Wave Mode Couplings in a Free-Electron Laser with Axial Magnetic Field in the Presence of Self-Fields

Behrouz Maraghechi, Haleh Mahdavi, Taghi Mohsenpour

Department of Physics, Amirkabir University of Technology, P.O. Box 15875-4413, Tehran, Iran

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The one-dimensional analysis of the collective interaction in a free electron laser (FEL) with combined helical wiggler and axial guide magnetic field in the presence of self-fields is presented. Contrary to the previous investigations relativistic terms to all orders of the wiggler amplitude is retained in the linearized equations. A dispersion relation for the unstable couplings of waves is derived. This dispersion relation is solved numerically to investigate the usual FEL instabilities with relativistic terms included. It was found that self-fields lower the maximum growth rate and narrows the width of the unstable spectrum.

Keywords: Free electron laser, helical wiggler, self-fields, dispersion relation, instability

1. Introduction

The free-electron laser (FEL) theory in the collective or Raman regime relies on the unstable coupling between the radiation and the negative-energy space-charge wave [1-3]. Mehdian et al. devised a relativistic theory for an FEL and derived a dispersion relation (DR) [4]. Recently, Mohsenpour et al. solved the DR numerically and found additional couplings between waves [5].

In this high-gain regime, due to the high density and low energy of the electron beam, an axial magnetic field is usually employed to focus on the beam. In such a configuration, equilibrium self-electric and self-magnetic fields, due to the charge and current densities of the beam have considerable effects on equilibrium orbits. It has been shown that self-fields can induce chaos in the single-particle trajectories in the vicinity of the gyroresonance. Freund et al. have calculated the first-order self-magnetic field, generated by the wiggler-induced transverse velocity [6]. Recently, the effects of self-fields on the stability of equilibrium trajectories have been studied in a FEL with a one-dimensional helical wiggler and an axial magnetic field [7-8].

The purpose of the present investigation is to use a relativistic theory to derive a DR for the interaction of all the waves in a relativistic electron beam that passes through a one-dimensional helical wiggler magnetic field and an axial magnetic field in the presence of self field. This DR is solved numerically, for group I orbits, to study the relativistic effects on the FEL instability, i.e., the unstable coupling between the negative energy space-charge wave and the electromagnetic radiation. This DR may also be studied further to investigate the couplings between other wave modes in the system [5].

author's e-mail: behrouz@aut.ac.ir

2. Self-Field Calculation

Consider a relativistic electron beam moving along the z axis of an idealized helical wiggler magnetic field described by

$$\mathbf{B}_w = B_w\left(\hat{\mathbf{x}}\cos k_w z + \hat{\mathbf{y}}\sin k_w z\right), \qquad (1)$$

and in the presence of an axial static magnetic field $B_0\,\hat{\mathbf{z}}$. Here, $k_w = 2\pi/\lambda_w$ is the wiggler wave number and λ_w is the wiggler wavelength. The transverse part of the steady-state helical trajectories of electrons, neglecting the self-fields of the beam, can be found as

$$\mathbf{v}_w = v_w\left(\hat{\mathbf{x}}\cos k_w z + \hat{\mathbf{y}}\sin k_w z\right), \qquad (2)$$

$$v_w = \frac{\Omega_w v_\parallel}{\left(\Omega_0 - k_w v_\parallel\right)}. \qquad (3)$$

where $\Omega_w = eB_w/(\gamma_0 m_0 c)$, $\Omega_0 = eB_{0z}/(\gamma_0 m_0 c)$, $\gamma_0 = \left[1-\left(v_w^2+v_\parallel^2\right)/c^2\right]^{-1/2}$, m_0 is the electron rest mass, e is the magnitude of the charge of an electron, and c is the speed of light in vacuum.

The self-electric and self-magnetic fields are induced by the steady-state charge density and current of the non-neutral electron beam. In order to model these self-fields, we make the assumption of a homogeneous electron density profile,

$$n_b(r) = \begin{cases} n_b = const, & r \le r_b \\ 0, & r > r_b \end{cases} \qquad (4)$$

where n_b is the number density of the electron and r_b is the beam radius. Solving Poison's equation yields the self-electric field in the form

$$\mathbf{E}_s = -2\pi n_b e r\,\hat{\mathbf{r}} = -2\pi n_b e\left(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}}\right). \qquad (5)$$

The self magnetic field is induced by the steady-state current density of the electron beam and may be obtained by Ampere's law,

$$\nabla \times \mathbf{B}_s = \frac{4\pi}{c}\mathbf{J}_b, \qquad (6)$$

where

$$\mathbf{J}_b = -en_b\left[v_w\left(\hat{\mathbf{x}}\cos k_w z + \hat{\mathbf{y}}\sin k_w z\right) + v_\parallel \hat{\mathbf{z}}\right] \quad (7)$$

is the beam current density. By the method of Ref. [8] $\mathbf{B}_s$ may be found as

$$\mathbf{B}_s = \frac{2\omega_b^2\left(v_\parallel/c\right)^2}{\Omega_0 k_w v_\parallel - \omega_b^2\left(1+v_\parallel^2/c^2\right) - k_w^2 v_\parallel^2}\mathbf{B}_w - 2\pi n_b e\frac{v_\parallel}{c} r\left(y\,\hat{\mathbf{x}} - x\,\hat{\mathbf{y}}\right), \quad (8)$$

where $\omega_b = \left(4\pi n_b e^2/\gamma m\right)^{1/2}$.

By solving the equation of motion of an electron, in the presence of self-fields $\mathbf{E}_s$ and $\mathbf{B}_s$, we will find the steady-state orbits

$$\mathbf{v}_0 = v_w\left(\hat{\mathbf{x}}\cos k_w z + \hat{\mathbf{y}}\sin k_w z\right) + v_\parallel \hat{\mathbf{z}}, \quad (9)$$

where

$$v_w = \frac{k_w \Omega_w v_\parallel^2}{\Omega_0 k_w v_\parallel - \omega_b^2\left(1+v_\parallel^2/c^2\right) - k_w^2 v_\parallel^2}. \quad (10)$$

Figure 1 shows the variation of axial velocity $v_\parallel/c$ with normalized cyclotron frequency $\Omega_0/k_w c$ for group I $\left(\Omega_0 < \mathrm{k_w v_\parallel}\right)$ and group II $\left(\Omega_0 > \mathrm{k_w v_\parallel}\right)$ orbits. The parameters are $\mathrm{k_w} = 2\ \mathrm{cm}^{-1}$, $\mathrm{n_0} = 10^{12}\ \mathrm{cm}^{-3}$, $\gamma_0 = 3$, and $\mathrm{B_w} = 1500\ \mathrm{G}$.

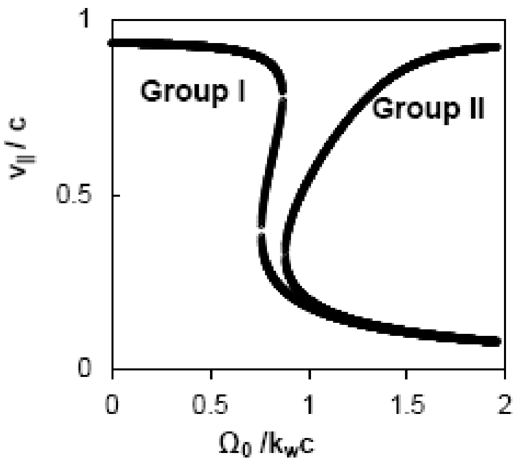

Fig.1 Axial velocity as a function of the normalized axial magnetic field.

3. Dispersion Relation

An analysis of the propagation of electromagnetic/electrostatic waves in the electron beam may be based on the continuity equation,

$$\frac{\partial n}{\partial t} + \nabla\cdot\left(n\,\mathbf{v}\right) = 0, \quad (11)$$

the relativistic momentum equation

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{e}{\gamma m_0}\left[\mathbf{E} - \frac{1}{c^2}\mathbf{v}\,\mathbf{v}\cdot\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right], \quad (12)$$

and the wave equation

$$\nabla\times\left(\nabla\times\mathbf{E}\right) + \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{\partial}{\partial t}\frac{4\pi e}{c^2}\left(n\,\mathbf{v}\right). \quad (13)$$

Here, n is the electron density, $\mathbf{v}$ is the electron velocity, γ is the Lorentz factor corresponding to $\mathbf{v}$, $\mathbf{E}$ is the electric field, and $\mathbf{B}$ is the magnetic field. With the unperturbed electron density n_0 taken to be independent of position and time, the electron and field variables may be expressed in the form

$$n = n_0 + \delta n, \quad (14)$$

$$\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v}, \quad (15)$$

$$\mathbf{E} = \mathbf{E}_0 + \delta\mathbf{E}, \quad (16)$$

$$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}, \quad (17)$$

$$\mathbf{R} = \mathbf{R}_0 + \delta\mathbf{R}, \quad (18)$$

where $\mathbf{R}$ is the radial distance from the axis.

The linearized equations for the continuity equation, the relativistic momentum equation, and the wave equation may be derived as

$$\frac{\partial\delta n}{\partial t} + n_0\nabla\cdot\delta\mathbf{v} + \mathbf{v}_0\cdot\nabla\delta n = 0, \quad (19)$$

$$\frac{\partial\delta\mathbf{v}}{\partial t} + \mathbf{v}_0\cdot\nabla\delta\mathbf{v} + \delta\mathbf{v}\cdot\nabla\mathbf{v}_0 = -\frac{e}{\gamma_0 m_0}\left(\delta\mathbf{E} - \frac{1}{c^2}\mathbf{v}_0\mathbf{v}_0\cdot\delta\mathbf{E} - \frac{1}{c^2}\mathbf{v}_0\delta\mathbf{v}_0\cdot\mathbf{E}_0 + \frac{1}{c}\delta\mathbf{v}\times\mathbf{B}_0 + \frac{1}{c}\mathbf{v}_0\times\delta\mathbf{B} - \frac{\gamma_0^2}{c^2}\left(\mathbf{E}_0 + \frac{1}{c}\mathbf{v}_0\times\mathbf{B}_0\right)\mathbf{v}_0\cdot\delta\mathbf{v}\right), \quad (20)$$

$$\nabla\times\left(\nabla\times\delta\mathbf{E}\right) + \frac{1}{c^2}\frac{\partial^2\delta\mathbf{E}}{\partial t^2} = \frac{\partial}{\partial t}\frac{4\pi e}{c^2}\left(\delta n\mathbf{v}_0 + n_0\,\delta\mathbf{v}\right). \quad (21)$$

By introducing a new set of basis vectors $\hat{\mathbf{e}} = \left(\hat{\mathbf{x}} + i\,\hat{\mathbf{y}}\right)/\sqrt{2}$, $\hat{\mathbf{e}}^* = \left(\hat{\mathbf{x}} - i\,\hat{\mathbf{y}}\right)/\sqrt{2}$, and $\hat{\mathbf{e}}_z = \hat{\mathbf{z}}$, the unperturbed parameters can be written as

$$\mathbf{R}_0 = i\sqrt{2}\,R_0\left[\exp(-ik_w z)\,\hat{\mathbf{e}} + \exp(ik_w z)\hat{\mathbf{e}}^*\right], \quad (22)$$

$$\mathbf{E}_0 = -i\sqrt{2}\,\pi e n_b R_0\left[\exp(-ik_w z)\,\hat{\mathbf{e}} - \exp(ik_w z)\hat{\mathbf{e}}^*\right], \quad (23)$$

$$\mathbf{v}_0 = \left(v_w/\sqrt{2}\right)\exp(-ik_w z)\hat{\mathbf{e}} + \left(v_w/\sqrt{2}\right)\times\exp(ik_w z)\hat{\mathbf{e}}^* + v_\parallel\hat{\mathbf{e}}_z, \quad (24)$$

$$\mathbf{B}_0 = \left[\left(-2\pi e n_b R_0\frac{v_\parallel}{c} + \lambda B_w\right)\Big/\sqrt{2}\right]\times\left(\exp(-ik_w z)\hat{\mathbf{e}} + \exp(ik_w z)\hat{\mathbf{e}}^*\right) + B_{0z}\hat{\mathbf{e}}_z, \quad (25)$$

where $R_0 = v_w/k_w v_\parallel$ is the radius of the equilibrium orbits of electrons and

$$\lambda = 1 + \frac{2\omega_b^2\left(v_\parallel/c\right)^2}{\Omega_0 k_w v_\parallel - \omega_b^2\left(1+v_\parallel^2/c^2\right) - k_w^2 v_\parallel^2}. \quad (26)$$

The perturbed state is assumed to consist of a longitudinal space-charge wave and right and left circularly polarized electromagnetic waves, referred here as radiation, with all perturbed waves propagating in the positive direction. Accordingly, solution of the system of equations (19)-(21) may be assumed as

$$\delta\mathbf{v} = \delta v_R\hat{\mathbf{e}} + \delta v_L\hat{\mathbf{e}}^* + \delta v_Z\hat{\mathbf{e}}_z, \quad (27)$$

$$\delta\mathbf{E} = \left(-2\pi e n_b\,\delta R_R + \delta E_R\right)\hat{\mathbf{e}} + \left(-2\pi e n_b\,\delta R_L + \delta E_L\right)\hat{\mathbf{e}}^* + \delta E_Z\hat{\mathbf{e}}_z, \quad (28)$$

$$\delta \mathbf{B} = \delta B_R \hat{\mathbf{e}} + \delta B_L \hat{\mathbf{e}}^* , \tag{29}$$

$$\delta \mathbf{R} = \delta R_R \hat{\mathbf{e}} + \delta R_L \hat{\mathbf{e}}^* , \tag{30}$$

$$\delta n = \tilde{n} \exp[i(kz - \omega t)] , \tag{31}$$

$$\delta v_R = \tilde{v}_R \exp[i(k_R z - \omega t)] , \tag{32}$$

$$\delta v_L = \tilde{v}_L \exp[i(k_L z - \omega t)] . \tag{33}$$

δv_z and δE_z are analogous to δn ; δE_R , δR_R and δB_R are analogous to δv_R ; δE_L , δR_L and δB_L are analogous to δv_L ; the wave numbers are related to by

$$k_R = k - k_w , \tag{34a}$$

$$k_L = k + k_w . \tag{34b}$$

Substituting Eqs. (27)-(33) in Eqs. (19)-(21) gives the expression for perturbed fields as follows:

$$\begin{aligned} &\left[D_R^0 + \psi_R v_w^2/c^2\right]\delta E_R + \left[\xi_{L1} v_w^2/c^2\right]\delta E_L \\ &+ \left[\xi_{R3} v_w/c + M_1 v_w^3/c^3\right]\delta E_z = 0, \end{aligned} \tag{35}$$

$$\begin{aligned} &\xi_{R1}\left(v_w^2/c^2\right)\delta E_R + \left[D_L^0 + \psi_L v_w^2/c^2\right]\delta E_L \\ &+ \left[\xi_{L3} v_w/c + M_2 v_w^3/c^3\right]\delta E_z = 0, \end{aligned} \tag{36}$$

$$\begin{aligned} &\xi_{R2}\left(v_w/c\right)\delta E_R + \xi_{L2}\left(v_w/c\right)\delta E_L \\ &- \omega \varepsilon^0 \delta E_z = 0, \end{aligned} \tag{37}$$

where

$$\begin{aligned} D_R^0 &= \left(k_R^2 c^2 - \omega^2\right)\left[\omega - k_R v_\parallel - \Omega_0 + \omega_b^2/\gamma_\parallel^2 \right. \\ &\left. \times \left(k_R v_\parallel - \omega\right)\right] + \omega_b^2\left(\omega - k_R v_\parallel\right), \end{aligned} \tag{38}$$

$$\begin{aligned} D_L^0 &= \left(k_L^2 c^2 - \omega^2\right)\left[\omega - k_L v_\parallel - \Omega_0 - \omega_b^2/\gamma_\parallel^2 \right. \\ &\left. \times \left(k_L v_\parallel - \omega\right)\right] + \omega_b^2\left(\omega - k_L v_\parallel\right), \end{aligned} \tag{39}$$

$$\varepsilon^0 = (\omega - k v_\parallel)^2 - \omega_p^2/\gamma_\parallel^2 , \tag{40}$$

and ψ_R , ψ_L , M_1 , M_2 , ξ_{R1} , ξ_{R2} , ξ_{R3} , ξ_{L1} , ξ_{L2} , ξ_{L3} are defined in the Appendix. Here, D_R^0 , D_L^0 , and ε^0 are the uncoupled dispersion relations, i.e., in the absence of the wiggler, for the right and left circularly polarized electromagnetic waves, and the space-charge wave, respectively. Equations (35) and (36) show that the DR for the right and left waves alone, in the absence of the other two waves, are

$$D_R = D_R^0 + \psi_R\left(v_w^2/c^2\right) = 0 , \tag{41}$$

$$D_L = D_L^0 + \psi_L\left(v_w^2/c^2\right) = 0 , \tag{42}$$

which indicate that the wiggler has direct effect on the right and left waves and the wiggler effect on their DRs are of the second order in the wiggler amplitude. On the other hand, Eq. (40) shows that the DR for the space-charge wave in the absence of the right and left wave is $\varepsilon^0 = 0$, which indicates that the wiggler has no direct effect on the space-charge wave. The reason is that the transverse helical motion of electrons, due to the wiggler, has no effect on the longitudinal oscillations of the space-charge wave.

The necessary and sufficient condition for a nontrivial solution consists of the determinant of coefficients in Eqs. (35)-(37) equated to zero. Imposing this condition yields the dispersion relation

$$\begin{aligned} \omega \varepsilon^0 D_R D_L = &-\left[D_R \xi_{L2}\left(\xi_{L3} + M_2 \frac{v_w^2}{c^2}\right) \right. \\ &\left. + D_L \xi_{R2}\left(\xi_{R3} + M_1 \frac{v_w^2}{c^2}\right)\right]\frac{v_w^2}{c^2} + \left[\xi_{L1}\xi_{R2} \right. \\ &\times\left(\xi_{L3} + M_2 \frac{v_w^2}{c^2}\right) + \xi_{R1}\xi_{L2}\left(\xi_{R3} + M_1 \frac{v_w^2}{c^2}\right) \\ &\left. + \omega \varepsilon^0 \xi_{L1}\xi_{R1}\right]. \end{aligned} \tag{43}$$

Equation (43) is the DR for coupled electrostatic and electromagnetic waves propagating along a relativistic electron beam in the presence of a wiggler magnetic field and an axial guide magnetic field. This DR (43) will be solved numerically, in the next section, to investigate the relativistic and self-field effects on the FEL instability.

4. Coupling Between Waves for Group I Orbits

In the stable group I orbits the wiggler induced velocity v_w is not so large, therefore, we should expect to

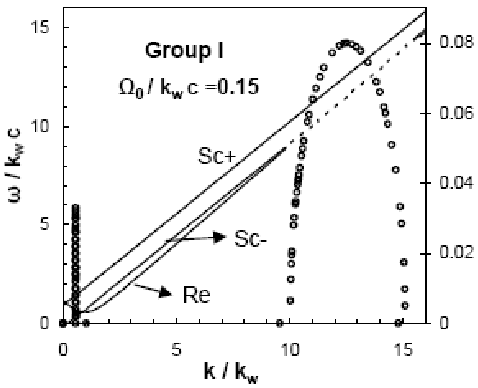

Fig.2 Coupling of right escape and negative space-charge waves for group I orbits.

observe the weakest couplings that can be induced by a relatively small v_w . In group I orbits with larger axial magnetic field and in group II orbits, near the resonance region with $\Omega_0/k_w c < 1.7$, v_w is larger and we should have additional couplings that are driven by the relativistic effects of v_w . The roots of the DR (43) are found numerically for group I orbits with $\Omega_0/k_w c = 0.1$. The rest of the parameters are the same as in Fig. 1. The positive and negative-energy space-charge waves $(Sc_\pm)$ and the escape branch of right circular wave (R_e) are shown in Fig. 2. There are two couplings between the R_e mode and the Sc_- mode and are shown by dotted lines. The wide spectrum coupling at large k values is the well known FEL resonance. Solid lines show $\omega/k_w c$ on the left vertical axis. Circles show the normalized imaginary part of wave number $\mathrm{Im}\, k/k_w$ for the two couplings in Fig. 2. In the group I orbits, induced velocity

v_w is low and therefore, only the well known FEL coupling between the right circularly polarized electromagnetic wave and the negative-energy space-charge wave is possible. In order to correct self-field analysis, it is convenient to introduce an effective wiggler magnetic field $(B_w)_{eff} = \lambda B_w$, where λ is given by equation (26). In the absence of self-fields λ is unity. For group I orbits, λ is considerably below unity, therefore, the coupling agent becomes weak and maximum growth rate is decreased, from 0.11 to 0.08, in comparison with the absence of self-fields. The width of the unstable spectrum in Fig. 2 is $10 < k/k_w < 15$, which it was $7.2 < k/k_w < 13.5$ in the absence of self-fields [5]. Therefore, self-fields have lowered the growth rate, have made the unstable spectrum narrower, and have moved it toward the longer wavelengths.

Due to the large density and low energy of the electron beam in this analysis, an axial magnetic field is employed to focus the beam against its self-fields. Moreover, since the beam is intense the electrostatic potential of the space-charge wave is not negligible compared to the pondermotive potential. Therefore, the FEL operates in the high-gain Raman regime and in the one-pass amplifier mode. The problem under consideration is in the linear stage of the FEL instability, which can be described by the parametric instability, i.e., Raman backscattering of the pump wave (wiggler), in the beam frame, into a forward scattered space-charge wave and a backscattered electromagnetic radiation.

In order to check the validity of our results a one dimensional nonlinear computer simulation is performed with the same parameters as in Fig. 2. The simulation code is the same as in Ref. 9. The results show that the small signal domain occupies the entire injection length from $z = 0$ to $z = 31.4\text{cm}$ along the undulator. The linear domain starts from $z = 38\text{cm}$ and ends at $z = 75\text{cm}$ and the radiation saturates at $z = 86\text{cm}$. The growth rate in the linear domain is $\operatorname{Im} k/k_w = 0.107$, which is in a very good agreement with our theory, without the self-fields, with $\operatorname{Im} k/k_w = 0.11$.

5. Appendix: Definition of Quantities

The following quantities are used in equations (34)-(36)

$$\psi_R = \left(\frac{k_R^2 c^2 - \omega^2}{2}\right)\left\{\left(\frac{\omega_b^2}{k_w v_\parallel}\right)(1-\gamma^2) + \gamma^2\left[\Omega_0(1-\lambda) + \lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right) + \lambda k_w v_\parallel + \left(\frac{\omega_b^2}{k_w^2 c^2}\right)k_w v_\parallel\right] + \frac{\omega_b^2}{k_R v_\parallel - \omega}\right\} - \left(\frac{\omega\,\omega_b^2}{2}\right),$$

$$\xi_{L1} = \left(\frac{k_L^2 c^2 - \omega^2}{2}\right)\left\{-\left(\frac{\omega_b^2}{k_w v_\parallel}\right)(1+\gamma^2) + \gamma^2\left[\Omega_0(1-\lambda) + \lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right) + \lambda k_w v_\parallel + \left(\frac{\omega_b^2}{k_w^2 c^2}\right)k_w v_\parallel\right] + \frac{\omega_b^2}{k_L v_\parallel - \omega}\right\} - \left(\frac{\omega\,\omega_b^2}{2}\right),$$

$$\xi_{R3} = \left(\frac{\omega}{\sqrt{2}}\right)\Bigg\{k c\left(\omega - k_R v_\parallel - \Omega_0\right) + \left(k v_\parallel - \omega\right)\Bigg(k_w c + \lambda c\left(\Omega_0 - \frac{\omega_b^2}{k_w v_\parallel}\left(1+\frac{v_\parallel^2}{c^2}\right) - k_w v_\parallel\right)\Big/ v_\parallel - \left(\frac{\omega_b^2}{k_w c}\right)(2+\gamma^2) + \gamma^2\frac{v_\parallel}{c}\left[\Omega_0(1-\lambda) + \lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right) + \lambda k_w v_\parallel + \left(\frac{\omega_b^2}{k_w^2 c^2}\right) \times k_w v_\parallel\right] - kc\left(\frac{\omega_b^2}{\gamma_\parallel^2 k_w v_\parallel}\right) - kc\left(\frac{\omega_b^2}{\gamma_\parallel^2 (k_R v_\parallel - \omega)}\right) - \omega_b^2\frac{v_\parallel}{c},$$

$$M_1 = \left(\frac{\omega}{2\sqrt{2}}\right)kc\left\{-2\gamma^2\left(\frac{\omega_b^2}{k_w v_\parallel}\right) + 2\gamma^2\left[\Omega_0(1-\lambda) + \lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right) + \lambda k_w v_\parallel + \left(\frac{\omega_b^2}{k_w^2 c^2}\right)k_w v_\parallel\right] + \left(\frac{\omega_b^2}{k_R v_\parallel - \omega}\right) + \left(\frac{\omega_b^2}{k_L v_\parallel - \omega}\right)\right\},$$

$$\xi_{R1} = \left(\frac{k_R^2 c^2 - \omega^2}{2}\right)\left\{\left(\frac{\omega_b^2}{k_w v_\parallel}\right)(1+\gamma^2) - \gamma^2\left[\Omega_0(1-\lambda) + \lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right) + \lambda k_w v_\parallel + \left(\frac{\omega_b^2}{k_w^2 c^2}\right)k_w v_\parallel\right] + \frac{\omega_b^2}{k_R v_\parallel - \omega}\right\} - \left(\frac{\omega\,\omega_b^2}{2}\right),$$

$$\psi_L = \left(\frac{k_L^2 c^2 - \omega^2}{2}\right)\left\{\left(-\frac{\omega_b^2}{k_w v_\parallel}\right)(1-\gamma^2) - \gamma^2\left[\Omega_0(1-\lambda) + \lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right) + \lambda k_w v_\parallel + \left(\frac{\omega_b^2}{k_w^2 c^2}\right)k_w v_\parallel\right] + \frac{\omega_b^2}{k_L v_\parallel - \omega}\right\} - \left(\frac{\omega\,\omega_b^2}{2}\right),$$

$$\xi_{L3} = \left(\frac{\omega}{\sqrt{2}}\right)\left\{k\,c\left(\omega - k_L v_\parallel + \Omega_0\right) + \left(\omega - k v_\parallel\right)\left(k_w c\right.\right.$$

$$+\lambda c\left(\Omega_0 - \frac{\omega_b^2}{k_w v_\parallel}\left(1+\frac{v_\parallel^2}{c^2}\right) - k_w v_\parallel\right)\Big/ v_\parallel - \left(\frac{\omega_b^2}{k_w c}\right)\left(2+\gamma^2\right)$$

$$+\gamma^2\frac{v_\parallel}{c}\left[\Omega_0\left(1-\lambda\right)+\lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right)+\lambda\,k_w v_\parallel\right.$$

$$\left.\left.+\left(\frac{\omega_b^2}{k_w^2 c^2}\right)k_w v_\parallel\right]\right) + kc\left(\frac{\omega_b^2}{\gamma_\parallel^2\,k_w v_\parallel}\right) - kc\left(\frac{\omega_b^2}{\gamma_\parallel^2\left(k_L v_\parallel - \omega\right)}\right)$$

$$\left. -\omega_b^2\frac{v_\parallel}{c}\right\},$$

$$M_2 = \left(\frac{\omega}{2\sqrt{2}}\right)kc\left\{2\gamma^2\left(\frac{\omega_b^2}{k_w v_\parallel}\right) - 2\gamma^2\left[\Omega_0\left(1-\lambda\right)\right.\right.$$

$$\left.+\lambda\left(\frac{\omega_b^2}{k_w v_\parallel}\right)\left(1+\frac{v_\parallel^2}{c^2}\right)+\lambda k_w v_\parallel + \left(\frac{\omega_b^2}{k_w^2 c^2}\right)k_w v_\parallel\right]$$

$$\left.+\left(\frac{\omega_b^2}{k_L v_\parallel - \omega}\right)+\left(\frac{\omega_b^2}{k_R v_\parallel - \omega}\right)\right\},$$

$$\xi_{R2} = \left(k_R^2 c^2 - \omega^2\right)\frac{\lambda c}{\sqrt{2}}\left(\Omega_0 - \frac{\omega_b^2}{k_w v_\parallel}\left(1+\frac{v_\parallel^2}{c^2}\right) - k_w v_\parallel\right)\Big/ v_\parallel$$

$$+\left(\frac{\omega_b^2}{\sqrt{2}}\right)\left(k_R c - \omega\frac{v_\parallel}{c}\right),$$

$$\xi_{L2} = -\left(k_L^2 c^2 - \omega^2\right)\frac{\lambda c}{\sqrt{2}}\left(\Omega_0 - \frac{\omega_b^2}{k_w v_\parallel}\left(1+\frac{v_\parallel^2}{c^2}\right) - k_w v_\parallel\right)\Big/ v_\parallel$$

$$+\left(\frac{\omega_b^2}{\sqrt{2}}\right)\left(k_L c - \omega\frac{v_\parallel}{c}\right),$$

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7. References

[1] T. Kwan and J.M. Dawson, Phys. Fluids **22** 1089 (1979).

[2] H.P. Freund, P. Sprangle, D. Dillenburg, E.H. da Jornada, R.S. Schneider, and B. Liberman, Phys. Rev. A **26** 2004 (1982).

[3] J.E. Willett, B. Bolon, U.-H. Hwang, and Y. Aktas, J. Plasma Phys. **66** 301 (2001).

[4] H. Mehdian, J.E. Willett, and Y. Aktas, Phys. plasmas **5** 4079 (1988).

[5] T. Mohsenpour, B. Maraghechi, and S. Mirzanejhad, Phys. Plasmas **14**, 053106 (2007).

[6] H.P. Freund, R. H. Jackson, and D. E. Pershing, Phys. Fluids B **5**. 2318 (1993).

[7] S. Mirzanejhad, B. Maraghechi, and T. Mohsenpour, Phys. Plasmas **11**, 4776 (2004).

[8] M. Esmaeilzadeh, J. E. Willett, and L. J. Willett, J. Plasma Physics **71,** 367 (2005).

[9] H. P. Freund, Phys. Rev. A **23,** 1977 (1983).