} - \frac{\boldsymbol{E}(x,t)\cdot\boldsymbol{B}(x,t)}{B_{x0}}\frac{dx}{dt}, \tag{20}$$

we can put Eq. (17) into the following form

$$\frac{d\varepsilon}{dt} = -e\frac{\partial F}{\partial t} \tag{21}$$

where ε is defined as

$$\varepsilon = \gamma h - eF. \tag{22}$$

We call ε energy as in Ref. 3. If F is in the region $0 < F < F_{\rm m}$, particles with energies in the region $-eF_{\rm m} < \varepsilon < 0$ are trapped.

We assume that F at time t is written as

$$F(x,t) = F^{(0)}(x)[1 + \alpha n_{\rm t}(t)], \tag{23}$$

where $n_{\rm t}(t)$ is the number density of trapped electrons at time t, $F^{(0)}(x)$ is given by Eq. (9), and α is of constant order,

$$\alpha \sim \frac{1}{n_{\rm t}}\frac{F_{\rm m}^{(\rm t)}}{F_{\rm m}^{(0)}} \sim \frac{1}{n_{\rm e0}}\frac{\omega_{\rm pe}^2}{|\Omega_{\rm e}|^2}\frac{B_0}{B_{\rm m}^{(0)}}. \tag{24}$$

Substituting Eq. (23) into Eq. (21), we have

$$\frac{d\varepsilon}{dt} \simeq -e\alpha F^{(0)}(x)\frac{dn_{\rm t}}{dt}. \tag{25}$$

This indicates that if $dn_{\rm t}/dt > 0$, the energy ε decreases, which gives rise to deep trapping of electrons; just as a particle oscillating in a potential well with damping. We therefore find that the trapped electrons become more deeply trapped owing to the electromagnetic fields that they produce themselves.

The increase of F enhances the acceleration of trapped electrons. From Eq. (22), We can write γ of the particle at time t and position x as

$$\gamma(x,t) = \frac{\varepsilon(t) + eF(x,t)}{m_{\rm e}c(c + v_{\parallel}E_{y0}/B_{x0})}. \tag{26}$$

Since $\varepsilon(t) \lesssim 0$ for trapped particles and $F(x,t) \le F_{\rm m}(t)$, where $F_{\rm m}(t)[\equiv F(x_{\rm m},t)$ with x_m the center of the main pulse] is the maximum F at time t, the upper limit of γ of trapped electrons is given as

$$\gamma_{\rm lim}(t) = \frac{eF_{\rm m}(t)\cos\theta}{m_{\rm e}c(c\cos\theta - v_{\rm sh})}, \tag{27}$$

where we have used $v_{\parallel} \simeq c$. The upper limit of γ at time t is proportional to $F_{\rm m}(t)$.

We now consider the time variation of γ of a trapped particle. We suppose that the particle gets trapped at time t_0 and its γ takes maximum values $\gamma_{\rm m}$ at times $t_1, t_2, \cdots$, at which the particle is near the position $x = x_{\rm m}$ with $v_{\parallel} \simeq c$. The value of $\gamma_{\rm m}$ can then be estimated from Eq. (26) as

$$\gamma_{\rm m}(t_n) = \frac{[\varepsilon(t_n) + eF_{\rm m}(t_n)]\cos\theta}{m_{\rm e}c(c\cos\theta - v_{\rm sh})} \tag{28}$$

where n is an integer. To obtain $\varepsilon(t_n)$, we integrate Eq. (21) from t_0 to t_n, giving

$$\varepsilon(t_n) - \varepsilon(t_0) \sim -e\alpha\langle F^{(0)}\rangle[n_{\rm t}(t_n) - n_{\rm t}(t_0)] \tag{29}$$

where $\langle F^{(0)}\rangle$ is the average of $F^{(0)}$ over the main pulse region. Using the approximation $\langle F^{(0)}\rangle \sim F_{\rm m}^{(0)}/2$, we can write $\gamma_{\rm m}(t_n)$ as

$$\gamma_{\rm m}(t_n) \sim \frac{e[F_{\rm m}(t_n) + F_{\rm m}(t_0) + \varepsilon(t_0)]\cos\theta}{2m_{\rm e}c(c\cos\theta - v_{\rm sh})}. \tag{30}$$

From this, we can expect that if $F_{\rm m}$ increases, $\gamma_{\rm m}$ also increases.

4. Particle Simulations

In this section, using a one-dimensional (one space coordinate and three velocities), relativistic, electromagnetic particle code with full ion and electron dynamics, we simulate an oblique shock wave and confirm that the number of trapped particles $n_{\rm t}$, the magnitude of F, and the maximum energies of electrons grow with time.

As in the theory in Sec. 2, the shock wave propagates in the x direction in an external magnetic field $\boldsymbol{B}_0 = B_0(\cos\theta, 0, \sin\theta)$. The propagation angle is set to be $\theta = 45°$. The total system length is $L = 16384\Delta_g$, where Δ_g is the grid spacing. The number of ions and electrons are $N_{\rm i} = N_{\rm e} \simeq 1.0\times10^7$. The mass ratio is $m_{\rm i}/m_{\rm e} = 100$. The ratio of gyro and plasma frequencies of electrons is $|\Omega_{\rm e}|/\omega_{\rm pe} = 3.0$ in the upstream region. The light speed is $c/(\omega_{\rm pe}\Delta_g) = 4.0$ and the electron and ion thermal velocities in the upstream region are $v_{\rm Te}/(\omega_{\rm pe}\Delta_g) = 0.5$ and $v_{\rm Ti}/(\omega_{\rm pe}\Delta_g) = 0.05$, respectively. The Alfven speed is then $v_{\rm A}/(\omega_{\rm pe}\Delta_g) = 1.2$. We present the simulation results for the shock wave with $v_{\rm sh}$ being 96% of the $c\cos\theta$.

Figure 1 shows electron phase space plots (x,γ) and magnetic field profiles of a shock wave at times $\omega_{\rm pe}t = 480$ and 1300. In the top panel ($\omega_{\rm pe}t = 480$), we find some electrons are trapped and accelerated to ultrarelativistic energies with $\gamma > 50$ in the main pulse region, $478 < x/(c/\omega_{\rm pe}) < 485$. At $\omega_{\rm pe}t = 1300$, more particles are trapped and are accelerated to higher energies; the maximum value of γ reaches $\gamma \simeq 200$.

Figure 2 displays the profiles of F (dashed line) and B_z (solid line). F and B take their maximum values at almost the same positions. The peak value of F at $\omega_{\rm pe}t = 480$, at which the number of trapped particles is small, is observed to be $eF/(m_{\rm e}c^2) \sim 9$. This is in good agreement with the theoretical value, $eF/(m_{\rm e}c^2) \sim 7$, which was obtained from Eq. (9) using $n_{\rm t} \simeq 0$ and the observed value of $B_{\rm m}$. At $\omega_{\rm pe}t = 1300$, at which more electrons are trapped, the peak value of F is greater.

In order to compare the simulation and the theory for F, we plot in Fig. 3 the time variations of the maximum values of ϕ and F and of the number density of trapped electrons. (Here, $n_{\rm t}$ is approximated as $n_{\rm t}/\Delta_m$, where Δ_m is the width of the main pulse region and $N_{\rm t}$ is the number of energetic electrons with $\gamma > 10$ in the main pulse region; the trapped electrons can have such high energies, while the transmitted ones have energies of, at most, $\gamma = 10$.) The value of ϕ_m is almost constant although $n_{\rm t}$ increases with time. The magnitude of F, however, increases with an increase in $n_{\rm t}$. The increment of $F_{\rm m}$ from $\omega_{\rm pe}t = 400$ to 1300 is observed to be $e\Delta F_{\rm m}/(m_{\rm e}c^2) \simeq 3$, which is about 30% of the value estimated from Eq. (13). Substituting the observed value of the increment of $n_{\rm t}$ for this period, $n_{\rm t}/n_{\rm e0} \sim 5$ and half width of the region where the trapped particles exsit, $\Delta_{\rm h} \sim 2c/\omega_{\rm pe}$, in Eq. (13) gives $e\Delta F_{\rm m}/(m_{\rm e}c^2) \sim 10$. It is thus clearly shown that $F_{\rm m}$ grows owing to the effect of the trapped electrons. [The oscillations of $\phi_{\rm m}$, $F_{\rm m}$, and $n_{\rm t}$ with the period $\omega_{\rm pe}t \simeq 70$ [$\simeq (2\pi/3)(\omega_{\rm pe}/\Omega_{\rm i})$] are due to the ion reflection at the shock front [5, 6, 7].]

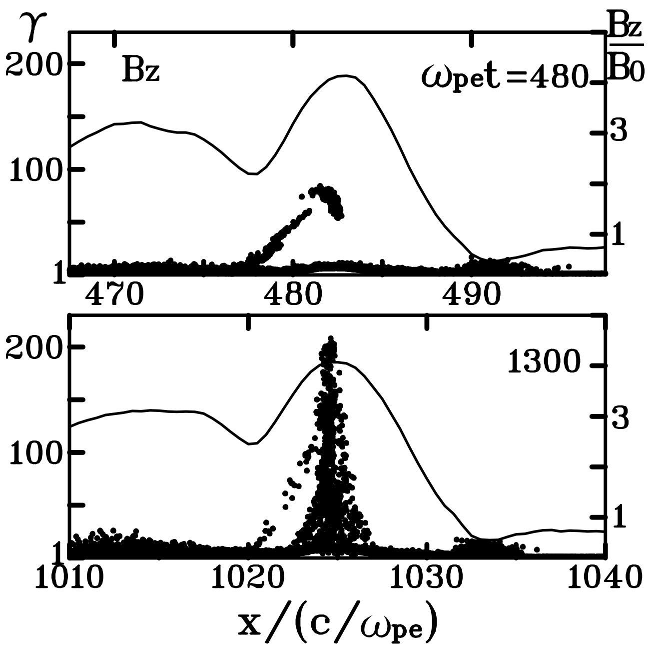

Fig. 1 Phase space plots (x, γ) of electrons and magnetic field profiles at $\omega_{\rm pe}t = 480$ and 1300.

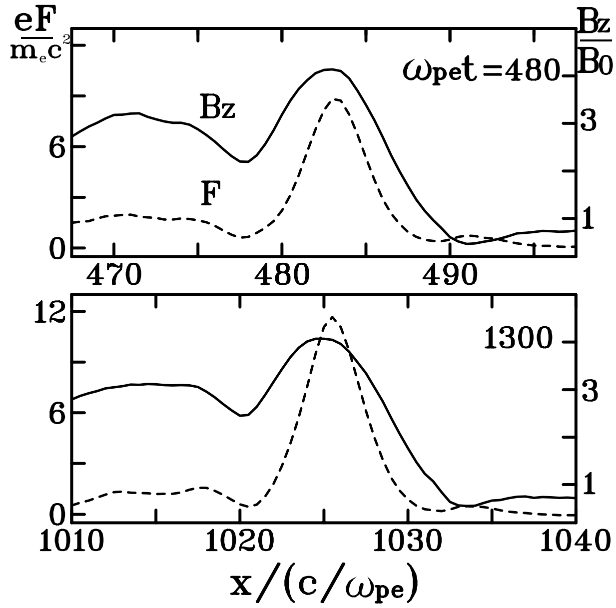

Fig. 2 Profiles of B_z (solid lines) and F (dashed lines) at $\omega_{\rm pe}t = 480$ and 1300.

From Fig. 3, we can confirm that the assumption used in the theory is appropriate, because condition (1) is satisfied. Using the observed value of $n_{\rm t}$, we can estimate $n_{\rm t}/(dn_{\rm t}/dt) \sim 1000/\omega_{\rm pe}$, which is much greater than $\Delta_{\rm m}/v_{\rm sh} \sim 10/\omega_{\rm pe}$.

We now present results showing that the increase

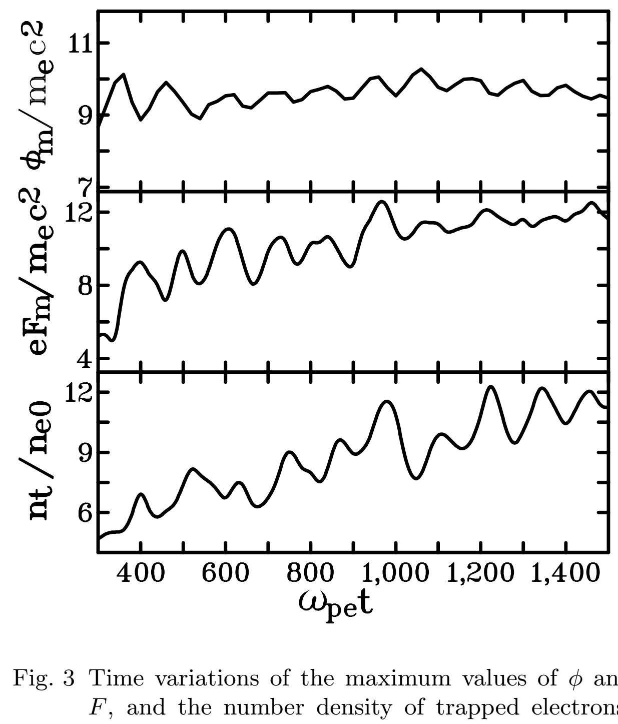

Fig. 3 Time variations of the maximum values of ϕ and F, and the number density of trapped electrons, $n_{\rm t}$. The values of $F_{\rm m}$ and $n_{\rm t}$ increase with time, while $\phi_{\rm m}$ is almost constant.

of F can enhance the electron acceleration. Figure 4 shows time variations of the observed value of the maximum γ of the electrons (black line). The maximum γ increases on average with time, due to the increase of F_{m}. The gray line in Fig. 4 indicates the theory (27) for the upper limit of γ at time t, where we have substituted the observed value of $F_{\mathrm{m}}(t)$ in Eq. (27) and have averaged over the time period of the amplitude oscillation due to the ion reflection, $\omega_{\mathrm{pe}}t = 70$. The profiles of γ_{m} and γ_{lim} are similar and their values are in the same order of magnitude. We can therefore confirm that the increase of γ_{m} is caused by that of F_{m}.

We next confirm that the theory (28) can explain the simulation result. Figure 5 shows the trajectory of a trapped electron, where the time variations of $x - x_{\mathrm{m}}$, γ, and $v_{\parallel}$ are plotted. The electron encounters the shock wave at $\omega_{\mathrm{pe}}t \simeq 300$. At $\omega_{\mathrm{pe}}t \simeq 400$, it is reflected at the end of the main pulse and gets trapped in the main pulse region. It moves forward relative to the shock wave with $v_{\parallel} \simeq c$. Its kinetic energy becomes maximum near the center of the main pulse, $x \simeq x_{\mathrm{m}}$, at $\omega_{\mathrm{pe}}t = 700$. The electron is then reflected backward in the shock transition region at $\omega_{\mathrm{pe}}t \simeq 1000$. It soon reaches the end of the main pulse and is again reflected forward. Its kinetic energy becomes maximum at $\omega_{\mathrm{pe}}t \simeq 1400$. Note that the second peak of γ at $\omega_{\mathrm{pe}}t \simeq 1400$ is higher than the first one at $\omega_{\mathrm{pe}}t \simeq 700$. This is due to the increase of F. The difference between the two maximum γ's, $\Delta\gamma_{\mathrm{m}} \simeq 16$, can be explained by Eq. (28); substituting the observed value of the increase of F_{m} from $\omega_{\mathrm{pe}}t = 700$ to $\omega_{\mathrm{pe}}t = 1400$ in Eq. (28), we have $\Delta\gamma_{\mathrm{m}} \simeq 20$.

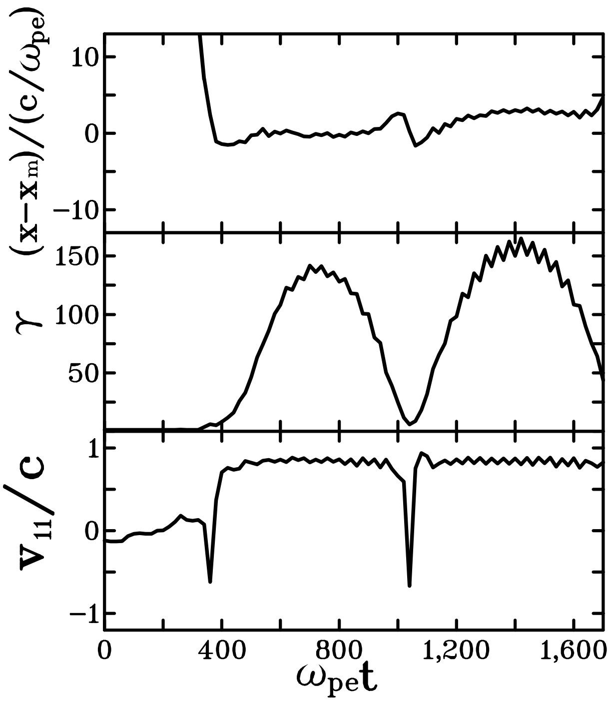

Fig. 5 Time variations of $x - x_{\mathrm{m}}$, γ, and $v_{\parallel}$ of a trapped electron.

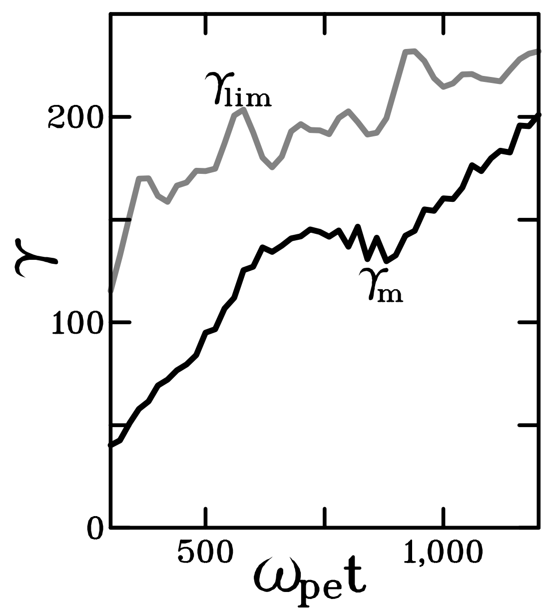

Fig. 4 Time variations of the maximum value of γ of electrons. The simulation result (black line) is similar to the theory (gray line) for the upper limit of γ given by Eq. (25).

5. Summary

A magnetosonic shock wave propagating obliquely to an external magnetic field can trap electrons and accelerate them to ultrarelativistic energies. Once the electrons are trapped, they cannot readily escape from the wave and the number of trapped electrons continually increases with time. The parallel electric field and its integral F along the magnetic field play crucial roles in this trapping and acceleration mechanism.

In order to investigate the effect of the trapped electrons on electromagnetic fields in a shock wave, we derive a theoretical expression for F including the number of the trapped electrons n_{t} as a factor. It is found that the magnitude of F increases with n_{t}. We then suggest that owing to the increase of F, the electrons are trapped deeper and are accelerated to higher kinetic energies.

Particle simulations demonstrate that both F and n_{t} increase with time and that associated with this increase, the kinetic energies of the trapped electrons grow. The theoretical predictions have thus been verified by the simulations.

We note that the theory and simulations are both one dimensional in the present and previous studies. As future work, it would be important to study multidimensional effects on the trapping and acceleration mechanisms.

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