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MHD Flow Field and Momentum Transfer Process of Magneto-Plasma Sail

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Magneto-Plasma Sail is a propulsion system making use of the solar wind in the interplanetary space. This propulsion system creates an artificial magnetosphere as a sail catching the momentum of the solar wind. An artificial magnetosphere created by a superconducting coil is drastically inflated by plasma injection and this technique is called "Magnetic Field Inflation" which allows the spacecraft to produce a large magnetosphere without huge coils. Magneto-Plasma Sail is expected to reduce the mission time for deep space explorations. However, the propulsive performance has not been quantified because the momentum transfer from the solar wind to the spacecraft has not been clarified even in the ideal magnetohydrodynamic framework. In this study, the flow field of Magneto-Plasma Sail was simulated based on the ideal magnetohydrodynamics. The flow field was characterized by Alfvenic Mach number of the injected plasma flow, and it was clarified that MHD waves played important roles in the momentum transfer process. For generating thrust, the MHD wave has to propagate from the solar wind to the spacecraft and therefore the sub-wave speed flow region has to be created between the spacecraft and the solar wind.

Keywords: Space propulsion, Magnetohydrodynamics, Solar wind

1. Introduction

Magneto-Plasma Sail is a propulsion system making use of the solar wind, which is a super-sonic and super-Alfvénic plasma flow, in the interplanetary space (as shown in Fig. 1). This propulsion system, originally called M2P2 by Winglee [1], et al., creates an artificial magnetosphere as a sail catching the momentum of the solar wind. An artificial magnetosphere created by a superconducting coil is drastically inflated by plasma injection from the spacecraft and this technique is called "Magnetic Field Inflation" (as shown in Fig. 2). The magnetic field inflation enables the spacecraft to produce an enough large magnetosphere without huge superconducting coils.

Magneto-Plasma Sail is expected to be an innovative propulsion system reducing the mission time for deep space explorations. Several researches about Magneto-Plasma Sail have been conducted based on the magnetohydrodynamics (MHD) [1, 2] and the ion kinetic theory [2, 3], however the propulsive performance of Magneto-Plasma Sail has not been quantified because the momentum transfer from the solar wind to the spacecraft has not been clarified even in the framework of the ideal MHD.

In this study, the flow field of Magneto-Plasma Sail is numerically simulated and the momentum transfer from the solar wind to the spacecraft is investigated in the framework of the ideal MHD.

In the ideal magnetohydrodynamic fluid, disturbances are propagated by the MHD waves; the slow wave, Alfvén wave and fast wave. Therefore, in the flow field of

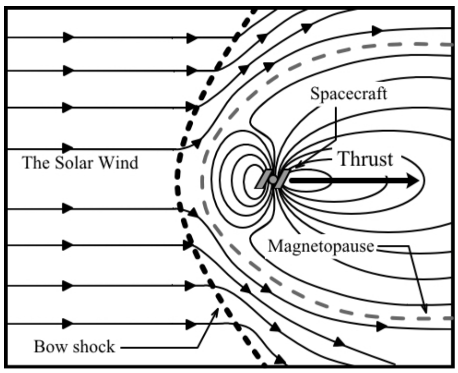

Fig. 1 Concept of Magneto-Plasma Sail making use of the solar wind.

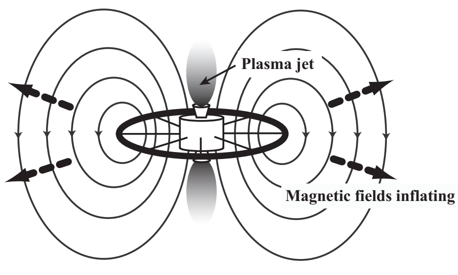

Fig. 2 Magnetic field inflation.

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Magneto-Plasma Sail, the MHD waves transmit the information of the solar wind to the spacecraft, i.e. it is considered that the MHD waves play an important role in the momentum transfer process from the solar wind to the spacecraft. However, this logic has not been exactly addressed.

We characterize the flow field of Magneto-Plasma Sail using the Alfvénic Mach number of the injected plasma flow to confirm the roles of the MHD waves in the momentum transfer process and the momentum transfer process is clarified by confirming the action-reaction forces in the flow field.

2. Numerical Model

The governing equations are the ideal magnetohydrodynamic (MHD) equations. The normalized ideal MHD equations are as follows;

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \nabla \cdot \boldsymbol{F} = \boldsymbol{S}, \tag{1}$$

$$\boldsymbol{Q} = \begin{bmatrix} \rho \\ \rho \boldsymbol{v} \\ \boldsymbol{B} \\ e \end{bmatrix}, \boldsymbol{S} = - \begin{bmatrix} 0 \\ \boldsymbol{B} \\ \boldsymbol{v} \\ \boldsymbol{v} \cdot \boldsymbol{B} \end{bmatrix} \nabla \cdot \boldsymbol{B},$$

$$\boldsymbol{F} = \begin{bmatrix} \rho \boldsymbol{v} \\ \rho \boldsymbol{v}\boldsymbol{v} + \boldsymbol{I}\left(p + B^2/2\right) - \boldsymbol{B}\boldsymbol{B} \\ \boldsymbol{v}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{v} \\ \left(e + p + B^2/2\right)\boldsymbol{v} - \left(\boldsymbol{v} \cdot \boldsymbol{B}\right)\boldsymbol{B} \end{bmatrix}.$$

$\rho, p, \boldsymbol{v}, \boldsymbol{B}$ and $\boldsymbol{I}$ are the density, pressure, flow velocity vector, magnetic flux vector and the unit tensor, respectively. The energy density e is $\rho v^2/2 + p/(\gamma - 1) + B^2/2$, here γ is the specific heat ratio ($= 5/3$). $\boldsymbol{S}$ is called the Powell's source term [4] which is effective to treat the numerical error in $\nabla \cdot \boldsymbol{B}$. In this study, all discussions are conducted using the non-dimensional values because discussions in the real dimension are not important for our study purpose.

The ideal MHD equations are solved by the finite-volume method. The numerical flux is evaluated by the TVD Lax-Friedrich scheme [5] and the MUSCL method is used with the MINMOD limiter to achieve the 2nd-order spatial accuracy. The time integration is done by the ADI-SGS implicit scheme [6]. To treat the numerical error in $\nabla \cdot \boldsymbol{B}$, not only the Powell's source term but also the projection method [7] are used.

3. Numerical Settings

The configuration of Magneto-Plasma Sail is assumed as shown in Fig. 2. The numerical settings are shown in Fig. 3; the flow field and magnetic field are assumed to be axis-symmetric. The spacecraft is located as a sphere of unit in radius at the origin and the dipole magnetic field is set around the spacecraft in the initial state. The dipole magnetic moment vector is aligned parallel to both the x axis and the solar wind flow direction. Two plasma jets for the magnetic field inflation are injected from two nozzles and the nozzle area is defined on the spacecraft surface as shown in Fig. 3 (b).

The physical states of the solar wind are set as follows;

$$\rho = 1.0, T = 0.024, M = 10.0, B = 0.0, \tag{2}$$

here T, M are the temperature and the sonic Mach number, respectively. The physical states of the injected plasma jet are set at the nozzle exit as follows;

$$T^* = 0.0048, M^* = 1.0, M_{alf}^{*2} = 10^0 - 10^{-5}, \tag{3}$$

here * is a superscript for distinguishing the values at the nozzle exit from the local values. M_{alf} is the Alfvenic Mach number ($= v/(B/\sqrt{\rho})$) and the simulations are conducted for 7 cases of M_{alf}^{*2} between 10^0 and 10^{-5}. The density is determined by M_{alf} and the magnetic flux density. The dipole magnetic field is set by following equation.

$$B(\boldsymbol{r}) = 3\frac{\boldsymbol{m_d} \cdot \boldsymbol{r}}{r^5}\boldsymbol{r} - \frac{\boldsymbol{m_d}}{r^3}, \tag{4}$$

here $\boldsymbol{r}$ is the position vector, and $\boldsymbol{m_d}$ is the dipole magnetic moment vector and $|\boldsymbol{m_d}| = 500$.

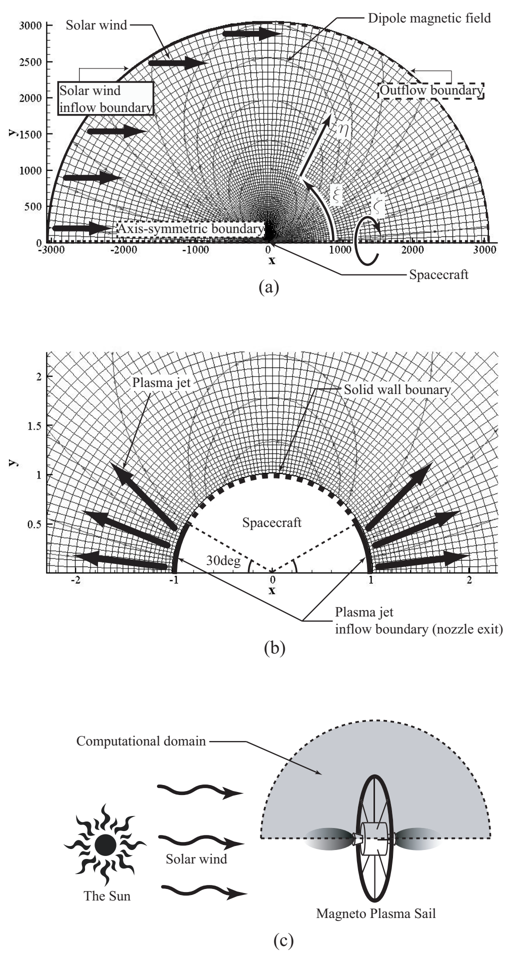

Fig. 3 Numerical settings; (a), (b) show computational grids and boundary conditions, and (c) shows a schematic picture of simulations.

Boundary conditions are implemented using ghost cells. The solar wind inflow boundary condition is implemented by fixing all physical states at the ghost cells, and the outflow boundary condition is implemented by linearly extrapolating all physical states into the ghost cells. The axis-symmetric condition is implemented at the x axis; all physical states at the ghost cells are symmetrically extrapolated about the axis. At the plasma jet inflow boundary (the nozzle area), only tangential magnetic field components at the ghost cells are extrapolated from the cells in the computational domain and other magnetic field components are fixed at the initial states. The flow velocity vector at the ghost cells is always set to be aligned parallel to the magnetic field, and the pressure, density and injection velocity are fixed at the initial states. At the spacecraft surface except the nozzle, the symmetric boundary condition is implemented as the ideal superconducting wall boundary condition; all physical states at the ghost cells are symmetrically extrapolated about the surface boundary.

4. Thrust of Magneto-Plasma Sail

Thrust of Magneto-Plasma Sail is evaluated by calculating three forces. These three forces are based on the momentum conservation law, the Maxwell stress and the Lorentz force, and these forces should be balanced from the law of action and reaction. The balance of these forces is confirmed to clarify the momentum transfer process and verify the self-consistency of simulation results. In this section, the method for calculating these forces are briefly explained.

4.1 Momentum Change of Flow; F_{mc}

This force is based on the momentum change in the plasma flow. The momentum change is calculated by numerically integrating the momentum flux over a curvilinear surface surrounding the spacecraft.

4.2 Maxwell Stress; F_{ms}

This force is based on the Maxwell stress tensor, which is a stress tensor caused by an electromagnetic field. The Maxwell stress tensor $\boldsymbol{T_{ms}}$ is $\boldsymbol{BB} - \boldsymbol{B} \cdot \boldsymbol{B}/2$. An electromagnetic force acting on the spacecraft can be calculated by integrating the Maxwell stress tensor over the spacecraft surface.

4.3 Lorentz Force; F_{lf}

This force is based on the Lorentz force caused by the induced magnetic field. The interaction of the plasma flow with the magnetic field induces strong currents at the magnetopause and in the magnetosphere, and the induced currents generate induced magnetic fields. This induced magnetic field exerts the Lorentz force on the on-board coil currents which produce the magnetic field of the spacecraft. The electromagnetic force acting on the spacecraft is evaluated by directly calculating the Lorentz force between the induced currents in the flow field and the coil currents generating the dipole magnetic field.

5. Simulation Results and Discussion

Results of calculated thrust are shown. F_{mc}, F_{ms} and F_{lf} are plotted as a function of M^{*2}_{alf} in Fig. 4. These forces are normalized by F_0 which is the thrust in the case of the original dipole magnetic field, i.e. without the magnetic field inflation. As shown in Fig. 4, profiles of F_{mc}, F_{ms} and F_{lf} are almost equal. This means a momentum change of the solar wind is transferred to the spacecraft by the electromagnetic force, which is exerted by the induced magnetic field on the on-board coil currents.

In the case of $M^{*2}_{alf} \geq 10^{-3}$, thrust of Magneto-Plasma Sail is almost zero. However when M^{*2}_{alf} becomes lower than 10^{-3}, the thrust rapidly increases with decreasing M^{*}_{alf} until M^{*2}_{alf} reaches 2.5×10^{-4}. In the case of $M^{*2}_{alf} < 2.5 \times 10^{-4}$, the thrust decreases with decreasing M^{*}_{alf} and then the thrust of Magneto-Plasma Sail in the case of $M^{*2}_{alf} = 10^{-5}$ is almost the same as the thrust in the case of the original dipole magnetic field.

The relation between thrust and the Alfvénic Mach number of the injected plasma jet is discussed by characterizing the flow field.

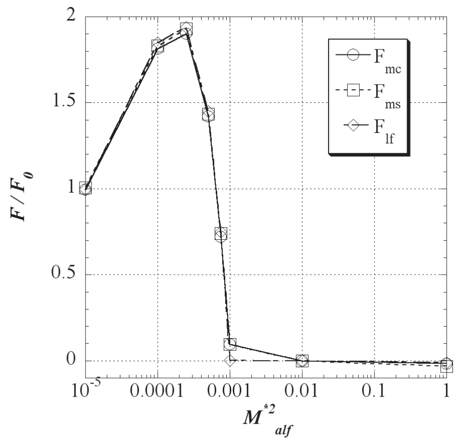

Fig. 4 Thrust of Magneto-Plasma Sail as a function of the Alfvenic Mach number of the injected plasma jet.

Figure 5 shows the flow field in the case of $M^{*2}_{alf} = 2.5 \times 10^{-4}$; the upper half of this figure shows the pressure contours and streamlines of the solar wind, and the lower half of this figure shows the fast-Mach number contours and streamlines of the injected plasma flow, here the fast-Mach number M_f is a Mach number related to the fast wave, which is the fastest MHD wave. In this case, Magneto-Plasma Sail can generate thrust. The bow shock is formed by the super-sonic solar wind. The injected plasma flow collides with the solar wind at the magnetopause which is the boundary of the magnetosphere, and after that, the injected plasma flow is swept away downwind. It is seen that most of the injected plasma flow is sub-fast wave flow, and the magnetopause is located inside the sub-fast wave flow region.

Next, the flow field in the case of $M_{alf}^{*2} = 10^0$ is shown in Fig. 6. In this case, Magneto-Plasma Sail can not produce thrust. As shown in this figure, the magnetosphere is drastically inflated because higher Alfvénic Mach number means higher dynamic pressure of the plasma flow. The bow shock is formed same as in the case of $M_{alf}^{*2} = 2.5 \times 10^{-4}$. However the injected plasma flow accelerates to the super-fast wave flow during expansion before colliding with the solar wind, and then decelerates to the sub-fast wave flow forming a shock wave called the termination shock. The termination shock surrounds the spacecraft. This flow structure indicates that the MHD waves play important roles in the momentum transfer. Any MHD waves can not propagate from the solar wind to the spacecraft against the radially expanding super-fast wave flow, and so the solar wind can not influence the flow field inside of the termination shock. That is to say Magneto-Plasma Sail can not produce thrust.

When the Alfvénic Mach number of the plasma jet gets higher, the magnetosphere gets larger and the sub-fast wave flow region around the spacecraft gets smaller. In the case that the Alfvénic Mach number of the plasma jet becomes higher than the certain value, the injected plasma flow accelerates to the super-fast wave flow by the acceleration expansion before colliding with the solar wind, and then Magneto-Plasma Sail can not get thrust. However, low Alfvénic Mach number of the plasma jet means weak magnetic field inflation, therefore high thrust can not be expected. Our simulation results indicate thrust can not be improved by the plasma jet whose Alfvénic Mach number is lower than $M_{alf}^{*2} = 10^{-5}$.

The momentum change of the solar wind is transferred to the spacecraft by the Lorentz force, which is exerted by the induced magnetic field. It can be explained based on the induced magnetic field why the super-fast wave expanding flow disturbs the momentum transfer. The induced magnetic field created by the induced current sheets at the magnetopause and the magnetic neutral sheet exerts the Lorentz force on the spacecraft. However when the termination shock is formed, the strong induced current sheet at the termination shock cancels the induced magnetic field generated by the current sheets at the magnetopause and neutral sheet inside of the termination shock; Fig. 7 shows the induced current density contours in the case of $M_{alf}^{*2} = 10^0$. Therefore, the Lorentz force is not exerted on the spacecraft, i.e. Magneto-Plasma Sail can not generate thrust.

The MHD wave has to propagate from the interaction region between the magnetic field and the solar wind to the spacecraft for thrust production, i.e. a sub-fast wave flow field has to be created between the solar wind and the spacecraft. However, plasma jets at high Alfvénic Mach number have to be used to drastically inflate the magnetosphere. One of the candidates for improving Magneto-Plasma Sail is the utilization of a one-way single plasma jet. In the case of the one-way single plasma jet, even if the plasma jet is the high Alfvénic Mach number flow, the super-fast expanding flow is formed on only one side of the spacecraft. In addition, when the single jet is injected toward the Sun, it is expected that the spacecraft can utilize not only thrust by catching the solar wind but also thrust by the momentum of the plasma jet as shown in Fig. 8.

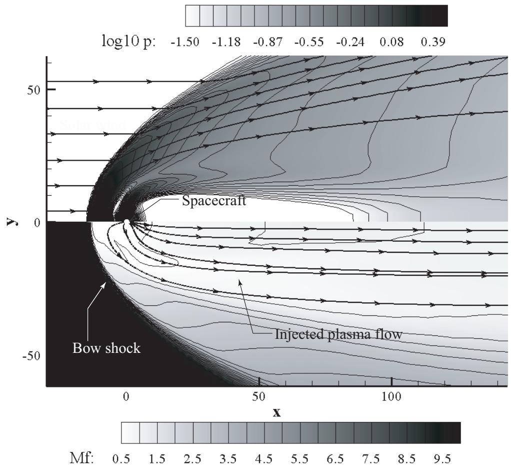

Fig. 5 Flow Field of Magneto-Plasma Sail in the case of $M_{alf}^{*2} = 2.5 \times 10^{-4}$, (upper-half) pressure contours and streamlines of the solar wind, (lower-half) fast-Mach number contours and streamlines of the injected plasma.

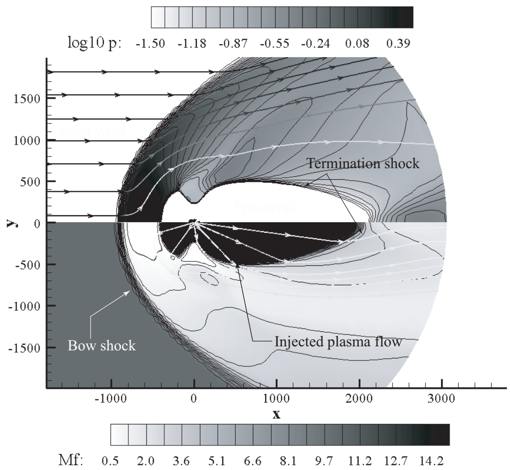

Fig. 6 Flow Field of Magneto-Plasma Sail in the case of $M_{alf}^{*2} = 10^0$, (upper-half) pressure contours and streamlines of the solar wind, (lower-half) fast-Mach number contours and streamlines of the injected plasma.

6. Conclusion

The flow field around Magneto-Plasma Sail was simulated based on the ideal MHD to investigate the momentum transfer from the solar wind to the spacecraft. The simulation was conducted for various Alfvén Mach number of the injected plasma jet, and thrust and the flow field were characterized by the Alfvén Mach number.

The balance of three forces acting on the spacecraft

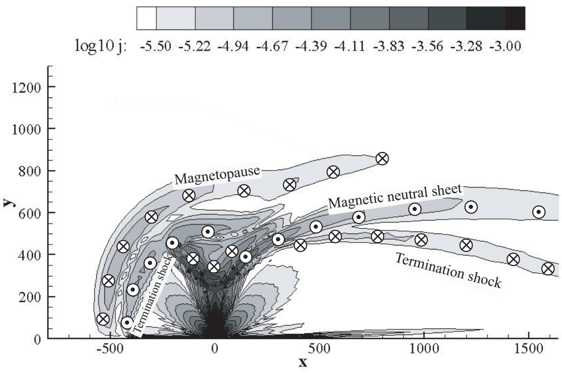

Fig. 7 Induced current density contours in the case of $M_{alf}^{*2} = 10^0$.

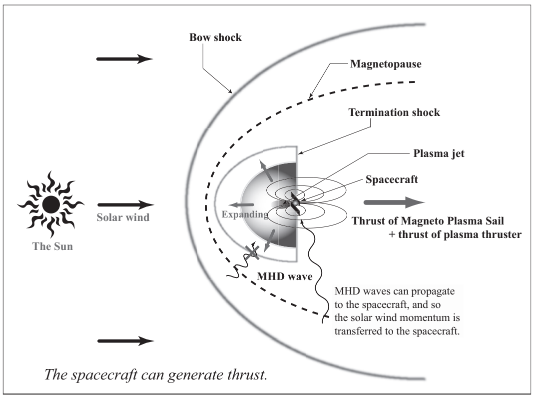

Fig. 8 Schematic figure of the concept of Magneto-Plasma Sail with plasma propulsion.

was confirmed; the first force is the momentum change of the plasma flow, the second force is the Maxwell stress and the third force is the Lorentz force. This result means a momentum change in the solar wind is transferred to the spacecraft as the Lorentz force, which is exerted by the induced magnetic field on the on-board coil currents, and as a result, the spacecraft produces thrust.

The simulation results indicated that Magneto-Plasma Sail could generate thrust only when a sufficiently low Alfvén Mach number plasma jet was used. When high Alfvén Mach number plasma jet is used, Magneto-Plasma Sail can not generate thrust in the ideal MHD limit. In this case, the injected plasma jet accelerates into a super-fast wave flow before colliding with the solar wind. This result showed the MHD wave played an important role in the momentum transfer. The MHD wave has to propagate from the interaction region between the magnetic field and the solar wind to the spacecraft for thrust production. This consequence can be also explained based on the induced magnetic field; inside of the termination shock, the induced magnetic field created by current sheets at the magnetopause and magnetic neutral sheet is canceled by a strong current sheet at the termination shock, and therefore the Lorentz force does not act on the spacecraft. The momentum transfer process is summarized in Fig. 9.

In this study, the interplanetary magnetic field in the solar wind is not taken into account. The effect of the magnetic dissipation such as the electric resistivity reconnects the interplanetary magnetic field to the magnetic field of the spacecraft, and it is concerned that the magnetosphere is deformed by the magnetic reconnection and then thrust is affected by the deformations. The effects of the magnetic reconnection have to be investigated in future works.

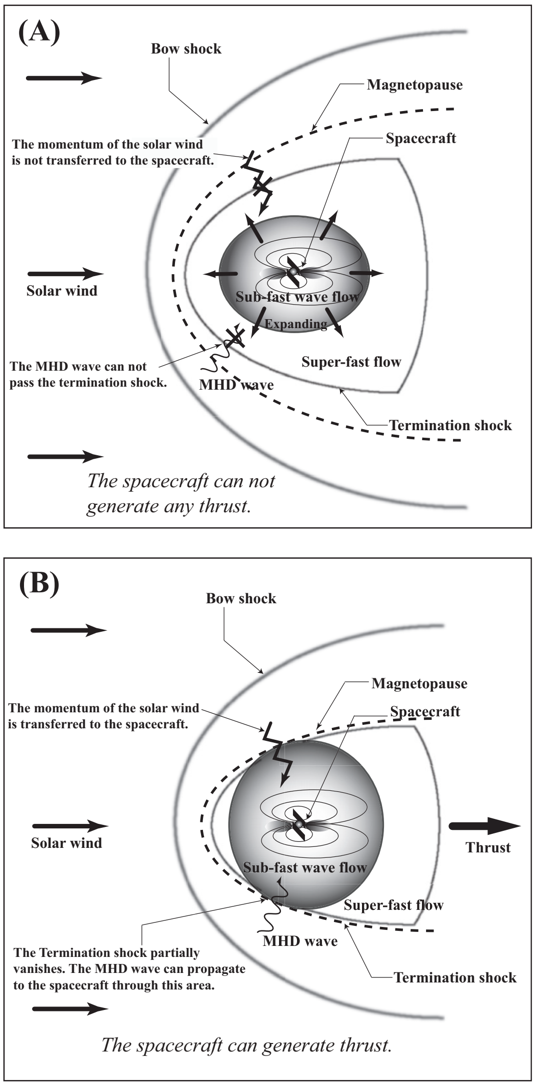

Fig. 9 Momentum transfer from the solar wind to the spacecraft

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