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Monopole Drift-Wave Vortices in a Viscous Magnetized Plasma

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Monopole nonlinear drift-wave vortices in a viscous magnetized plasma are investigated by applying a time-dependent finite element method of two-dimensional space to Hasegawa-Mima equation with viscosity term. Under a periodic boundary condition in the direction of drift-wave propagation and a natural boundary condition transverse to the propagation, the monopole drift-wave vortices of positive potential tilt in the clockwise direction. The tilting angle depends on the value of initial localized potential.

Keywords: Hasegawa-Mima equation, time-dependent finite element method, monopole drift-wave vortices, wake release, boundary condition.

1. Introduction

Hasegawa-Mima equation [1] is an important equation in case of discussing plasma transport in a magnetized plasma. This equation includes the drift-wave propagation term and the nonlinear term leading to vortex formation. Hasegawa-Mima equation with viscosity term is written as follows.

$$\frac{\partial}{\partial T}(\Phi - \nabla_\perp^2 \Phi) + V_* \frac{\partial \Phi}{\partial Y} = \left(\frac{\partial \Phi}{\partial X}\frac{\partial}{\partial Y} - \frac{\partial \Phi}{\partial Y}\frac{\partial}{\partial X}\right)\nabla_\perp^2 \Phi - N\nabla_\perp^2(\nabla_\perp^2 \Phi)$$

Here, normalization takes place as follows.

$$\omega_c t \to T \quad , \quad \frac{x}{\rho_s} \to X \quad , \quad \frac{e\phi}{KT_e} \to \Phi$$

$$\frac{v_*}{C_s} = -\frac{\partial}{\partial\left(\frac{x}{\rho_s}\right)} \ln n_0(x) \to V_*$$

Time t is normalized by a reciprocal number of angular cyclotron frequency ω_c. Spaces x and y are normalized by ion-acoustic Larmor radius ρ_s. v_* is drift-wave propagation velocity in the positive y direction and is normalized by ion acoustic speed C_s. Electric potential ϕ is normalized by electron temperature T_e. N is normalized viscosity coefficient.

2. Application of Time-dependent Finite Element Method

Time-dependent finite element method is applicable to Hasegawa-Mima equation in linear case without any problem. Figure 1 is an example of linear monopole drift-wave propagation without viscosity. The linear monopole drift-wave releases a wake in the backward direction because of dispersion effect of linear drift wave. In this article, normalized drift-wave speed is kept at a constant value of V_*=0.1, and we use a periodic boundary condition in the direction of drift-wave propagation and a natural boundary con-

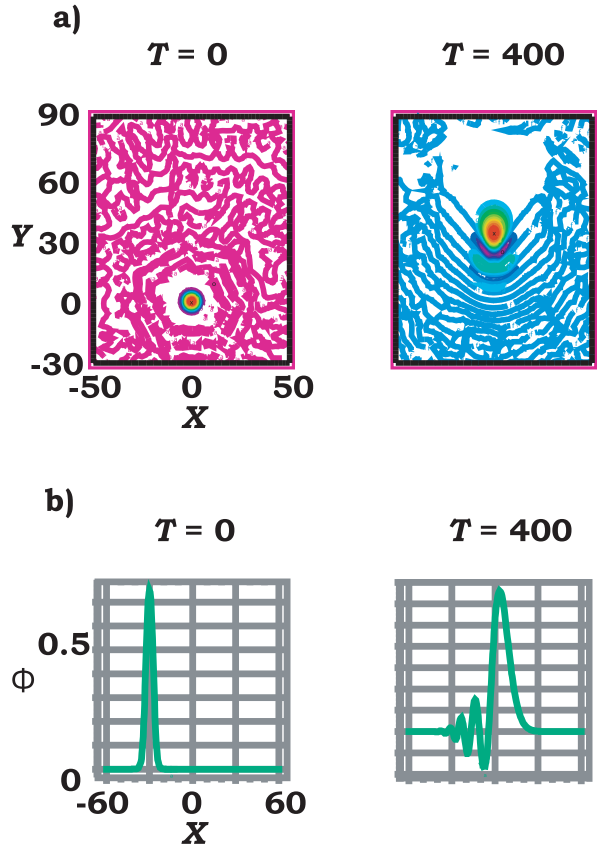

Fig. 1 Linear monopole drift-wave propagation. a) Two-dimensional structure of Potential Φ, and b) density profile along $Y = 0$, at times $T = 0$ and $T = 400$. Linear monopole drift-wave emits a wake in the backward direction. V_*=0.1. N=0.

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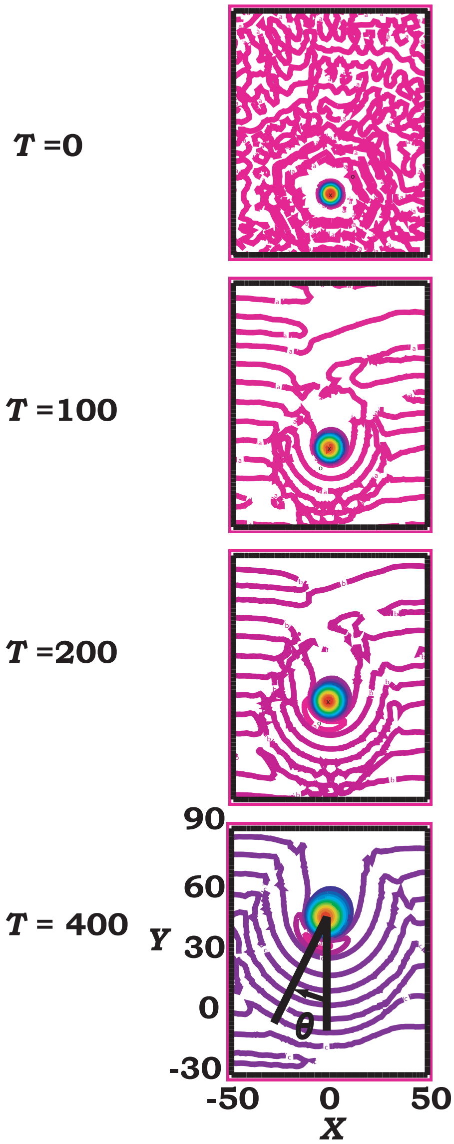

Fig. 2 Propagation of a nonlinear drift-wave vortex in case of the initial monopole positive potential Φ_0=20. The tilting angle θ=22° at T = 400. V_*=0.1. N=0.1. θ depends on the value of initial localized potential Φ_0.

dition transverse to the propagation.

In case of taking a nonlinear drift-wave monopole into account, the time-dependent finite element method gives rise to unstable calculation, where N=0. The viscosity term with N=0.1, however, stabilizes the calculation as shown in Fig. 2. Nonlinear monopole drift-wave vortices have been investigated in case with no viscosity [2, 3, 4]. W. Horton showed only calculated figures of the tilting angle θ=90° [2]. J.S. Hesthaven et al. [3] and J.J. Rasmussen et al. [4] did not discuss the dependence of the tilting angle θ on the amplitude of monopole vortices. Here, we show their dynamics in case with finite viscosity and discuss the dependence of θ on the initial monopole potential Φ_0 of monopole vortices.

3. Dynamics of Nonlinear Monopole Drift-Wave Vortices

Figure 2 shows the propagation of a nonlinear drift-wave vortex in case of the initial monopole positive potential Φ=20. The monopole drift-wave vortex of positive potential tilts in the clockwise direction. The tilting angle θ=22° at T = 400 in this case. θ grows initially and decreases very slowly. The release of a wake from the monopole is suppressed compared with the linear case because of the vortex rotation.

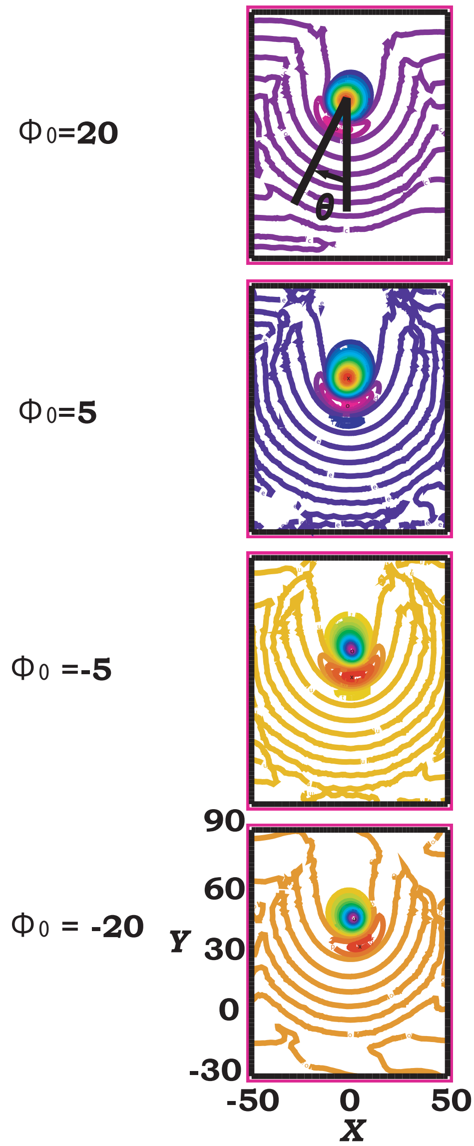

Fig. 3 The tilting angle θ dependence on the value of initial localized potential Φ_0 at a time T = 400. V_*=0.1. N=0.1.

The tilting angle θ depends on the value of initial localized potential Φ_0 at a time T = 400 as shown in Fig. 3. The positive and negative monopole vortices tilt in the clockwise and counterclockwise directions respectively. The graph of tilting angle θ versus the value of initial localized potential Φ_0 at a time $T = 400$ is shown in Fig. 4.

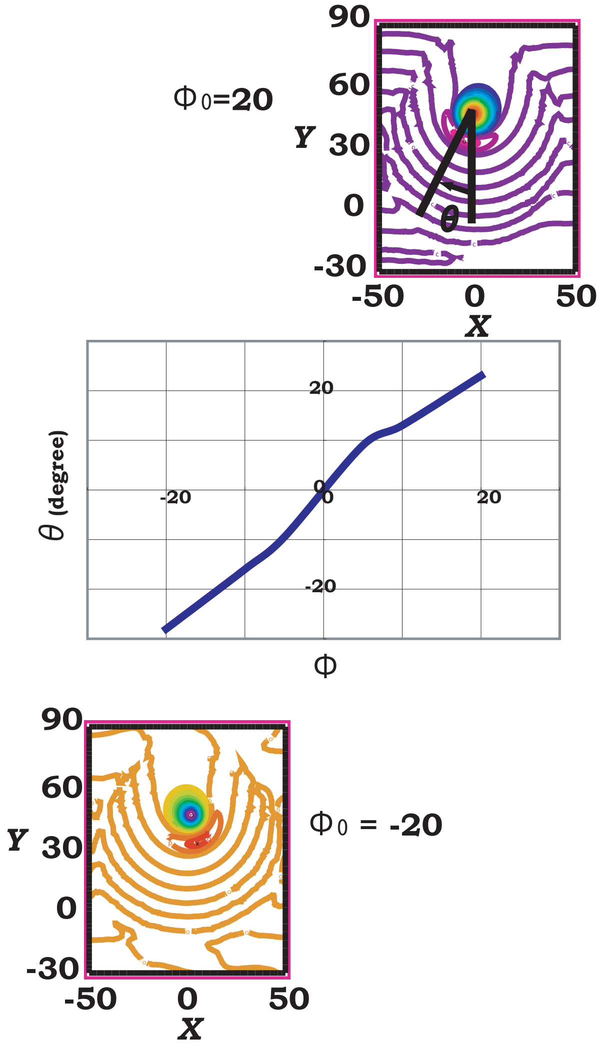

Fig. 4 The graph of tilting angle θ dependence on the value of initial localized potential Φ_0 at a time $T = 400$. V_*=0.1. N=0.1.

4. Conclusions

In summary, a time-dependent finite element method of two-dimensional space is successfully applied to Hasegawa-Mima equation with viscosity term. The viscosity term is necessary in order to suppress calculation instability. The positive and negative nonlinear monopole vortices in a viscous magnetized plasma tilt in the clockwise and counterclockwise directions respectively.

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