# Effect of Nuclear Elastic Scattering on Fusion Product Spectrum in Self-Sustaining D$^3$He Plasmas

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(Received: 1 September 2008 / Accepted: 7 January 2009)

The modification of fusion-product emission spectrum from Gaussian distribution due to nuclear elastic scattering (NES) of fuel ion by fusion-produced 14.7-MeV proton in a self-sustaining deuterium-helium-3 (D$^3$He) plasma is accurately evaluated by simultaneously solving the Boltzmann-Fokker-Planck (BFP) equations for deuteron, triton, alpha-particle, proton and $^3$He. It is shown that the emission spectrums of fusion products, i.e. proton, alpha-particle, triton, $^3$He, neutron, are significantly modified from Gaussian distributions. The influence of the plasma parameters on the modification is discussed.

Keywords: nuclear elastic scattering, D$^3$He plasma, fusion product emission spectrum, knock-on tail, Boltzmann-Fokker-Planck equation

## 1. Introduction

In thermonuclear plasmas Coulomb scattering is the dominant slowing-down process characterized by a highly forward-peaked angular distribution, small energy transfer, and large cross section. On the other hand, the nuclear elastic scattering (NES) is a non-Coulombic, large-energy-transfer (LET) scattering process [1,2], and in some cases it has been recognized as an important slowing-down mechanism for energetic ions. In deuterium-helium-3 (D$^3$He) plasmas, it is well known that the NES plays an important role in plasma burning and sustaining operations [3-10]. Devaney and Stein [1] first pointed out the necessity of taking into account the nuclear-force contribution to ion-ion scattering in high-temperature plasmas. In order to examine the NES effect, several formulations to describe the discrete nature of LET scattering have been developed [3-5]. For DD and D$^3$He plasmas, using the continuous [3,4] and multi-group [5] slowing-down models, energy-transfer processes of fusion-produced ions [6] and its effect on the plasma ignition properties [7,8] were investigated.

It is also well known that a knock-on tail is created in ion velocity distribution function due to NES of thermal component by energetic ions [9-15]. In magnetically-confined deuterium-tritium (DT) plasmas, from the viewpoint of plasma diagnostics [16-18], the knock-on tail formation in fuel-ion velocity distribution function due to NES by fusion-produced alpha-particles and the resulting modification of the neutron emission spectrum were computed [11]. The knock-on tail formation in fuel-ion velocity distribution functions in DT plasma due to NES of fuel ion by alpha-particle was experimentally ascertained by looking at the deviation of emitted neutron spectrum from Gaussian distribution [13].

The knock-on tail formation in D$^3$He plasma has also been investigated by solving the Boltzmann-Fokker-Planck (BFP) equation [9,10]. On the basis of the BFP model, the influences of the knock-on tail formation on fusion reaction rate coefficient [9,10] and confinement condition [10] were examined. In the previous calculations to examine the knock-on tail formation [9,10], we have assumed mono-energetic sources for all of the fusion-produced ions, i.e. proton, alpha-particle, triton, $^3$He. However, the spectrum of fusion-produced ions would spread widely toward high and low energy regions due to knock-on tail formation. The fractional energy deposition from fusion-produced ion to bulk ions and electrons is changed depending on the relative velocity (collision frequency) between energetic and bulk particles. The transport processes of energetic ion in the fusion devices may also be affected by the particle energy. It is important to accurately grasp the modification of the emission spectrum of fusion-produced ions (especially 14.7-MeV proton) in D$^3$He plasmas.

In this paper, we consider a self-sustaining D$^3$He plasma and by simultaneously solving the BFP equations for deuteron, triton, alpha-particle, proton and $^3$He velocity distribution functions, the fusion product emission spectrums are accurately evaluated. A significant modification in the fusion-produced 14.7-MeV proton emission spectrum from Gaussian distribution is shown.

## 2. Analysis Model

In the calculation, we consider all of the following reactions occurring in a self-sustaining D$^3$He plasma

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simultaneously.

$$D + {}^3He \rightarrow \alpha + p + 18.3\text{MeV} \quad , \tag{1}$$

$$D + D \rightarrow {}^3He + n + 3.2\text{MeV} \quad , \tag{2}$$

$$D + D \rightarrow T + p + 4.1\text{MeV} \quad , \tag{3}$$

$$D + T \rightarrow \alpha + n + 17.6\text{MeV} \quad . \tag{4}$$

## 2.1 Boltzmann-Fokker-Planck model

To evaluate the steady-state ion distribution functions, we solve the following BFP equations for ion species $a$ in the $D^3He$ burning plasma ($a$ = D, T, $\alpha$, p, $^3$He).

$$\sum_j \left( \frac{\partial f_a(v)}{\partial t} \right)_j^C + \sum_i \left( \frac{\partial f_a(v)}{\partial t} \right)_i^{NES} + \frac{1}{v^2} \frac{\partial}{\partial v} \left( \frac{v^3 f_a(v)}{2\tau_C^*(v)} \right) + S_a(v) - L_a(v) = 0 \quad , \tag{5}$$

where $f_a(v)$ is the velocity distribution of particle $a$. The first term in Eq.(5) represents the effect of the Coulomb scattering [19] with background charged particles, i.e. D, T, $\alpha$, p, $^3$He, and electron. The second term accounts for the NES of species $a$ by background ions [14,20]. We consider NES between 1) proton and deuteron, and 2) proton and $^3$He, i.e. $(a,i)$=(p,D), (p,$^3$He) (d,p) and ($^3$He,p). In this paper, the NES cross sections are taken from the work of Perkins and Cullen [21]. In Fig.1 we show the dependence of NES cross section $\sigma_{NES}$ on relative energy $E_r$. The third term of Eq.(5) represents the diffusion in velocity space due to thermal conduction [14,15,20].

Particle source $S(v)$ and loss $L(v)$ terms take different form for every ion species. For deuteron, the source and loss terms are described so that the fueling, transport loss and the loss due to $D^3He$, DD and DT reaction are balancing each other [14,15,20];

$$S_D(v) - L_D(v) = \frac{S_D}{4\pi v^2}\delta(v - v_D^{fueling}) - \varsigma_D^{^3He(d,p)^4He} f_D(v) - \varsigma_D^{D(d,p)T} f_D(v) - \varsigma_D^{D(d,n)^3He} f_D(v) - \varsigma_D^{T(d,n)^4He} f_D(v) - \frac{f_D(v)}{\tau_p^*(v)} \,. \tag{6}$$

Here $v_D^{fueling}$ indicates the speed of the fueled deuteron, which is much smaller than the thermal speed. The fueling rate $S_D$ is determined so that the deuteron density is kept constant. For example, the $^3$He(d,p)$^4$He reaction rate coefficient is written as

$$\langle \sigma v \rangle_{^3He(d,p)^4He} = \frac{4\pi}{n_D n_{^3He}} \int dv_D v_D^2 \times \varsigma_D^{^3He(d,p)^4He}(v_D) f_D(v_D) \quad , \tag{7}$$

with

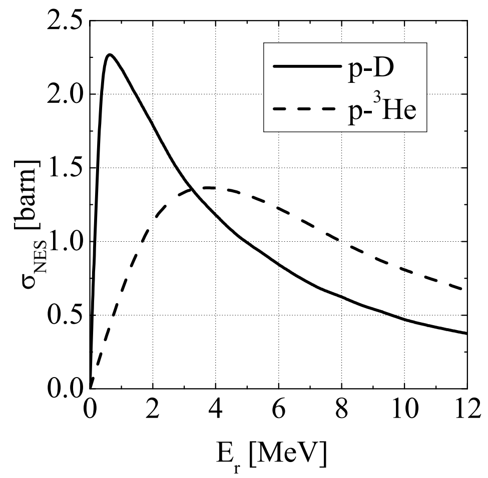

Fig.1 p-D and p-$^3$He NES cross sections as a function of relative energy.

$$\varsigma_D^{^3He(d,p)^4He} = \frac{2\pi}{v_D} \int dv_{^3He} v_{^3He} f_{^3He}(v_{^3He}) \times \left[ \int_{|v_D - v_{^3He}|}^{v_D + v_{^3He}} dv_r v_r^2 \sigma_{D^3He}(v_r) \right] \quad . \tag{8}$$

The fusion cross sections have been taken from the work of Bosch [22].

For $^3$He, the source and loss terms are written so that the fueling, production due to D(d,n)$^3$He reaction, the loss due to transport and $^3$He(d,p)$^4$He reaction are balancing. For alpha-particle, proton and triton, the source and loss terms are described in the same manner so that the production due to fusion reactions and transport loss are balancing [9,10].

## 2.2 Emission spectra of reaction products

When we consider the reaction $A + B \rightarrow a + b$, the energy of produced particle $a$ in the laboratory system is represented in the following expression [23];

$$E_a = \frac{1}{2} m_a V_c^2 + \frac{m_b}{m_a + m_b}(Q + E_r) + V_c \cos\theta_c \sqrt{\frac{2 m_a m_b}{m_a + m_b}(Q + E_r)} \quad . \tag{9}$$

Here $V_c$ is velocity of center of mass (C.M.), $Q$ is the reaction $Q$-value, $E_r$ is relative kinetic energy, and $\theta_c$ is the angle between the center-of-mass velocity and velocity of particle $a$ in C.M. frame. The spectrum of particle $a$ is described as follows;

$$\left( \frac{dN}{dE} \right)_a = \iiint f_A(\boldsymbol{v}_A) f_B(\boldsymbol{v}_B) \frac{d\sigma_{AB}}{d\Omega} \delta(E - E_a) v_r d\boldsymbol{v}_A d\boldsymbol{v}_B d\Omega \quad . \tag{10}$$

Here $v_r$ is relative velocity between particle A and B, $d\sigma/d\Omega$ is differential cross section, and $N$ is generation rate of particle $a$ per unit volume. Throughout the calculation, we assumed $d\sigma/d\Omega$ is isotropic in the C.M. system.

## 3. Results and Discussion

In Fig.2 and Fig.3, the deuteron and $^3$He distribution functions obtained by solving Eq.(5) for several electron temperatures, i.e. $T_e$ = 60keV, 80keV, 100keV are shown. The distribution function in energy space $f_a(E)$ is related to $f_a(v)$ as $f_a(E) = (4\pi v/m_a) f_a(v)$. In this calculation, ion densities $n_D = 2n_{^3He} = 2\times10^{20}\,\mathrm{m}^{-3}$, energy and particle confinement times $\tau_E = (1/2)\tau_P = 3\,\mathrm{sec}$ are assumed. The dotted lines represent Maxwellian distribution at the temperature $T_i$ ($T_i = (T_D n_D + T_{^3He} n_{^3He})/(n_D + n_{^3He})$). Here, deuteron ($^3$He) temperature $T_D$ ($T_{^3He}$) is determined by comparing bulk component in the obtained distribution function with Maxwellian by means of the least-square fitting. It is found that the non-Maxwellian tail is formed due to NES in both deuteron and $^3$He distribution functions. The relative intensity of the knock-on tail becomes large with increasing temperature. This is because the density of proton which causes the knock-on tail formation in fuel-ion distribution functions increases in high-temperature range. The knock-on tail in $^3$He distribution function is small compared with that of deuteron. This is because the p-$^3$He NES cross sections are smaller than p-D ones [2].

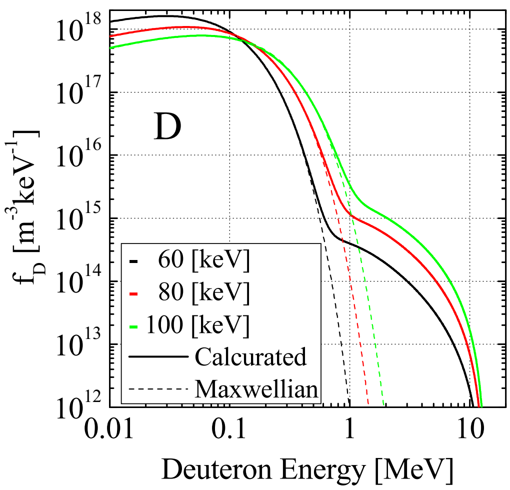


Fig.2 Deuteron distribution functions for 60, 80 and 100keV electron temperatures.

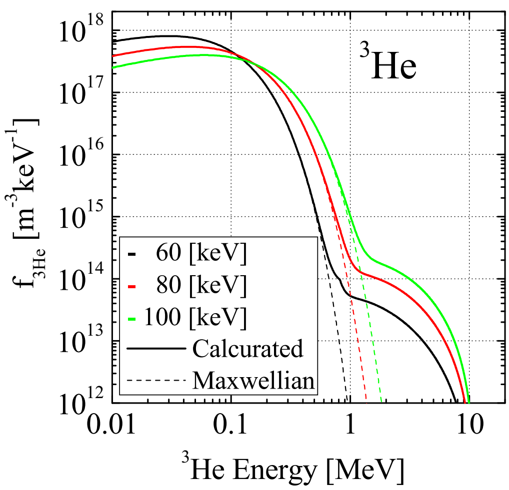


Fig.3 Helimu-3 distribution functions for 60, 80 and 100keV electron temperatures.

In Fig.4(a), we next show the proton emission spectrum as a function of proton energy in the laboratory system for several electron temperatures, i.e. $T_e$ = 60keV, 80keV and 100keV. The dotted lines in Fig.4(a) are Gaussian distributions. Because the relative intensity of knock-on component in fuel-ion distribution function increases with increasing temperature (as was shown in Figs.2 and 3), the broadening in the emission spectrum is also enhanced for high temperature range.

In order to evaluate the degree of the broadening of the emission spectrum quantitatively, we introduce the following parameter;

$$F_{>15MeV} = \frac{\int_{15MeV}^{\infty} E\left(\frac{dN}{dE}\right)dE}{\int_0^{\infty} E\left(\frac{dN}{dE}\right)dE} \times 100\,[\%] \quad . \qquad (11)$$

The $F_{>15MeV}$ expresses the fraction of the power carried by energetic ($>15MeV$) proton to total generation power. In Fig.4(b), the parameter when knock-on tail is considered $F_{>15MeV}$ (solid lines) and the enhancement of the parameter due to the spectrum modification from the Gaussian distribution $F_{>15MeV}/F_{>15MeV}^{Gauss}$ (dotted lines) are shown as a function of electron temperature $T_e$ for 15MeV respectively. Here $F_{>15MeV}^{Gauss}$ represents the fraction when Gaussian distribution is assumed for emission spectrum. It is found that $F_{>15MeV}$ becomes large for high-temperature range. This is because relative intensity of the knock-on tail component in distribution functions increases with increasing temperature. On the other hand it is found that $F_{>15MeV}/F_{>15MeV}^{Gauss}$ decreases with increasing $T_e$. This is because in low-temperature range the half width of the Gaussian distribution decreases and the deviation of the spectrum from the Gaussian distribution function becomes more conspicuous.

In Figs.5(a) and (b) we next shows the proton emission spectrum for several deuteron densities, i.e. $n_D$ = $1\times10^{20}\,\mathrm{m}^{-3}$, $2\times10^{20}\,\mathrm{m}^{-3}$, and $3\times10^{20}\,\mathrm{m}^{-3}$, and $F_{>15MeV}$ and $F_{>15MeV}/F_{>15MeV}^{Gauss}$ as a function of deuteron density $n_D$. We can find that $F_{>15MeV}/F_{>15MeV}^{Gauss}$ increases with decreasing $n_D$. This is because slowing down of knock-on tail component is weakened and relative intensity of knock-on tail component becomes large for

Table.1 $F_{>15MeV}$ and $F_{>15MeV} / F_{>15MeV}^{Gauss}$ for several temperatures, densities and cutoff energies.

| | $F_{>15MeV}$ [%] | $F_{>15MeV}^{Gauss}$ [%] | $F_{>15MeV} / F_{>15MeV}^{Gauss}$ |
|---|---|---|---|
| $T_e$ [keV] | | | |
| 60 | 39.1 | 27.5 | 1.42 |
| 80 | 43.9 | 31.7 | 1.38 |
| 100 | 47.1 | 34.8 | 1.36 |
| $n_D$ [$m^{-3}$] | | | |
| $1\times10^{20}$ | 41.1 | 28.9 | 1.42 |
| $2\times10^{20}$ | 43.9 | 31.7 | 1.38 |
| $3\times10^{20}$ | 44.5 | 32.3 | 1.38 |

low density region. The numerical data of $F_{>15MeV}$ and $F_{>15MeV} / F_{>15MeV}^{Gauss}$ for several temperatures and densities are summarized in Table.1.

## 4. Conclusion

We have evaluated the effect of NES on fusion product spectrums in D$^3$He plasmas. As a result of knock-on tail formation in fuel-ion distribution function due to NES of bulk component by D$^3$He-fusion-produced protons, the emission spectrum of fusion-produced proton

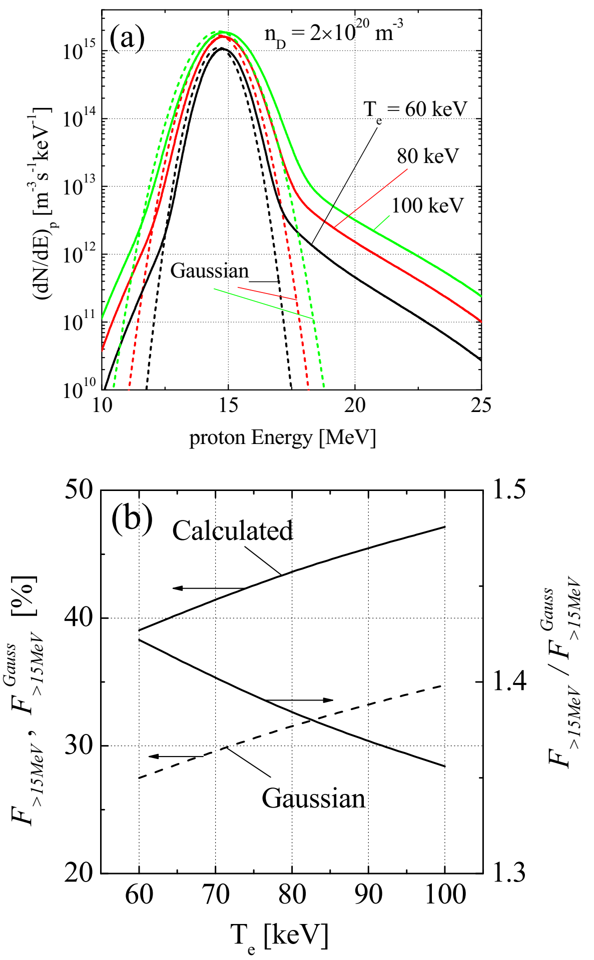


Fig.4 (a) proton emission spectrum, (b) fraction of power carried by energetic (>15$MeV$) protons as a function of electron temperature.

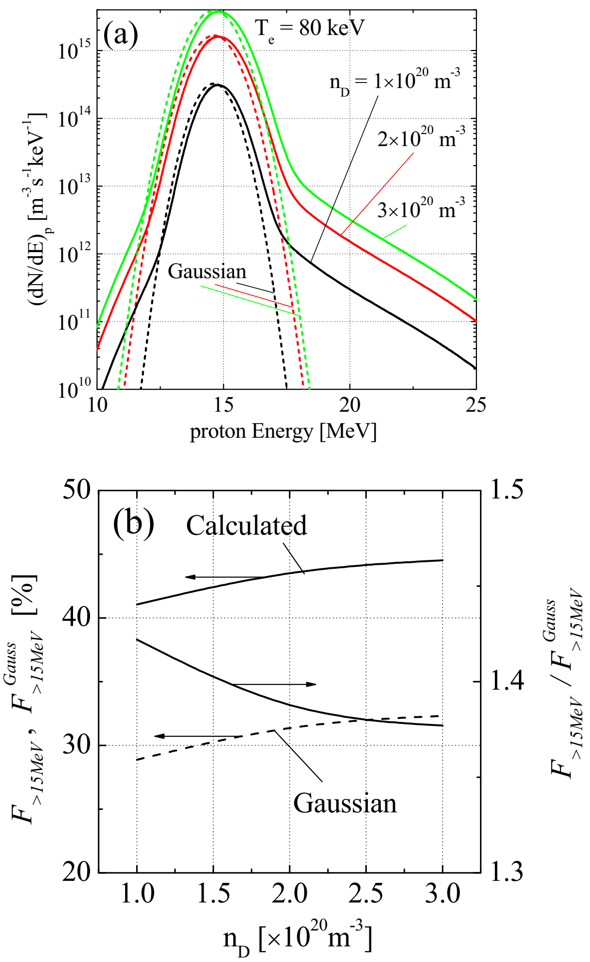


Fig.5 (a) proton emission spectrum, (b) fraction of power carried by energetic (>15$MeV$) protons as a function of deuteron density.

spreads widely toward both high and low energy range, and is significantly modified from the Gaussian distribution. If we do not accurately take account of the modification of the spectrum, we would underestimate the energetic (>15MeV) proton generation significantly. The increment in the fraction of the energetic (>15MeV) proton may influence the plasma heating and sustaining characteristics. In this paper, we have neglected the knock-on tail formation caused by D$^3$He-fusion-produced alpha-particles. Owing to the NES of fuel-ions by energetic alpha-particles, the modification of the spectrum would be slightly enhanced.

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